学位論文

# Proton Decay in High-scale Supersymmetry

(高いスケールの超対称性と陽子崩壊)

平成 25 年 12 月　博士（理学）申請

東京大学大学院理学系研究科
物理学専攻

永田　夏海

Abstract

Recent discovery of a Higgs boson with a mass of $\sim 126$ GeV, as well as no evidence for new physics beyond the Standard Model so far, may imply that the supersymmetry (SUSY) breaking scale is somewhat higher than the electroweak scale. Indeed, although SUSY models with high-scale SUSY breaking challenge their role in providing a solution to the hierarchy problem, the models turn out to be quite fascinating from both theoretical and phenomenological points of view. Firstly, since the flavor changing neutral current processes and/or the electric dipole moments induced by SUSY particles are suppressed by their masses, the SUSY flavor and CP problems are relaxed when their masses are considerably heavy. Secondly, the heavy sfermions yield sufficient radiative corrections to lift the Higgs mass up to 126 GeV. As for the cosmology, the gravitino problem is avoided because of the high-scale SUSY breaking. In addition, the gauge coupling unification is achieved with great accuracy if the fermionic SUSY partners lie around TeV scale. Since chiral symmetries can protect the fermion masses, it is possible for them to be much lighter than the scalar particles. These fermionic SUSY particles contain candidates for dark matter in the Universe. A mass spectrum which satisfies these phenomenological requirements is naturally obtained on the assumption of a hidden sector where SUSY is broken dynamically. Thus, with the recent LHC results considered, the high-scale SUSY scenario is even promising.

In this thesis, we propose that the minimal SUSY Standard Model with high-scale SUSY breaking has additional attractive points, which are brought to light when it is considered in the context of the grand unified theory (GUT). We first evaluate masses of superheavy gauge and Higgs multiplets showing up around the GUT scale by using the renormalization group method, and reveal that all of these particles, especially the color-triplet Higgs multiplets, can lie around a scale of $\sim 10^{16}$ GeV, contrary to previous results based on the low-scale SUSY breaking. This observation indicates two new aspects on the GUT scale particles; the color-triplet Higgs mass is larger by an order of magnitude and the GUT scale is slightly lower than those in the case of low-scale SUSY. Taking into account these features, we next study proton decay in the high-scale SUSY scenario. We have found that thanks to heavy color-triplet Higgs multiplets and high-scale SUSY breaking, the rate of the disastrous dimension-five proton decay induced by the color-triplet Higgs exchange is significantly reduced, and the current experimental limits on the $p \to K^+\bar{\nu}$ channel can be evaded. Further, a lower GUT scale leads to an enhancement of the rate of the $p \to \pi^0 e^+$ mode. Other proton decay modes may also be considerably accelerated if there exists large flavor violation in the sfermion mass matrices, which can be allowed in the case of high-scale SUSY breaking.

For a variety of decay modes, we have found that the resultant proton decay rates may lie in the region which can be reached in the future proton decay experiments. Therefore, although the high-scale SUSY scenario is hard to be probed in the collider experiments, the proton decay searches may give us a chance to verify the scenario as well as the existence of supersymmetry and the grand unification.

# Contents

# Chapter 1

# Introduction

## 1.1 Introduction

The discovery of the Higgs boson [1, 2], which was announced on July 4th, 2012 by the ATLAS and CMS Collaborations [3], has opened the way for physics beyond the Standard Model (SM). It is certainly a striking hint for understanding the high-energy physics. By using the data collected at the early stage of the LHC running (the center of mass energy is 7 and 8 TeV for the 2011 and 2012 runs, respectively), the Collaborations have presented the results showing that the Higgs boson has properties totally consistent with those of the SM Higgs boson [4, 5, 6, 7]; all of the signal strengths they have measured so far can be explained with the SM Higgs boson having a mass of $\sim 126$ GeV [8]. It is a great triumph of the Standard electroweak model [9, 10, 11, 12], which is based on the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge theory and the SM Higgs mechanism. In fact, it is more than just a confirmation of the model. It also implies that elementary scalar particles may play an important role in realizing our complicated world with apparent broken symmetries. In this sense, the discovery of the Higgs boson is extremely important in probing more fundamental theories than the SM.

Now that we have strong evidences for the unification mechanism of gauge interactions by means of elementary scalar particles, why don't we go further in this direction? It seems possible that not only the electromagnetic and weak interactions, but also the strong interaction is unified together on the basis of the Higgs mechanism. Actually, such efforts have been made since the earliest work was presented by H. Georgi and S. L. Glashow in 1974 [13]. In their model, the SM gauge group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ is embedded into a single gauge group; SU(5). Quarks and leptons in the SM are incorporated into irreducible representations of SU(5). The unified gauge group is spontaneously broken into the SM gauge group by the vacuum expectation value (VEV) of the SU(5) adjoint Higgs boson, and the gauge fields corresponding to the broken symmetries acquire masses of the symmetry breaking scale. The gauge coupling constants of the SM gauge group are supposed to be unified at the scale and become a single gauge coupling constant of the SU(5) gauge group. After their original work, models based on various simple Lie groups such as SO(10) [14, 15], $\mathrm{E}_6$ [16], etc., have been proposed. Such models are often called *grand unified theories (GUTs)*.

As mentioned above, the gauge coupling constants should be unified at a certain scale in the GUTs. Indeed, by using the renormalization group equations (RGEs), one finds that the couplings come close to each other at at very high energy scales [17], which supports the gauge coupling unification. Further, the GUTs can solve several problems that are not explained within the SM. Among them, the explanation for the electric charge quantization is remarkable. Since U(1) charges are essentially continuous parameters, there is no reason for the quarks and leptons in the SM to have charges being exactly integer multiple of 1/3, though it is experimentally confirmed with great precision; *e.g.*, a limit on the sum of the proton and electron charges is given as $|Q_p+Q_e| < 1\times10^{-21}\, e$ [18]. In the GUTs, on the other hand, it is naturally accounted for because the U(1) generator is actually a generator of a non-Abelian group like SU(5). Moreover, the miraculous anomaly cancellation in the SM suggests that there exist some relations among quarks and leptons, and such relations are naturally realized in the GUTs. Hence, not only conceptual considerations but also phenomenological analyses extremely support the GUTs.

In spite of their successes, the GUTs also give us a new problem. The theories introduce some large scale for the grand unification, so called the GUT scale. Then, one may ask a question: why are the masses of the SM particles so much lower than the GUT or the Planck scale? Since in the SM all the particles have masses proportional to the VEV of the Higgs boson, the question can be restated in the following manner: why does the Higgs field have the VEV $v \simeq 246$ GeV, which is extremely lower than the fundamental scales? This VEV is proportional to the mass of the Higgs boson (or Higgs bosons) responsible for the electroweak symmetry breaking. Unlike the SM fermions and gauge bosons, the scalar fields are not protected from acquiring huge bare masses by any symmetry in the SM, and thus within the SM it seems hopeless to answer the above question. This issue is called the *hierarchy problem* [19, 20, 21, 22, 23] or the *naturalness problem* [24], and has motivated us to seek a theory which can solve the problem.

As discussed in Chapter 2, the supersymmetric (SUSY) extensions of the SM are considered to be a solution to the problems. Supersymmetry is a symmetry with respect to the exchange of bosonic and fermionic particles. Particles which form an irreducible representation of SUSY are referred to as superpartners of each other. It turns out that superpartners have exactly the same quantum numbers, in particular, have an equal mass. So, if the Higgs boson has a chiral fermion as its superpartner, the chiral symmetry guarantees that the mass of the Higgs boson vanishes as long as SUSY is respected. If the nature is actually described by a SUSY SM, SUSY, of course, must be spontaneously broken at an energy-scale higher than the electroweak scale. Since the Higgs field can acquire its bare mass term only after the SUSY is broken, it is protected from acquiring a VEV comparable to the fundamental scale. Therefore, once you explain why the SUSY breaking scale is much lower than the fundamental scale, you can also understand the hierarchy between the electroweak and the fundamental scales. In fact, SUSY accommodates a mechanism in which such a scale difference dynamically shows up due to non-perturbative effects. This kind of mechanism is called the *dynamical SUSY breaking* [25, 26, 27, 28, 29, 30, 31]. Consequently, SUSY SMs have sufficient ingredients to solve the hierarchy problem.

As the SM is extended with SUSY, the GUTs are also reformulated to be supersymmetric. They are called the supersymmetric grand unified theories (SUSY GUTs) [32, 33]. A remarkable feature of the SUSY GUTS came to light soon after their original versions were presented; if SUSY appears around the electroweak scale, the gauge coupling unification is realized with great accuracy [34, 35, 36, 37, 38]. This observation extremely motivates us to study the SUSY GUTs and, of course, the SUSY SMs.

As is often the case with SUSY models, the SUSY GUTs are usually discussed within the context of the weak-scale supersymmetry. Recently, experiments at the LHC provide limits on the weak-scale SUSY models. The ATLAS and CMS Collaborations have been searching for the

SUSY particles and imposed severe constraints on their masses, especially those of squarks and gluino [39, 40]. The mass limits have began to exceed 1 TeV and, thus, there seems to be some discrepancy between the electroweak and SUSY breaking scales. Moreover, the observed mass of the Higgs boson around 126 GeV might also indicate the SUSY scale is considerably higher than the electroweak scale; in the minimal supersymmetric standard model (MSSM), the mass of the lightest Higgs boson is below the $Z$-boson mass at tree level, so sufficient mass difference between stops and top quark is required in order to raise the Higgs boson mass through the radiative corrections [41, 42, 43, 44, 45]. Unless the Higgs sector is modified nor the left- and right-handed stops adequately mix with each other, such a large quantum effect is only provided with heavy stops having masses of much higher than the electroweak scale.

