論文の内容の要旨

論文題目
Beliaev Theory of Spinor Bose-Einstein Condensates and Its Applications

（スピノルボース・アインシュタイン凝縮体におけるベリアエフ理論とその応用）

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In quantum fluids, the phase coherence between the constituent particles can amplify the effects of a microscopic interaction so that they are observable in macroscopic properties. The discovery of superfluid ^{4}He [1], the first such system in Nature, was followed by the exploration of a wide range of systems from the various superfluids and superconductors in condensed-matter physics to neutron stars and color superconductors in cosmology and high-energy physics. Among them, materials such as superfluid ^{3}He and *p*-, *d*-wave superconductors display a number of remarkable features due to the interplay between their internal degrees of freedom and the coherent motion of the center-of-mass degree of freedom [2].

However, the parameters of the above systems, e.g., the magnitude of the interparticle interaction, are predetermined by Nature and cannot be manipulated at will. Furthermore, in such complex systems the primary physical principles governing the system's properties are sometimes masked by side effects, making it difficult for them to be unveiled. On the other hand, since the first experimental realizations of Bose-Einstein condensates (BECs) [3], ultracold atoms have attracted attention of scientists not only in atomic physics but also in many other fields of physics. Compared to other systems, the interaction between ultracold atoms can be described to a good approximation in terms of a small number of well-defined parameters. This is because the atomic gases are so dilute that the average distance between atoms is much larger than the effective range of interaction, making the details of the interaction irrelevant. Since the properties of ultracold atoms can be readily manipulated and measured in experiments, they are considered to be an ideal table-top quantum simulator to study universal properties of other physical systems.

For alkali-metallic atoms, due to the hyperfine interaction the eigenvalues of F and F_z are the good quantum numbers with $F=I+J$ being the total spin of the atom. Here I and J are the nuclear and electronic spins, respectively. For these atoms, the hyperfine states $|F, m_F\rangle$ ($m_F = F,\dots,-F$) play the role of internal degrees of freedom in a way similar to the spin degrees of freedom of a single electron. Similar to other quantum fluids with internal degrees of freedom, spinor gases, in particular spinor BECs, have exhibited a number of fascinating features, e.g., the coherent spin-mixing dynamics [4], formation of spin domains and topological defects [5], etc. due to the interplay between the phase coherence and the magnetic order. Remarkably, these phenomena can be well described at least qualitatively based on the mean-field theory.

However, I have discovered that there are particular features of spinor BECs that the mean-field theory cannot give an accurate description even qualitatively. This is because the essence of these features is based on the effects of quantum fluctuations that are ignored in the mean-field theory. For example, let me consider a condensate of atoms in the $F = 2$ hyperfine spin manifold. Depending on the relative strengths of spin-dependent interactions, the ground state can be one of a number of phases as shown in Fig. 1. The distinct symmetries of these phases in spin space implies that the phase transitions are first order, suggesting that they should be accompanied by metastable states. However, the Bogoliubov excitation spectrum indicates that there is no metastable state associated with these phase transitions. I point out that this inconsistency in fact originates from the fact that the Bogoliubov spectrum [6] is obtained by considering a small amplitude expansion of the order parameter around the mean field with the linearized Gross-Pitaevskii energy functional which consists of only terms up to the fourth order in the order parameter. This energy functional is equivalent to the Landau's $\phi^2+\phi^4$ model of second-order phase transitions. In contrast, the description of first-order phase transitions requires higher-order terms beyond ϕ^4, and in gaseous BECs these higher-order terms only arise from quantum fluctuations. In other words, in the system under consideration the metastable states, if they exist, are induced by fluctuations.

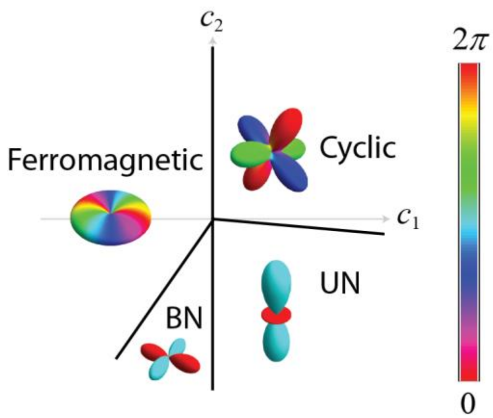

Figure 1: Ground-state phase diagram of $F = 2$ Bose-Einstein condensates. Depending on the relative strengths of the spin-dependent interactions c_1 and c_2, the ground state can be one of the four phases: ferromagnetic, cyclic, uniaxial-nematic (UN), and biaxial-nematic (BN) phases. The insets show the surface plot of $|\psi(\theta,\phi)|^2 \equiv \left|\sum_{m=-2}^{2} \xi_m Y_2^m(\theta,\phi)\right|^2$, where $\boldsymbol{\xi} \equiv (\xi_2, \cdots, \xi_{-2})^{\mathrm{T}}$ is the spinor order parameter and Y_2^m's denote the spherical harmonic functions of rank 2. Here the hue indicates the phase of $\psi(\theta,\phi)$ according to the color gauge on the right.

Consequently, in this thesis I have developed the spinor version of the Beliaev theory [7], which was originally proposed for a scalar BEC [8]. By taking into account the contributions from the second-order Feynman diagrams, I have derived analytically the excitation spectrum that contains a correction to the Bogoliubov spectrum due to quantum fluctuations. From the obtained Beliaev spectrum, I have shown that the metastable states indeed appear around the phase boundaries [9]. This result has shed light on the pivotal role of quantum fluctuations in the study of the phase diagram and the stability of a spinor condensate. The presence of a metastable condensate also suggests an interesting possibility of a decay of the metastable state into the ground state via macroscopic quantum tunneling (MQT) in which all atoms tunnel simultaneously from one phase to the other. I have evaluated the time scale of the MQT for the cyclic-uniaxial nematic phase transition as it is relevant to experiments of the spin-2 ^{87}Rb BEC.