Actually, the SUSY models with a high-SUSY breaking scale have a lot of attractive features [46, 47, 48, 49, 50, 51]. Firstly, since the flavor changing neutral current (FCNC) processes and/or the electric dipole moments induced by SUSY particles are suppressed by their masses, the SUSY flavor and CP problems [52] are relaxed when the masses are considerably heavy. Secondly, as mentioned above, the heavy sfermions yield sufficient radiative corrections to lift the Higgs mass up to 126 GeV [53, 54, 55]. Thirdly, the gauge coupling unification can be realized with great accuracy since the sfermions form complete SU(5) multiplets. As for the cosmology, the gravitino problem [56, 57, 58, 59, 60, 61] is avoided because of the high-scale SUSY breaking, and the thermal leptogenesis scenario well works with high reheating temperature [62]. Further, this high-scale SUSY scenario naturally accommodates dark matter (DM) candidates [63, 64], which might be detected in future dark matter experiments directly and indirectly. Thus, with the recent LHC results considered, the high-scale SUSY scenario is even promising from a phenomenological point of view.

This high-scale SUSY scenario does not give any satisfactory solution to the hierarchy problem. It requires at least the $10^{-4}$ level of fine-tunings among the parameters in the Higgs sector. Instead, this scenario relaxes a lot of problems in the low-scale SUSY without spoiling the virtue of SUSY SMs, and has quite simple structure. After all, the hierarchy problem is just a problem of naturalness, not a problem of internal consistency of the theory. It is not clear that the naturalness criterion excludes the $10^{-4}$ level of fine-tunings. Eventually, only the experiments can settle the problem. Hence, it is important to study the theoretical prediction of the high-scale SUSY scenario and test it in a variety of experiments. If the scenario is confirmed by experiments, it certainly would shed light on our understanding of the electroweak scale as well as the naturalness criterion.

In this thesis, we discuss the high-scale SUSY scenario in the context of the GUTs. We first evaluate masses of superheavy gauge and Higgs multiplets showing up around the GUT scale by using the renormalization group method, and reveal that all of these particles, especially the color-triplet Higgs multiplets, can lie around a scale of $\sim 10^{16}$ GeV, contrary to previous results based on the low-scale SUSY breaking. This observation indicates that the threshold corrections to the gauge coupling constants at the GUT scale are smaller than those in the case of the low-energy SUSY. In this sense, the gauge coupling unification is not only preserved but can be improved in the high-scale SUSY scenario. By taking the mass spectrum into account, we then consider proton decay, which is generally predicted in the GUTs. It is induced by the exchanges of the color-triplet Higgs multiplets and the $X$-bosons, and their effects are described in terms of the dimension-five and -six effective operators, respectively. In the minimal SUSY SU(5) GUT, the former process yields the dominant decay modes, such as $p \to K^+\bar{\nu}$ [65, 66]. The lifetime of the channel is estimated as $\tau(p \to K^+\bar{\nu}) \lesssim 10^{30}$ yrs [67, 68] in the low-scale

SUSY scenario. On the other hand, the Super-Kamiokande experiment gives stringent limits on the channels: $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs [69, 70]. This contradiction makes it widely believed that the minimal SUSY SU(5) GUT has been already excluded and, therefore, needs some extensions in order to suppress the dimension-five proton decay. However, since the effects of the dimension-five operators are suppressed by the sfermion masses, it turns out that the minimal SUSY SU(5) GUT can evade the constraints from the proton decay experiments in the case of high-scale SUSY. Interestingly enough, these results again indicate that the high-scale SUSY scenario is rather supported than the traditional low-energy SUSY models.

As we will see below, the resultant proton lifetime lies in the regions which may be reached in the future proton decay experiments. Therefore, although the high-scale SUSY scenario is hard to be probed in the collider experiments, the proton decay searches may give us a chance to verify the scenario as well as the existence of supersymmetry and the grand unification.

This thesis is organized as follows. In the following sections in this chapter, we briefly review the SM and the MSSM. In Chap. 2, we discuss both the theoretical and phenomenological aspects of the high-scale SUSY scenario. In Chap. 3, we review the minimal SUSY SU(5) GUT and investigate it in the case of high-scale SUSY. Then, we study proton decay in the subsequent chapter. Finally, Chap. 5 is devoted to conclusions and discussion.

## 1.2 Standard Model

First, we briefly review the Standard Model (SM) to clarify our notation and conventions to be used in the rest of the thesis. The SM is a gauge theory based on the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge group, which describes the strong and electroweak interactions.

The SM contains three families of spin-1/2 quarks and leptons, a spin-zero Higgs boson, and the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge bosons. The $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ electroweak symmetry is not manifest at low-energies; it is spontaneously broken by the VEV of the Higgs field $H$, which is a $\mathrm{SU}(2)_L$ doublet. After the electroweak symmetry breaking, the $W^\pm$ and $Z^0$ gauge bosons acquire masses, while the photon field $A$ remains massless. All of the fields in the SM and their quantum numbers are displayed in Table 1.1. In the table, the index $i$ denotes a generation index $i = 1, 2, 3$, and all of the fermion fields are defined in terms of the two-component left-handed spinors; thus, $u^\dagger_{Ri}$, $d^\dagger_{Ri}$, and $e^\dagger_{Ri}$ are the conjugates of the right-handed quarks and leptons. The notation and conventions for the two-component spinors are summarized in Appendix A.1.

The Higgs sector in the SM is described by the following Lagrangian density:

$$\mathcal{L}_{\text{Higgs}} = |D_\mu H|^2 - V(H) \ , \tag{1.1}$$

where $H$ is the Higgs doublet

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \ , \tag{1.2}$$

and $D_\mu$ is the covariant derivative. The Higgs potential $V(H)$ is given by

$$V(H) = -m_H^2 (H^\dagger H) + \frac{\lambda_H}{2} (H^\dagger H)^2 \ , \tag{1.3}$$

with $m_H^2$ assumed to be positive. Using the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ symmetry, we can rotate a VEV of

Table 1.1: Fields in the SM and their quantum numbers.

| | Field | $\mathrm{SU}(3)_C$ | $\mathrm{SU}(2)_L$ | $\mathrm{U}(1)_Y$ |
|---|---|---|---|---|
| Quarks | $Q_{L_i} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$ | $\mathbf{3}$ | $\mathbf{2}$ | $1/6$ |
| | $u^\dagger_{Ri}$ | $\overline{\mathbf{3}}$ | $\mathbf{2}$ | $-2/3$ |
| | $d^\dagger_{Ri}$ | $\overline{\mathbf{3}}$ | $\mathbf{2}$ | $1/3$ |
| Leptons | $L_{L_i} = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}$ | $\mathbf{1}$ | $\mathbf{2}$ | $-1/2$ |
| | $e^\dagger_{Ri}$ | $\mathbf{1}$ | $\mathbf{1}$ | $+1$ |
| Higgs | $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ | $\mathbf{1}$ | $\mathbf{2}$ | $+1/2$ |
| Gauge fields | $G^A_\mu$ | $\mathbf{8}$ | $\mathbf{1}$ | $0$ |
| | $W^a_\mu$ | $\mathbf{1}$ | $\mathbf{3}$ | $0$ |
| | $B_\mu$ | $\mathbf{1}$ | $\mathbf{1}$ | $0$ |

the Higgs field into the following form without loss of generality:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \ , \tag{1.4}$$

where $v = \sqrt{2m_H^2/\lambda_H} \simeq 246$ GeV.

In the unitary gauge, the Higgs field has the following form

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \ . \tag{1.5}$$

The field $h(x)$ describes a massive neutral scalar particle, which is called the Higgs boson. It was recently discovered at the LHC [1, 2], and its mass turns out to be $\sim 126$ GeV [8]. In this gauge, the massless Nambu-Goldstone (NG) bosons are eliminated and the $W^\pm$ and $Z$ bosons have explicit mass terms.

The mass terms for the quarks and leptons are prohibited by the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge symmetries. Therefore, the quarks and leptons are massless before the electroweak symmetry breaking. The masses are generated through the Yukawa couplings to the Higgs doublet:

$$\mathcal{L}_{\text{Yukawa}} = -y_u^{ij} \epsilon^{\alpha\beta} (Q'_{Li} u'^\dagger_{Rj})_\alpha H_\beta - y_d^{ij} (H^\dagger)^\alpha (Q'_{Li} d'^\dagger_{Rj})_\alpha - y_e^{ij} (H^\dagger)^\alpha (L'_{Li} e'^\dagger_{Rj})_\alpha + \text{h.c.} \ , \tag{1.6}$$

where $\alpha, \beta$ denote the $\mathrm{SU}(2)_L$ indices and $\epsilon^{\alpha\beta}$ is the antisymmetric tensor with $\epsilon^{12} = -\epsilon^{21} = 1$. Color indices and spinor indices are suppressed in the above equation. Further, to show explicitly that the fermions are in the interaction eigenstate basis, we put primes on them. After the Higgs field acquires the VEV, the Yukawa interactions give rise to the $3 \times 3$ mass matrices for quark and leptons

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} y_u v, \qquad \mathcal{M}_d = \frac{1}{\sqrt{2}} y_d v, \qquad \mathcal{M}_e = \frac{1}{\sqrt{2}} y_e v. \tag{1.7}$$

Any complex matrix $M$ can be diagonalized as $M = U^\dagger D U'$ where $U$ and $U'$ are unitary matrices and $D$ is a diagonal matrix with its elements being real and non-negative. Let us diagonalize the mass matrices by the following unitary transformations:

$$\begin{aligned} u'_L &= V_u\ u_L, \qquad & u'_R &= U_u\ u_R\ , \\ d'_L &= V_d\ d_L, \qquad & d'_R &= U_d\ d_R\ , \\ e'_L &= V_e\ e_L, \qquad & e'_R &= U_e\ e_R\ , \end{aligned} \tag{1.8}$$

where the mass eigenstates are written without primes. Kinetic terms are invariant under the transformations. In general, $V_u$ is different from $V_L$, so the quark doublet is written as

$$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = \begin{pmatrix} V_u u_L \\ V_d d_L \end{pmatrix} = V_u \begin{pmatrix} u_L \\ V_{\mathrm{CKM}}\ d_L \end{pmatrix}\ , \tag{1.9}$$

where $V_{\mathrm{CKM}}$ is the *Cabibbo-Kobayashi-Maskawa (CKM) matrix* [71, 72] defined by

$$V_{\mathrm{CKM}} \equiv V_u^\dagger V_d \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\ . \tag{1.10}$$

By expressing the interaction Lagrangian in the SM with the mass eigenstates, one can find that there exist flavor-changing charged currents caused by the $W^\pm$ exchange, while there are no flavor-changing neutral currents at tree level.

The CKM matrix $V_{\mathrm{CKM}}$ includes four physical parameters. We parametrize it using three mixing angles and the single CKM phase as [8, 73]

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}\ , \tag{1.11}$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, and $\delta$ is the CKM phase. The angles are chosen such that $s_{ij} > 0$ and $c_{ij} > 0$.

For leptons, on the other hand, we can always choose the same unitary transformation for the left-handed neutrino fields as those for the left-handed charged lepton fields since neutrinos are exactly massless in the SM. Hence, the SM leptons can be both mass and weak eigenstates simultaneously. The observations of neutrino oscillations, however, imply that neutrinos have small but non-vanishing masses and flavor mixing [74, 75, 76]. The mixing matrix in this case is often called the *Pontecorvo-Maki-Nakagawa-Sakata (PMNS)* mixing matrix.

As shown in Table D.1 in Appendix D, the eigenvalues of the mass matrices in Eq. (1.7) are quite hierarchical. To show it clearly, we illustrate their values in Fig. 1.1. Here, all of the masses

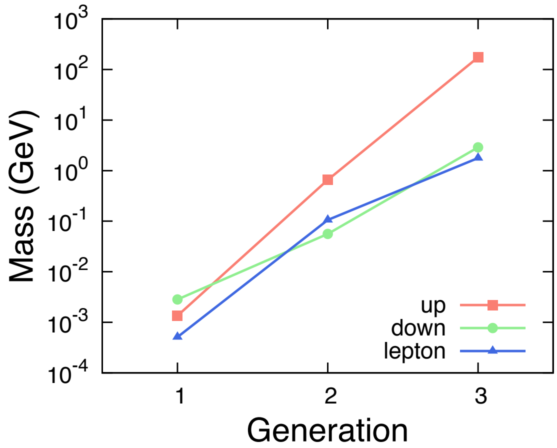


Figure 1.1: Masses of quarks and charged leptons at $m_Z$ scale. Here, red boxes, green circles, and blue triangles indicate masses of up-type quarks, down-type quarks, and charged leptons, respectively.

are renormailized in the $\overline{\text{MS}}$ scheme at the scale of $\mu = m_Z$. The red boxes, green circles, and blue triangles indicate the masses of up-type quarks, down-type quarks, and charged leptons, respectively. From this figure, it is found that the mass hierarchy among the up-type quarks is stronger than those among the down-type quarks and the charged leptons. In addition, the masses of the down-type quark and charged lepton of the same generation have similar values. The implication of the observation will be discussed in Sec. 3.3.

## 1.3 Minimal Supersymmetric Standard Model

Next, we give a brief review of the *minimal supersymmetric standard model (MSSM)*, which is the simplest supersymmetric extension of the SM. For more details, see, for example, Ref. [77].

The field content of the MSSM is summarized in Table. 1.2. Here, tildes (˜) indicate the superpartners of the corresponding SM fields. Again, all of the fermion fields are defined in terms of the two-component left-handed spinors. Notice that the MSSM requires two Higgs doublets, $H_u$ and $H_d$. One of the reasons for this is to realize the cancellation of gauge anomalies. In SUSY theories, a Higgs field requires the existence of its superpartner fermion field with the same quantum numbers, called a *higgsino*. This new fermion field would spoil the anomaly cancellation, which is satisfied in the SM. To avoid this difficulty, it is sufficient to add another Higgs doublet superfield with the opposite hypercharge. With this minimal extension, we have two Higgs doublets and two higgsino doublets.

The MSSM superpotential is

$$W_{\text{MSSM}} = f_u^{ij}(Q_i \cdot H_u)\overline{U}_j - f_d^{ij}(Q_i \cdot H_d)\overline{D}_j - f_e^{ij}(L_i \cdot H_d)\overline{E}_j + \mu(H_u \cdot H_d) \ , \tag{1.12}$$

with $(A \cdot B) \equiv \epsilon^{\alpha\beta}(A)_\alpha(B)_\beta$. The last term is the so-called $\mu$ *term*, which is the only dimensionful parameter in the MSSM superpotential. Note that the holomorphy of the superpotential requires both $H_u$ and $H_d$ in order to keep the Yukawa couplings for quarks and leptons. This is another reason for the necessity of at least two Higgs doublets in the SUSY SMs.

Table 1.2: Fields in the MSSM and their quantum numbers.

| Superfield | Boson | Fermion | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|---|---|
| $Q_i = (U_i\ D_i)$ | $\widetilde{Q}_{Li} = (\widetilde{u}_{Li}\ \widetilde{d}_{Li})$ | $Q_{Li} = (u_{Li}\ d_{Li})$ | $\mathbf{3}$ | $\mathbf{2}$ | $1/6$ |
| $\overline{U}_i$ | $\widetilde{u}^*_{Ri}$ | $u^\dagger_{Ri}$ | $\overline{\mathbf{3}}$ | $\mathbf{2}$ | $-2/3$ |
| $\overline{D}_i$ | $\widetilde{d}^*_{Ri}$ | $d^\dagger_{Ri}$ | $\overline{\mathbf{3}}$ | $\mathbf{2}$ | $1/3$ |
| $L_i = (N_i\ E_i)$ | $\widetilde{L}_{Li} = (\widetilde{\nu}_{Li}\ \widetilde{e}_{Li})$ | $L_{Li} = (\nu_{Li}\ e_{Li})$ | $\mathbf{1}$ | $\mathbf{2}$ | $-1/2$ |
| $\overline{E}_i$ | $\widetilde{e}^*_{Ri}$ | $e^\dagger_{Ri}$ | $\mathbf{1}$ | $\mathbf{1}$ | $+1$ |
| $H_u$ | $H_u = (H_u^+\ H_u^0)$ | $(\widetilde{H}_u^+\ \widetilde{H}_u^0)$ | $\mathbf{1}$ | $\mathbf{2}$ | $+1/2$ |
| $H_d$ | $H_d = (H_d^0\ H_d^-)$ | $(\widetilde{H}_d^0\ \widetilde{H}_d^-)$ | $\mathbf{1}$ | $\mathbf{2}$ | $-1/2$ |
| $G$ | $G_\mu^A$ | $\widetilde{g}^A$ | $\mathbf{8}$ | $\mathbf{1}$ | $0$ |
| $W$ | $W_\mu^a$ | $\widetilde{W}^a$ | $\mathbf{1}$ | $\mathbf{3}$ | $0$ |
| $B$ | $B_\mu$ | $\widetilde{B}$ | $\mathbf{1}$ | $\mathbf{1}$ | $0$ |

The gauge invariance and holomorphy of the superpotential allows other renormalizable terms, which are not included in the above superpotential. Such terms violate either lepton number or baryon number. In general, we have

$$W_{\not{L}} = \frac{1}{2}\lambda^{ijk}(L_i \cdot L_j)\overline{E}_k + \lambda'^{ijk}(L_i \cdot Q_j^a)\overline{D}_{ka} - \kappa^i(L_i \cdot H_u)\ , \tag{1.13}$$

$$W_{\not{B}} = \frac{1}{2}\lambda''^{ijk}\epsilon^{abc}\ \overline{U}_{ia}\overline{D}_{jb}\overline{D}_{kc}\ , \tag{1.14}$$

where $a, b, c = 1, 2, 3$ denote the color indices and $\epsilon^{abc}$ is the totally antisymmetric tensor with $\epsilon^{123} = +1$. Since lepton number and/or baryon number violating processes have not been observed yet experimentally, these operators must be extremely suppressed.[1] To eliminate such disastrous terms, in the MSSM, a new symmetry called *R-parity* is assumed. The $R$-parity is a discrete $Z_2$ symmetry defined for each particle as

$$P_R = (-1)^{3(B-L)+2s}\ , \tag{1.15}$$

where $s$ is the spin of the particle. In terms of the chiral superfields, this $Z_2$ symmetry corresponds to the following transformations:

$$\begin{aligned} Q_i,\ \overline{U}_i,\ \overline{D}_i,\ L_i,\ \overline{E}_i &\to -Q_i, -\overline{U}_i, -\overline{D}_i, -L_i, -\overline{E}_i\ , \\ H_u,\ H_d &\to H_u,\ H_d\ , \end{aligned} \tag{1.16}$$

with $\theta_\alpha, \bar{\theta}^{\dot{\alpha}} \to -\theta_\alpha, -\bar{\theta}^{\dot{\alpha}}$ . This definition gives the SM particles and the Higgs bosons $P_R = +1$, while the SUSY particles are assigned $P_R = -1$.

The $R$-parity conservation makes the *lightest SUSY particle (LSP)* stable. Therefore, if the LSP is neutral, it becomes a candidate for dark matter in the Universe. Moreover, in collider

[1] For a review of the $R$-parity violating SUSY models, see Ref. [78].

experiments, all of the SUSY particles are only produced in even numbers, and they eventually decay into the LSP with the emission of the SM particles.