Besides the first-order phase transitions with fluctuation-induced metastabilities, I have found that there is in spinor BECs another class of first-order phase transitions that have no metastable state around their phase boundaries [9]. I show that in this case the absence of metastability holds to all orders of approximation. Such phase transitions are characterized by the fact that the Hamiltonian possesses a special symmetry at the phase boundary so that the energy landscape becomes flat. The ground state would then abruptly change to an unstable state without undergoing any transient regime of metastability as the system crosses the phase boundary. This is in contrast to the case of conventional first-order phase transitions where the energy landscape features a double-well structure at the transition point, and therefore, supports the coexistence of two phases. On the other hand, the flat energy landscape results in the criticality in the dynamics of the condensate through these phase transitions. I have studied the critical features in the context of both instantaneous and slow quenches of a system's parameter [10]. Consequently, despite being first-order phase transitions, their dynamics is similar to that of second-order phase transitions. Some of the quantum phase transitions in both spin-1 and spin-2 BECs are within reach of current experiments, bringing hope that our theoretical predictions can be verified experimentally.

Another remarkable effect of quantum fluctuations that I have found is the emergence of a nonzero energy gap of the so-called quasi-Nambu-Goldstone (quasi-NG) modes, which are the extra gapless excitations at the mean-field level that are not generated by spontaneous symmetry breaking. This is similar to the quantum symmetry breaking or quantum anomaly in high-energy physics in which the symmetry of the vacuum's manifold is broken only if the one-loop quantum correction to the tree approximation is taken into account. After first being introduced in the context of gauge theories [11], quasi-NG modes have become an important element in the theories of technicolor and supersymmetry [12]. They are also predicted to appear in the weak coupling limit of the A phases of superfluid ^{3}He and spin-1 color superconductors [13]. Despite their prevalence in various fields of physics, no experimental evidence of the quasi-NG modes has hitherto been observed. Recently, it was found that the nematic phase of spin-2 BECs can be a host of quasi-NG modes [14], leading to a renewed interest in this special kind of excitations. At the mean-field level, all nematic phases are degenerate and quasi-NG modes are gapless. However, the zero-point fluctuations lift this degeneracy in a way similar to the vacuum alignment in quantum field theory [15]. Therefore, it is predicted that with the quantum corrections the quasi-NG modes would acquire a nonzero energy gap whose magnitude is of the same order as the zero-point energy. I have proved explicitly the above conjecture of quasi-NG modes becoming gapful by deriving the analytic expression for the energy gap in terms of the fundamental interaction parameters [16]. Regarding the magnitude of the emergent energy gap, I find that it depends on the relative strengths of the spin-dependent interactions.

From the obtained magnitude of the energy gap, I have been able to evaluate the critical temperature above which a topological defect such as a vortex of spin nematicity would decay by emitting these thermally excited quasi-NG modes. Conversely, below this temperature the vortex would be stabilized by suppressing the emission of these excitations. The magnitude of the energy gap and the critical temperature can be increased to a regime accessible with typical ultracold atomic experiments by, for example, adjusting the relative strengths of the scattering lengths. In addition to the emergence of a finite energy gap, I have found that the propagation velocity of the quasi-NG modes is suppressed due to the particle-number density fluctuations. This is in contrast to the enhancement of the sound velocity, and a qualitative account of the difference has been given in terms of the particle-number density correlation.

Finally, I have calculated the damping rates of various types of quasiparticles in spin-2 BECs including phonons, magnons, and quasi-NG modes [17]. At the level of the Bogoliubov theory, all of these quasiparticles have infinitely long lifetimes. However, with higher-order approximations, their lifetimes become finite since they can decay through numerous channels of collision with the condensate atoms (see Fig. 2). By using Fermi's golden rule to calculate the transition probabilities, I find that the damping of the quasi-NG modes is suppressed due to the energy conservation. In contrast, the damping rates of phonons and magnons are found to be finite with their own scaling laws with respect to the momentum. It is worth noting that the damping rates of these quasiparticles can be reproduced by using the developed spinor Beliaev theory. The obtained analytic expressions for the damping rates as functions of the fundamental interactions also suggest an alternative approach to measuring precisely the magnitudes of the spin-dependent interactions. I have proposed a scheme to measure the lifetime of magnons in the spin-2 ^{87}Rb condensate by temporarily switching on an external magnetic field.

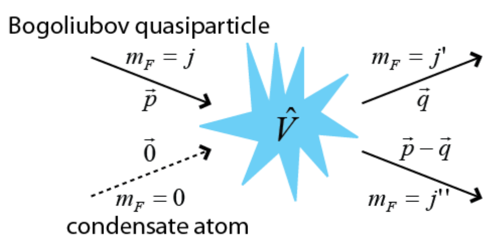

Figure 2: A general decay channel of a quasiparticle with momentum $\boldsymbol{p}$ and spin state $m_F=j$. Its interaction with a condensate atom (the dashed line) generates two quasiparticles with momenta $\boldsymbol{q}$ and $\boldsymbol{p}$-$\boldsymbol{q}$ and spin states $m_F=j'$ and $m_F=j''$, respectively. Here $\hat{V}$ denotes the interaction Hamiltonian.

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