In addition to the supersymmetric, $R$-parity conserving terms in Eq. (1.12), the MSSM contains the soft SUSY breaking terms:

$$\begin{aligned}
\mathcal{L}_{\text{soft}} = &-\frac{1}{2}\big(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}^a\widetilde{W}^a + M_3\widetilde{g}^A\widetilde{g}^A + \text{h.c.}\big) \\
&- \big(\widehat{a}_u^{ij}(\widetilde{Q}_{Li}\cdot H_u)\widetilde{u}_{Rj}^* - \widehat{a}_d^{ij}(\widetilde{Q}_{Li}\cdot H_d)\widetilde{d}_{Rj}^* - \widehat{a}_e^{ij}(\widetilde{L}_{Li}\cdot H_d)\widetilde{e}_{Rj}^* + \text{h.c.}\big) \\
&- \widetilde{Q}_{Li}^*(\widehat{m}^2_{\widetilde{Q}_L})_{ij}\widetilde{Q}_{Lj} - \widetilde{L}_{Li}^*(\widehat{m}^2_{\widetilde{L}_L})_{ij}\widetilde{L}_{Lj} - \widetilde{u}_{Ri}(\widehat{m}^2_{\widetilde{u}_R})_{ij}\widetilde{u}_{Rj}^* - \widetilde{d}_{Ri}(\widehat{m}^2_{\widetilde{d}_R})_{ij}\widetilde{d}_{Rj}^* - \widetilde{e}_{Ri}(\widehat{m}^2_{\widetilde{e}_R})_{ij}\widetilde{e}_{Rj}^* \\
&- m^2_{H_u}H_u^{*\alpha}H_{u\alpha} - m^2_{H_d}H_d^{*\alpha}H_{d\alpha} - \big(B\mu(H_u\cdot H_d) + \text{h.c.}\big)\ . \qquad (1.17)
\end{aligned}$$

This is the most generic soft SUSY breaking Lagrangian that respect SUSY and the $R$-parity conservation in the MSSM. The parameters in the above equation in general introduce new sources of flavor or CP violation, which are severely restricted by low-energy precision experiments [52].

Both the SUSY-conserving and SUSY-breaking terms in Eqs. (1.12) and (1.17), respectively, yield the scalar potential of the MSSM, and its minimum point corresponds to the ground state of the theory. In the following discussion, we assume that only the Higgs bosons in the MSSM have the VEVs; especially, squarks and sleptons should not get VEVs. Then, let us investigate the Higgs potential in the MSSM which is given as

$$\begin{aligned}
V_{\text{Higgs}} =&(|\mu|^2 + m^2_{H_u})(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m^2_{H_d})(|H_d^0|^2 + |H_d^-|^2) \\
&+ [B\mu(H_u^+H_d^- - H_u^0H_d^0) + \text{h.c.}] \\
&+ \frac{1}{8}(g^2+g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{1}{2}g^2|H_u^+H_d^{0*} + H_u^0H_d^{-*}|^2\ , \qquad (1.18)
\end{aligned}$$

with $g$ and $g'$ the coupling constants of $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$, respectively. Here, we presume that a minimum of the potential at which we live breaks the electroweak symmetry into the electromagnetic symmetry. By using the $\mathrm{SU}(2)_L$ gauge transformation, one can always take $\langle H_u^+\rangle = 0$ without loss of generality. In addition, the stationary condition at this point turns out to be satisfied with $\langle H_d^-\rangle = 0$. These two conditions meet our requirement for the symmetry breaking pattern. So it is sufficient to consider the potential consisting of only the neutral scalar fields:

$$\begin{aligned}
V_{\text{neutral}} =&(|\mu|^2 + m^2_{H_u})|H_u^0|^2 + (|\mu|^2 + m^2_{H_d})|H_d^0|^2 - B\mu(H_u^0H_d^0 + \text{h.c.}) \\
&+ \frac{1}{8}(g^2+g'^2)(|H_u^0|^2 - |H_d^0|^2)^2\ . \qquad (1.19)
\end{aligned}$$

Here, we have adjusted the over-all phase of $H_u$ and $H_d$ such that $B\mu$ is real and positive. Let us write the stationary point of the potential, *i.e.*, the VEVs of the Higgs fields, as

$$\langle H_u^0\rangle = \frac{v_u}{\sqrt{2}}\ , \qquad \langle H_d^0\rangle = \frac{v_d}{\sqrt{2}}\ . \qquad (1.20)$$

It turns out that by adjusting the relative phase between $H_u$ and $H_d$ one can always make both $v_u$ and $v_d$ real and positive. They are related with the VEV of the SM Higgs field $v$ as

$$v^2 = v_u^2 + v_d^2 \simeq (246\ \text{GeV})^2\ . \qquad (1.21)$$

Conventionally, their ratio is denoted by

$$\tan\beta \equiv v_u/v_d \ , \tag{1.22}$$

and thus

$$v_u = v\sin\beta \ , \qquad v_d = v\cos\beta \ . \tag{1.23}$$

The masses of the $W$- and $Z$-bosons are given by

$$\begin{aligned} m_W^2 &= \frac{1}{4}g^2v^2 \ , \\ m_Z^2 &= \frac{1}{4}(g^2+g'^2)v^2 \ . \end{aligned} \tag{1.24}$$

The formulae are identical to those in the SM.

Next, we expand the potential (1.19) around the VEVs. Notice that the potential is invariant under the charge conjugation; $H_u^0, H_d^0 \to H_u^{0*}, H_d^{0*}$. This implies the real and imaginary parts of the fields are decoupled. So, let us first look into the mass matrix of the imaginary part of $H_u^0, H_d^0$. The eigenvalues of the matrix are found to be

$$0, \qquad m_A^2 \equiv 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 \ , \tag{1.25}$$

with the massless eigenstate being the neutral Nambu-Goldstone boson which is eaten by $Z^0$. The other eigenstate $A^0$ with the mass of $m_A$ is a neutral $CP$-odd state. Note that from the stationary condition it follows that

$$\begin{aligned} B\mu &= m_A^2\sin 2\beta \ , \\ m_{H_u}^2 - m_{H_d}^2 &= (m_A^2+m_Z^2)\cos 2\beta \ . \end{aligned} \tag{1.26}$$

In particular, if $B\mu = 0$, then $A^0$ also becomes massless. In this case, the potential has an additional symmetry, the U(1) *Peccei-Quinn symmetry* [79]; under an equal phase rotation of $H_u^0$ and $H_d^0$, the potential is invariant. Since the VEVs of the fields break the symmetry, $A^0$ is regarded as the Nambu-Goldstone boson corresponding to the symmetry. This is in fact the original version of *axion* [80, 81], which has been already excluded from various experiments.

The eigenvalues of the real part are, on the other hand, evaluated as

$$\begin{aligned} m_h^2 &= \frac{1}{2}\left(m_A^2+m_Z^2-\sqrt{(m_A^2-m_Z^2)^2+4m_A^2m_Z^2\sin^2 2\beta}\right) \ , \\ m_H^2 &= \frac{1}{2}\left(m_A^2+m_Z^2+\sqrt{(m_A^2-m_Z^2)^2+4m_A^2m_Z^2\sin^2 2\beta}\right) \ , \end{aligned} \tag{1.27}$$

where the eigenstates $h^0$ and $H^0$ are neutral and $CP$-even.

Finally, we carry out similar analysis on the charged Higgs fields. We obtain the mass eigenvalues

$$0, \qquad m_{H^\pm}^2 = m_A^2 + m_W^2 \ , \tag{1.28}$$

and the massless charged states are eaten by $W^\pm$. The massive states $H^\pm$ have the charges $\pm 1$, respectively.

From Eq. (1.27), it is found that the mass of the lighter $CP$-even Higgs boson $m_h$ is bounded from above as

$$m_h^2 < \min\{m_Z^2, m_A^2\} \ . \tag{1.29}$$

Especially, $m_h$ is smaller than the $Z$-boson mass at tree-level. This originates from the fact that the quartic coupling of the MSSM Higgs potential is given by the gauge couplings, as seen in Eq. (1.19). In this thesis, we often consider the case of $m_A^2 \gg m_Z^2$, which is called the *decoupling limit*. In this limit, the tree-level mass of the lightest Higgs boson becomes $m_h^2 = m_Z^2 \cos^2 2\beta$, and it has the same couplings to the SM particles as those of the SM Higgs boson; *i.e.*, $h^0$ can be regarded as the SM Higgs boson. The masses of the other Higgs bosons are almost degenerate, and thus they form an SU(2) doublet.

The above constraint on $m_h$ appears to contradict the mass of Higgs boson $\sim 126$ GeV. The mass formula is, however, modified once the quantum corrections are considered [41, 42, 43, 44, 45]. The dominant contribution to the corrections is given by the top- and stop-loop diagrams because of the large Yukawa coupling. In the decoupling limit, the resultant Higgs mass at one-loop level is given as

$$m_h^2 \simeq m_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}{m_t^2}\right) + \frac{X_t^2}{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}\left(1 - \frac{X_t^2}{12 m_{\widetilde{t}_1} m_{\widetilde{t}_2}}\right)\right] , \tag{1.30}$$

where $m_{\widetilde{t}_1}$ and $m_{\widetilde{t}_2}$ are the mass eigenvalues of stops, $m_t$ is the top mass, and $X_t \equiv A_t - \mu \cot\beta$ with $(a_u)_{33} \equiv A_t(\hat{f}_u)_{33}$ ($a_u$ is given in Eq. (1.39)). From this expression one finds that to realize the 126 GeV Higgs mass the second term must be large enough. It occurs when the stop masses are significantly larger than the top mass and/or the $X_t$ terms are sizable. As mentioned in the Introduction, this observation is one of the motivations for the high-scale SUSY scenario.

For later use, we here give a specific basis for the squark soft-mass matrices. The basis is called the *Super-CKM basis*, in which the quark mass matrix is diagonal and the squarks are rotated in parallel to their superpartners. As in Eq. (1.8), quark superfields are rotated as

$$U' = V_u U, \quad D' = V_d D, \quad \overline{U}' = U_u^* \overline{U}, \quad \overline{D}' = U_d^* \overline{D} , \tag{1.31}$$

where again the primed fields represent the interaction eigenstates. This rotation transfers the Yukawa matrices $f_u$ and $f_d$ into their diagonal form $\hat{f}_u$ and $\hat{f}_d$ as

$$\begin{aligned} (\hat{f}_u)_{ii} &= (U_u^\dagger f_u^T V_u)_{ii} \equiv f_{u_i} = \sqrt{2}\frac{m_{ui}}{v \sin\beta} , \\ (\hat{f}_d)_{ii} &= (U_d^\dagger f_d^T V_d)_{ii} \equiv f_{d_i} = \sqrt{2}\frac{m_{di}}{v \cos\beta} . \end{aligned} \tag{1.32}$$

In this basis, the mass matrices for the up- and down-type squarks are given as

$$\mathcal{L}_{\text{squark mass}} = -\widetilde{\mathcal{U}}^\dagger \mathcal{M}_{\tilde{u}}^2 \, \widetilde{\mathcal{U}} - \widetilde{\mathcal{D}}^\dagger \mathcal{M}_{\tilde{d}}^2 \, \widetilde{\mathcal{D}} , \tag{1.33}$$

with $\widetilde{\mathcal{U}} = (\widetilde{u}_L, \widetilde{c}_L, \widetilde{t}_L, \widetilde{u}_R, \widetilde{c}_R, \widetilde{t}_R)$ and $\widetilde{\mathcal{D}} = (\widetilde{d}_L, \widetilde{s}_L, \widetilde{b}_L, \widetilde{d}_R, \widetilde{s}_R, \widetilde{b}_R)$. These $6 \times 6$ mass matrices are explicitly given by

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} m_{\tilde{q}_L}^2 & m_{\tilde{q}_{LR}}^2 \\ m_{\tilde{q}_{LR}}^{2\dagger} & m_{\tilde{q}_R}^2 \end{pmatrix} , \tag{1.34}$$

where

$$
\begin{aligned}
m^2_{\tilde{q}_L} &= V_q^\dagger \widehat{m}^2_{\tilde{Q}_L} V_q + m_q^2 + m_Z^2 \cos 2\beta (T_{3q_L} - Q_{q_L} \sin^2\theta_W) \ , \\
m^2_{\tilde{q}_R} &= U_q^\dagger \widehat{m}^{2T}_{\tilde{q}_R} U_q + m_q^2 + m_Z^2 \cos 2\beta Q_{q_R} \sin^2\theta_W \ , \\
m^2_{\tilde{q}_{LR}} &= \begin{cases} \frac{1}{\sqrt{2}} v \sin\beta (U_u^\dagger \widehat{a}_u^T V_u)^\dagger - \mu m_u \cot\beta & \text{for} \quad q = u \\ \frac{1}{\sqrt{2}} v \cos\beta (U_d^\dagger \widehat{a}_d^T V_d)^\dagger - \mu m_d \tan\beta & \text{for} \quad q = d \end{cases} .
\end{aligned}
\tag{1.35}
$$

Similarly, the charged slepton mass matrix is given as

$$
\mathcal{M}^2_{\tilde{e}} = \begin{pmatrix} m^2_{\tilde{e}_L} & m^2_{\tilde{e}_{LR}} \\ m^{2\dagger}_{\tilde{e}_{LR}} & m^2_{\tilde{e}_R} \end{pmatrix} \ , \tag{1.36}
$$

where

$$
\begin{aligned}
m^2_{\tilde{e}_L} &= V_e^\dagger \widehat{m}^2_{\tilde{L}_L} V_e + m_e^2 + m_Z^2 \cos 2\beta (T_{3e_L} - Q_{e_L} \sin^2\theta_W) \ , \\
m^2_{\tilde{e}_R} &= U_e^\dagger \widehat{m}^{2T}_{\tilde{e}_R} U_e + m_e^2 + m_Z^2 \cos 2\beta Q_{e_R} \sin^2\theta_W \ , \\
m^2_{\tilde{e}_{LR}} &= \frac{1}{\sqrt{2}} v \cos\beta (U_e^\dagger \widehat{a}_e^T V_e)^\dagger - \mu m_e \tan\beta \ .
\end{aligned}
\tag{1.37}
$$

Again, the basis $\widetilde{\mathcal{E}} = (\widetilde{e}_L, \widetilde{\mu}_L, \widetilde{\tau}_L, \widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R)$ is taken so that the lepton Yukawa coupling matrix is diagonalized. The sneutrino mass matrix is just a $3\times3$ mass matrix as long as the right-handed neutrinos are not introduced:

$$
\mathcal{M}^2_{\tilde{\nu}} = m^2_{\tilde{\nu}_L} = V_e^\dagger \widehat{m}^2_{\tilde{L}_L} V_e + m_Z^2 \cos 2\beta T_{3\nu_L} \ . \tag{1.38}
$$

When the right-handed neutrinos are considered, we need to extend the expressions similarly to the case of the quark sector. In such a case, the basis for the lepton sector is called the *super-PMNS basis*.

In the rest of the thesis, we use the soft breaking parameters which have been rotated by $V_f$ as input parameters. Namely, we use

$$
\begin{aligned}
\widetilde{m}^2_{\tilde{q}_L} &\equiv V_q^\dagger \widehat{m}^2_{\tilde{Q}_L} V_q, \qquad \widetilde{m}^2_{\tilde{q}_R} \equiv U_q^\dagger \widehat{m}^{2T}_{\tilde{q}_R} U_q, \\
\widetilde{m}^2_{\tilde{e}_L} &\equiv V_e^\dagger \widehat{m}^2_{\tilde{L}_L} V_e, \qquad \widetilde{m}^2_{\tilde{e}_R} \equiv U_e^\dagger \widehat{m}^{2T}_{\tilde{e}_R} U_e, \\
a_u &\equiv U_u^\dagger \widehat{a}_u^T V_u, \qquad a_d \equiv U_d^\dagger \widehat{a}_d^T V_d, \qquad a_e \equiv U_e^\dagger \widehat{a}_e^T V_e \ .
\end{aligned}
\tag{1.39}
$$

Further, we take the generation basis for the matter fields so that the Yukawa coupling matrices for the up-type quarks and the charged leptons are diagonal. In this case, the Yukawa coupling matrix for the down-type quarks is written as

$$
f_d = V^*_{\rm CKM} \hat{f}_d \ . \tag{1.40}
$$

The sfermion matrices $\mathcal{M}_{\tilde{f}}$ $(f = u, d, e, \nu)$ are diagonalized with unitary matrices $R_f$ as

$$
R_f \mathcal{M}^2_{\tilde{f}} R_f^\dagger = \mathrm{diag}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}, \dots) \ , \tag{1.41}
$$

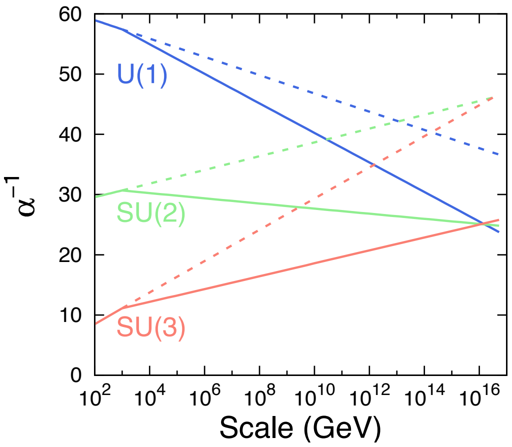


Figure 1.2: Running of gauge couplings. Solid (dashed) lines represent the gauge coupling running in the MSSM (SM). Blue, green, and red lines correspond to the U(1), SU(2), and SU(3) gauge couplings, respectively. SUSY breaking scale is set to be 1 TeV in the graph.

where sfermion mass eigenstates are denoted by $\widetilde{f}_I$:

$$\widetilde{f}_I = (R_f)_{I\widehat{i}} \widetilde{f}_{\widehat{i}} \ , \tag{1.42}$$

with the subscript $\widehat{i}$ representing the index of the Super-CKM basis; *e.g.*, for up-type squarks, $\widehat{i} = u_L, c_L, t_L, u_R, c_R, t_R$.

Before concluding the section, we comment on a remarkable feature of the MSSM, which is one of the reasons for the model to be regarded as the leading candidate of physics beyond the SM. As mentioned in the Introduction, with the particle content of the MSSM, the gauge coupling constants turn out to be unified at a high-energy scale with great accuracy. In Fig. 1.2, we compare the evolution of the gauge couplings in the MSSM (solid lines) with those in the SM (dashed lines). Here, the blue, green, and red lines correspond to the U(1), SU(2), and SU(3) gauge couplings, respectively. In the calculation, we use the two-loop RGEs for the gauge couplings, though we do not take into account the threshold corrections at the SUSY breaking scale, which is set to be 1 TeV. As you can see, the gauge couplings unify much better in the MSSM than in the SM.

In the computation, we assume that the SUSY breaking scale is slightly above the electroweak scale. As a result, we obtain the gauge coupling unification with good accuracy. Then, what happen if the SUSY breaking scale is much higher than the electroweak scale? One may think that the unification is always spoiled when the SUSY breaking scale is taken to be higher. We will see that this is not the case in the high-scale SUSY scenario, which we consider in the following discussion. This issue will be revisited in Sec. 3.2.

# Chapter 2

# High-scale Supersymmetry

As introduced in the previous chapter, the recent LHC results motivate us to study a SUSY scenario in which SUSY breaks at a much higher scale than the electroweak scale. From now on, we collectively refer to such scenarios as *high-scale SUSY scenarios*. In fact, such a scenario is attractive from both theoretical and phenomenological points of view. Theoretically, it is fascinating because it can be realized with quite a simple SUSY breaking mechanism. We only need a hidden sector in which SUSY is broken dynamically. Then we in general have a hierarchical spectrum among the SUSY particles; fermionic SUSY partners are a few orders of magnitude lighter than bosonic partners. This kind of scenario is discussed in Sec. 2.2. Of course, the breaking mechanism itself does not tell us where the SUSY breaking scale $M_S$ should be. The scale is to be set by phenomenological considerations. In particular, the observed value of the Higgs boson mass, $m_h \sim 126$ GeV, gives us significant information on $M_S$. As shown in Sec. 2.3, it is suggested that the SUSY scale is higher than the electroweak scale by several orders of magnitude, *e.g.*, $M_S \sim 10^2$–$10^5$ TeV. In this case, all the scalar particles except the lightest Higgs boson (*i.e.*, the 126 GeV Higgs boson) have masses of around $M_S$, while the fermionic SUSY partners (gauginos and perhaps higgsinos) are around $\mathcal{O}(1)$ TeV. Then, with such a mass spectrum it turns out that the model has the following features:

- The SUSY flavor and CP problems [52] are significantly relaxed.
- The cosmological gravitino problem [56, 57, 58, 59, 60, 61] is avoided.
- There are dark matter candidates [63, 64].
- The gauge coupling unification can be achieved with great accuracy.

Therefore, this high-scale SUSY scenario is also quite interesting from a phenomenological point of view. In the subsequent chapters, we will discuss additional features of the high-scale SUSY scenario in the context of the grand unified theories.

Although this model is a simple and phenomenologically attractive, it has a theoretical (or I might say aesthetic) shortcoming. Namely, it seems extremely challenging for the model to solve the so-called *hierarchy problem* [19, 20, 21, 22, 23] or *naturalness problem* [24]. Using the conventional ways of parametrizing the extent of naturalness, we find that the high-scale SUSY scenario requires at least the $10^{-4}$–$10^{-6}$ level of fine-tunings. Since these problems have been standing problems in particle physics and SUSY has been considered to be able to provide a solution to the problems, it is appropriate that we start with the discussion on this matter. It is presented in Sec. 2.1.

## 2.1 Naturalness vs. fine-tuning

In this section, we discuss the so-called *hierarchy problem* [19, 20, 21, 22, 23] or *naturalness problem* [24]. For the past years, the problem has been one of the motivations to look for physics beyond the SM. For recent discussion on the matter, see, for instance, Refs. [82, 83, 84, 85]. We also remark some possibilities for the Higgs sector in the SM to be finely tuned to realize the electroweak scale as it is.

### 2.1.1 Hierarchy problem

As mentioned in the Introduction, the grand unification of the strong and electroweak interactions requires a new mass scale much higher than the electroweak scale. But, what can naturally explain such an enormous hierarchy? This question is a start point of what we call the *hierarchy problem* [19, 20, 21, 22, 23]. To clarify the point, suppose that the grand unified gauge group is spontaneously broken down by the VEVs of elementary scalar fields. The massless Nambu-Goldstone bosons are eliminated by the Higgs mechanism [4, 5, 6, 7], and the gauge fields corresponding to the broken symmetries acquire masses of the order of the GUT scale. The theory much below the scale is described by an effective theory consisting of only the fields which do not get superheavy masses from the symmetry breaking. Such fields include massless gauge fields corresponding to the unbroken symmetries, as well as fermions protected by the chiral symmetries which are not broken by the GUT scale symmetry breaking. Then, the question is whether the effective theory could include elementary scalar particles which have considerably small masses compared to the GUT scale particles—like the SM Higgs boson. There seems to be no reason for such light scalars to appear in the effective theory, though they are actually required to break the electroweak symmetry.

The hierarchy problem becomes more problematic when the quantum effects are considered. Assume that there exists a scalar boson $H$ whose mass $m_H$ is much below the GUT scale, which mimics the SM Higgs boson in this example, and it interacts with a GUT scale scalar boson $\Phi$ with mass of $M_\Phi \gg m_H$ through the interaction Lagrangian $\mathcal{L}_{\rm int} = -\lambda_\Phi |H|^2|\Phi|^2$. The heavy scalar $\Phi$ gives the quantum correction $\delta m_H^2$ to the mass term for $H$, which is evaluated at one-loop level as

$$\delta m_H^2 = \frac{\lambda_\Phi}{16\pi^2}\left[\Lambda^2 - M_\Phi^2 \ln\left(\frac{\Lambda^2}{M_\Phi^2}\right)\right] , \tag{2.1}$$

where $\Lambda$ denotes the ultraviolet momentum cutoff. The first term diverges quadratically with respect to the cutoff $\Lambda$, though the term is dependent on the regularization scheme. The second term has more physical meanings; it shows that the quantum corrections to the mass of $H$ are as large as the mass of $\Phi$. After all, $\delta m_H^2$ is much larger than $m_H^2$ itself. For this reason, the natural mass scale of $H$ should be the GUT scale $\sim M_\Phi$ or the cutoff scale $\Lambda$, otherwise the significant cancellation with the bare mass of $H$ is required to make its physical mass much smaller than the GUT scale. The reason for the feature is that scalar masses are renormalized additively (not multiplicatively) and thus the quantum corrections are not necessarily related to their classical masses.

This quadratically divergent nature of scalar masses was already pointed out by V. F. Weisskopf in 1939 [86]. After several decades, K. G. Wilson discussed the problematic feature on scalar masses by means of the renormalization group analysis, and interpreted the results in terms of symmetries [87]. He noted that the scalar masses are not protected from large quantum corrections since they do not break either an internal or a gauge symmetry. This observation should be compared with the mass terms of the SM fermions and gauge bosons. Evidently, they are protected by the chiral and gauge symmetries, respectively, and thus they can naturally have masses much below the cutoff scale.

This argument leads to the *naturalness* criterion proposed by G. 't Hooft in 1979 [24]. He required the following conjecture for naturalness: *"at any energy scale $\mu$, a physical parameter or set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system."*[1] Here, all of the parameters having mass-dimensions are measured in units of $\mu$. This criterion reflects the Wilson's argument; *i.e.*, only the parameters that can appear in the low-energy effective theories are those who violate some symmetries. Let us consider the SM in terms of the naturalness criterion. The mass parameters of the SM fermions $m_f$ can be small since $m_f = 0$ enhances an additional chiral symmetry for $f_L$ and $f_R$. The gauge coupling constants can be small since their zero limit implies the conservation of the number of the gauge bosons. The masses of $W$ and $Z$ are of course understood in terms of the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge symmetry. Consequently, the only parameter which is unnatural in the SM is, as we have been discussed, the mass parameter of Higgs boson.

If the Higgs mass term is in fact natural in a high-energy theory beyond the Standard Model, the term must be protected by a certain symmetry in the theory. Since the symmetry is not manifest below the electroweak scale, it must be broken at a high-energy scale. Let $\Lambda$ be the symmetry breaking scale. Then, the naturalness argument requires that $m_H^2/\Lambda^2$ should not be much smaller than unity. As long as this condition is satisfied, the mass parameter $m_H^2$ is natural; at a scale $\mu > \Lambda$, $m_H^2/\mu^2$ can be small because the mass term breaks the symmetry present in the energy region. However, as noted by 't Hooft, the naturalness requirement can only determine the upper bound on $\Lambda$ as an order of magnitude. There might be an additional factor including some coupling constants and/or loop loop factors in the inequality, which introduces order-of-magnitude arbitrariness.

In the present discussion, we use the following conjecture to estimate the scale $\Lambda$: the quantum corrections to a parameter evaluated in an effective theory with the cutoff $\Lambda$ should not be greater than the parameter itself. Indeed, the conjecture has been historically successful. One well-known example is the electromagnetic self-energy to the mass of electron[2]. In the non-relativistic theory, the energy diverges at short-distant scales linearly, and it can be smaller than the electron mass only if some new physics which appears at a short-distant scale modifies the contribution to the self-energy. Actually, in the relativistic quantum field theory, positrons

[1] As mentioned by 't Hooft, this requirement is weaker than that proposed by P. Dirac in his Large Number Hypothesis [88, 89]. In the hypothesis, he assumed that any large number like the ratio between electric and gravitational interactions of a proton with an electron should be related with a single large number, and he supposed that it was the age of the universe, only which he could connect to such a large number at that time. Obviously, he regarded the ratio $m_e m_p/(\alpha M_P^2)$ as unnatural, where $m_p$, $\alpha$, and $M_P$ are the proton mass, the fine structure constant, and the reduced Planck mass $M_P \equiv (8\pi G_N)^{-1/2}$ with $G_N$ the Newton constant, respectively. From the viewpoint of the 't Hooft's naturalness criterion, on the other hand, the ratio should be regarded natural since the chiral symmetry protects the masses of electron and proton.

[2] This interesting example was discussed by H. Murayama in his lecture to clearly show the motivation for supersymmetry in terms of the self-energy of the Higgs boson [90].

ease the divergence so that the self-energy does not exceed the electron mass [86]. Similarly, the quadratic divergence in electromagnetic effects on the mass difference between the charged and neutral pions are cured by the $\rho$ meson appearing near the cutoff scale of the effective theory. The $K_L^0$-$K_S^0$ mass difference is another example, where the quantum corrections are eventually modified by the charm quark [91]. Now let us apply the conjecture to the mass of Higgs boson. By thinking of the SM as an effective theory valid up to the cutoff $\Lambda$, we have the corrections to the Higgs mass $m_h$ at one-loop level as [92]

$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2}(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2)\Lambda^2 \ . \tag{2.2}$$

Therefore, the conjecture requires $\Lambda \lesssim 500$ GeV, and if the naturalness argument is right, some new particles must show up around the scale.

Above the scale $\Lambda$, the new theory must introduce a certain symmetry which prohibit the Higgs mass term. One of the most promising candidates for the symmetry is *supersymmetry*. In the SUSY theories, the Higgs boson is accompanied by the corresponding superpartner fermion. Since the mass of the fermion can be protected by the chiral symmetry, the fermion can have a mass much below the more fundamental scale like the GUT scale. On the other hand, supersymmetry requires the fermion and the Higgs boson to have the same mass. Hence, the smallness of the Higgs mass might be explained by means of the combination of the chiral symmetry and supersymmetry, and its value is expected to be around the order of the breaking scale of the symmetries. In the rest of the section, we assume the presence of SUSY in the high-energy theory, and discuss its role in providing a solution to the hierarchy problem.

### 2.1.2 Quantification of naturalness

In the previous section, we discuss a conjecture to determine the naturalness breaking scale $\Lambda$ of the SM: the quantum correction to the Higgs mass, which is dependent on the scale $\Lambda$, should be less than the Higgs mass itself. It measures the degree of naturalness in terms of the low-energy effective theory. In this subsection, we consider a criterion to measure the degree of naturalness of a given high-energy theory.

A strategy is to regard the $Z$-boson mass as a function of the most fundamental parameters $a_i$ in the high-energy theory [93, 94]. The degree of fine-tuning is then defined by

$$\Delta \equiv \max\left\{ \left| \frac{a_i}{m_Z^2} \frac{\partial m_Z^2(a_i)}{\partial a_i} \right| \right\} \ , \tag{2.3}$$

where $100 \times \Delta^{-1}$ gives a percentage of fine-tuning. In the calculation of the parameters, one needs to include the renormalization effects generated during the RGE flow from the input scale to the electroweak scale.

Another fine-tuning parameter was introduced by R. Kitano and Y. Nomura in Ref. [95]. Let us briefly sketch their discussion in the case of the MSSM. We consider the decoupling limit for brevity. From Eqs. (1.25) and (1.26), it is found that for moderately large $\tan\beta$, which is favored from the viewpoint of naturalness in the MSSM, we have

$$m_h^2 = m_Z^2 \cos^2 2\beta \simeq -2(|\mu|^2 + m_{H_u}^2) + \mathcal{O}(1/\tan^2\beta) \ , \tag{2.4}$$

at tree-level. Even when the quantum corrections are taken into account, a similar relation is satisfied with the tree-level input parameters appropriately tuned. Namely, we obtain

$$m_h^2 \simeq -2(|\mu|^2 + m_{H_u}^2 + \delta m_{H_u}^2) \;, \tag{2.5}$$

where $m_{H_u}^2$ denotes the tree-level soft SUSY breaking parameter and $\delta m_{H_u}^2$ represents the radiative corrections to the parameter. With the leading logarithmic approximation, $\delta m_{H_u}^2$ is evaluated as

$$\delta m_{H_u}^2 \simeq -\frac{3f_t^2}{16\pi^2}\left(m_{\widetilde{Q}_{L3}}^2 + m_{\widetilde{u}_{R3}}^2 + m_{H_u}^2 + |A_t|^2\right) \ln\left(\frac{\Lambda^2}{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}\right) \;. \tag{2.6}$$

Now, we have a criterion to measure the degree of fine-tuning to realize the Higgs mass, *i.e.*,

$$\Delta_{\rm KN} \equiv \max\left\{\frac{2|\mu|^2}{m_h^2}, \frac{2|m_{H_u}^2|}{m_h^2}, \frac{2|\delta m_{H_u}^2|}{m_h^2}\right\} \;. \tag{2.7}$$

Here again, $100 \times \Delta_{\rm KN}^{-1}$ represents a percentage of fine-tuning. From Eq. (2.6), it is found that not only the parameters in the tree-level Higgs potential such as $\mu$ and $m_{H_u}^2$, but also the stop mass parameters are required to be small to make the theory less fine-tuned. Thus, there is a tension between the naturalness and the 126 GeV Higgs mass, which requires large stop mass parameters to realize significant quantum corrections as discussed in the previous chapter. Indeed, it is found that at least 1 % fine-tuning is required to realize the electroweak scale in the MSSM [96].

In fact, both of the fine-tuning parameters are based on the essentially same criterion: to measure the degree of sensitivities of the electroweak scale to the input parameters. Anyway, we should always keep in mind that we have no criterion to determine what degree of fine-tuning a natural theory allows. We cannot exclude a theory even if it requires, say, $\Delta^{-1} = 10^{-10}$ fine-tuning; there might exist a mechanism which naturally yields the apparent fine-tuning without conspiracy. In the next subsection, we discuss such possibilities.

### 2.1.3 Possibilities of fine-tunings

As is well-known, there is an unnatural parameter in our nature: the *cosmological constant*. The cosmological constant is regarded as the energy density of the vacuum $\rho_\Lambda$ in the Universe. The presence of a small but non-zero cosmological constant has been widely believed since the discovery of the accelerating expansion of the Universe [97, 98]. Recently, the Planck satellite has measured its value as [99]

$$\rho_\Lambda \sim 5.8 \times 10^{-30}\ {\rm g/cm^3} \sim (2.2 \times 10^{-3}\ {\rm eV})^4 \;. \tag{2.8}$$

Notice that its energy scale is extremely low compared with other physical scales such as the electroweak scale or the GUT scale. Let us consider its naturalness in terms of the naturalness conjecture discussed above. The parameter may be unnatural[3] since its vanishing limit does not seem to enhance any symmetry. Indeed, it suffers from large quantum corrections; its quantum fluctuation can be as large as $M_P^4$, which is larger than the observed value by about 120 orders of magnitude.

[3] Actually, 't Hooft realized the problem when he discussed the naturalness criterion [24]. Instead of giving an answer to the problem, however, he conjectured that only the gravitational interactions violate naturalness.

The cosmological constant is more problematic if the history of the Universe is also taken into account. According to the results presented in Ref. [99], it occupies about 70 % of the total energy density of the Universe, while about 30 % is in the form of dark matter. During the cosmic expansion, the former density remains unchanged; this is why it is called the cosmological "constant". On the contrary, the latter decreases as $\propto a(t)^{-3}$ with $a(t)$ the scale factor. Therefore, it is quite special for us to observe these quantities almost equal at the very present time [100].

These two peculiar features of the cosmological constant problem force us to change our ways of thinking. Should all of the problems be solved in a "natural" way in a fundamental theory? Is not there any other reason that explains an apparent fine-tuning in a system?

In 1987, S. Weinberg argued a possibility that the smallness of the cosmological constant might be simply because otherwise no observer can exist [101]. It turns out that if the vacuum energy density is larger than the matter energy density by more than several orders of magnitude, no gravitational bound states such as galaxies can form, and thus no intellectual species may appear. This implies that the unnaturally small value of the cosmological constant may be not the consequence of some fundamental mechanism but the necessary condition for observing it. This discussion leads to a following conjecture. Suppose that there are a vast number of universes (or sufficiently separated regions from each other) with different values of cosmological constants. Most of them can be much larger than that of our Universe, which accelerate the universes so rapidly that no complex structure can exist there. Then, the value of our cosmological constant should be a typical one that is seen by the largest number of observers in the universes. This sort of *anthropic principle* [102] has been enthusiastically discussed in this context so far [103, 104, 105].

In fact, this argument has a theoretical support. It consists of two ingredients; the *string landscape* [106, 107, 108, 109] and the *eternal inflation* [110, 111, 112, 113]. The former comes from string theory. In the theory, six extra spatial dimensions are predicted, which are assumed to be compactified so that we do not see them. These extra dimensions can have a huge number of meta-stable vacua, which correspond to different four-dimensional theories. Thus, this framework offers an ensemble of theories with a variety of theoretical parameters including cosmological constant. The latter concept indicates that inflation generically continues eternally. During the inflation, bubbles are formed in the inflating spacetime, whose vacua correspond to various theories. Since it continues eternally, all of the vacua prepared by the compactification of extra dimensions are eventually scanned, so that any theories can be realized in a certain bubble. This picture is usually referred to as the *multiverse*.

Such a parameter scan can be also realized if one considers the wormhole effect proposed by S. Coleman [114]. If the wormhole configurations are taken into account in the computation of the path integral, the functional integral turns out to be interpreted as a superposition of the functional integrals with various values of coupling constants in the original action. This may offer a mechanism in which some parameters are fixed to particular values because of the integration with respect to the parameters, without use of the anthropic reasoning. Recent discussion in this direction, see Ref. [115] and references therein.

As we have seen, there are actually some possibilities that can provide apparently tuned parameters; they might be the consequence of the anthropic principle or non-trivial topology of spacetime. If such mechanism works in the case of the Higgs mass parameter, the naturalness argument would be pointless from the beginning.

After all, we can understand the naturalness of a theory only after we have revealed all of the structure of the theory experimentally. Therefore, it is important to study even the possibilities for theories including apparent fine-tunings. This is our attitude in this thesis.

## 2.2 SUSY breaking without singlets

If SUSY is broken at a scale much lower than a more fundamental scale, say, the GUT scale, then one may ask why there is such a large hierarchy between the scales. The situation appears to give a similar problem to that for the electroweak symmetry breaking. However, it is found that SUSY theories might present a natural framework for the emergence of the large hierarchy [25]. Once we require that SUSY is unbroken at tree level, the non-renormalization theorems [116] ensures that SUSY is unbroken to all orders of the perturbation theory. So in this case SUSY can only be broken by small non-perturbative effects, and this breaking mechanism might account for the large hierarchy. Such a SUSY breaking scenario is called *dynamical SUSY breaking*. In fact, it has been found that the dynamical SUSY breaking occurs in various asymptotically-free SUSY gauge theories [26, 27, 28, 29, 30, 31].

Let us assume that SUSY breaking occurs dynamically in a *hidden sector* and its effects are communicated to the MSSM (*visible*) sector via some higher-dimensional operators and/or quantum corrections. Such a hidden sector naturally arises in string theories. This framework may give a unique feature compared with other SUSY-breaking models, since the dynamical SUSY breaking models generically contain no singlet field. The present and the following subsections, therefore, are devoted to study its phenomenological consequences observed in the visible sector. In this subsection, we discuss the mediation mechanism of SUSY breaking in this framework. To that end, we first give a brief review of a formalism of *supergravity (SUGRA)* by means of a superconformal compensator field developed in Refs. [117, 118, 119, 120]. Then, by using the formalism, we discuss the so-called the *anomaly mediation* mechanism of SUSY breaking proposed in Refs. [46, 121]. We will see that a hierarchy between the fermionic and bosonic SUSY partner fields naturally arises in this framework. This observation leads us to consider a high-scale SUSY breaking model which will be discussed in Sec. 2.3.

### 2.2.1 Supergravity

*Supergravity (SUGRA)* is a theory which is invariant under local SUSY transformations.[4] It has been widely known that ordinary SUGRA theories (or Poincaré SUGRA) can be obtained by fixing redundant symmetries (dilatation, an $\mathrm{U}(1)_R$ symmetry, superconformal spinor symmetry, and the special conformal symmetry) of the theories which respect the local superconformal symmetry. This procedure, which is called the *superconformal tensor calculus*, is systematically executed by means of a chiral spurion superfield[5] called *compensator*, whose component fields are used to fix the redundant symmetries. Let $\Sigma$ be the compensator. We assign Weyl weight $+1$ and the $\mathrm{U}(1)_R$ charge $2/3$ to the lowest component of $\Sigma$ ($x^\mu$ and $\theta_\alpha$ have Weyl weight $-1$ and $-1/2$, respectively). Further, each physical field has a Weyl weight equal to its mass dimension

[4] For reviews, see Refs. [122, 123, 124, 125, 126].

[5] More accurately, the use of the chiral compensator leads to the so-called old minimal version of supergravity [127, 128]. If one uses a different type of supermultiplet as a compensator, one obtains a different formulation of Poincaré SUGRA [120]; a real linear multiplet compensator gives the new minimal SUGRA [129] while a complex linear multiplet compensator gives Breitenlohner's formulation of SUGRA [130, 131].

*d*. Now consider an arbitrary global SUSY theory described by the Lagrangian

$$\mathcal{L} = \int d^4\theta K(\Phi^\dagger, e^{2gV}\Phi) + \left\{ \int d^2\theta W(\Phi) + f_{AB}(\Phi) W^{A\alpha} W^A_\alpha + \text{h.c.} \right\} . \tag{2.9}$$

Here, $W_\alpha = W^A_\alpha T^A$ denotes the gauge field-strength chiral superfield defined in terms of the vector superfield $V = V^A T^A$ by

$$W_\alpha = -\frac{1}{4}\overline{D}^2 e^{-2gV} D_\alpha e^{2gV} . \tag{2.10}$$

In addition, $K$, $W$, and $f_{AB}$ are called the *Kähler potential*, the *superpotential*, and the *gauge kinetic function*, respectively. Then, the corresponding Weyl invariant Lagrangian is written as follows:

$$\mathcal{L} = \frac{1}{2}\left[-3\, e^{-\frac{1}{3}K(\frac{\Phi^\dagger}{\Sigma^\dagger},\, e^{2gV}\frac{\Phi}{\Sigma})}\, \Sigma^\dagger\Sigma\right]_D + \left\{ \left[ W\left(\frac{\Phi}{\Sigma}\right)\Sigma^3 + f_{AB}\left(\frac{\Phi}{\Sigma}\right) W^{A\alpha} W^A_\alpha \right]_F + \text{h.c.} \right\} , \tag{2.11}$$

where $[\ldots]_D$ and $[\ldots]_F$ denote the superconformal invariant $F$- and $D$-type action formulae presented in Refs. [117, 118, 119, 120], respectively. Let us show some terms of them which we use in the following calculation. To that end, we express the components of a real vector superfield $V$ by $\{C, \xi_\alpha, M, A_\mu, \lambda_\alpha, D\}$ and of a chiral superfield $\Phi$ by $\{\phi, \chi, F\}$, from the lowest to the highest component. The above formulae are then given as

$$\begin{aligned} [V]_D &= \sqrt{-g}D - \frac{C}{3}(\sqrt{-g}R + \mathcal{L}_{\text{RS}}) + \ldots , \\ [\Phi]_F &= \sqrt{-g}(F - \phi\, \overline{\psi}_\mu \bar{\sigma}^{\mu\nu} \overline{\psi}_\nu) + \ldots , \end{aligned} \tag{2.12}$$

where $R$ is the scalar curvature, $g \equiv -\det(g_{\mu\nu})$, and $\mathcal{L}_{\text{RS}}$ denotes the massless Rarita-Schwinger Lagrangian for the gravitino $\psi_\mu$. The second term in the latter line shows the mass term of the gravitino field.

From Eq. (2.10), one readily finds that the chiral covariant derivative $D_\alpha$ should involve the compensator as

$$D_\alpha \to \frac{\Sigma^{\frac{1}{2}}}{\Sigma^\dagger} D_\alpha . \tag{2.13}$$

Thus, since $D^2\overline{D}^2 D^2 = -16 D^2 \Box$, we also find that the Weyl covariant version of the d'Alembertian is given by $\Box \to \Box/(\Sigma^\dagger\Sigma)$. Notice that since the gauge field-strength term has mass dimension $+3$ and $R$ charge $+2$, it does not contain the compensator field $\Sigma$.

For later use, we introduce "rescaled" fields $\hat{\Phi}_i \equiv \Phi_i/\Sigma$. Further, we redefine the compensator as $\Sigma \to W^{\frac{1}{3}}\Sigma$. Then, Eq. (2.11) is expressed as follows:

$$\mathcal{L} = \frac{1}{2}\left[-3\, e^{-\frac{1}{3}G(\hat{\Phi}^\dagger, \hat{\Phi})}\, \Sigma^\dagger\Sigma\right]_D + \left\{ \left[\Sigma^3\right]_F + \text{h.c.} \right\} , \tag{2.14}$$

where

$$G(\hat{\Phi}^\dagger, \hat{\Phi}) \equiv K(\hat{\Phi}^\dagger, \hat{\Phi}) + \ln W(\hat{\Phi}) + \ln W^*(\hat{\Phi}^\dagger) \tag{2.15}$$

is called the *Kähler function*. We drop the gauge fields for brevity. By using the formulae in Eq. (2.12), we find that the fixing condition

$$\Sigma = M_P e^{\frac{G}{6}}(1 + \theta\theta F_\Sigma) \tag{2.16}$$

makes the Einstein term canonical. By substituting it to the above equation, we obtain

$$\mathcal{L} = \int d^4\theta \left[ -3\, e^{-\frac{1}{3}G(\hat{\Phi}^\dagger,\hat{\Phi})}\Sigma^\dagger\Sigma \right] + \left\{ \int d^2\theta \Sigma^3 + \text{h.c.} \right\} + \dots \ , \tag{2.17}$$

where the ellipsis denotes terms including the graviton, gravitino, and the gravitational vector auxiliary fields. Now we focus on the bosonic terms. By using the equation of motions of the auxiliary fields $F_\Sigma$ and $\hat{F}_i$ (for a chiral field $\hat{\Phi}_i$), we express them as

$$F_\Sigma = M_P\, e^{\frac{G}{2}} + \frac{1}{3}\hat{F}_i G^i \ , \tag{2.18}$$

$$\hat{F}_i = -M_P\, e^{\frac{G}{2}} (G^{-1})_i{}^j G_j \ , \tag{2.19}$$

where

$$G^i \equiv \frac{\partial G}{\partial \hat{\phi}_i} \ , \qquad G_i \equiv \frac{\partial G}{\partial \hat{\phi}_i^\dagger} \ , \qquad G_i{}^j \equiv \frac{\partial^2 G}{\partial \hat{\phi}_i^\dagger \partial \hat{\phi}_j} \ , \tag{2.20}$$

and $G^{-1}$ is defined as the inverse matrix of $G_i{}^j$. Then we have the following scalar potential:

$$V = M_P^4 e^G \left[ G^i (G^{-1})_i{}^j G_j - 3 \right] \ . \tag{2.21}$$

Notice that the above equation contains a negative term. This in fact comes from the wrong-sign of kinetic term for the compensator. Thanks to this term, we can fine-tune the vacuum energy to obtain the vanishing cosmological constant, contrary to the case of global SUSY. This condition leads to

$$G^i (G^{-1})_i{}^j G_j = 3. \tag{2.22}$$

As in the case of ordinary SUSY, the non-zero VEVs of the auxiliary fields are regarded as order parameters of SUSY breaking in SUGRA. To relate them with physical quantities, let us evaluate the following equation:

$$\hat{F}_i^* G_i{}^j \hat{F}_j = M_P^2\, e^G G^i (G^{-1})_i{}^j G_j = 3 M_P^2\, e^G \ . \tag{2.23}$$

Here, we use the condition (2.22) in the last equality. Now notice that the right-hand side can be expressed by the gravitino mass $m_{3/2}$; from Eqs. (2.12) and (2.14), it is found that the gravitino mass is given by the lowest component of $\Sigma^3$, *i.e.*,

$$m_{3/2} = M_P e^{\frac{\langle G \rangle}{2}} \ . \tag{2.24}$$

Therefore, we have

$$m_{3/2}^2 = \frac{\langle F_i^* G_i{}^j F_j \rangle}{3 M_P^2} \ . \tag{2.25}$$

Here we restore the canonical mass dimension of the auxiliary fields. In particular, if a chiral superfield $Z$ in a hidden sector with the canonical Kähler potential acquires the $F$-term VEV $F_Z$, and if it is the dominant source of SUSY breaking, then the gravitino mass is given by

$$m_{3/2} = \frac{|F_Z|}{\sqrt{3} M_P} \ . \tag{2.26}$$

Since

$$m_{3/2} = M_P e^{\frac{\langle G \rangle}{2}} = M_P e^{\frac{\langle K \rangle}{2}} |\langle W \rangle| \ , \tag{2.27}$$

we notice that the superpotential has a non-zero VEV. This results from the fine-tuning to make the cosmological constant vanish. Now that $R$-charge of the superpotential is $+2$, we conclude that also the $R$-symmetry is broken. The breaking of the $R$-symmetry is a necessary condition for the generation of gaugino masses. In fact, the compensator field breaks both SUSY and the $R$-symmetry, since its lowest and highest components have VEVs simultaneously.

The $F$-term VEV of the compensator $\Sigma$ is also regarded as an order parameter of SUSY breaking and related with the gravitino mass. The relation is, however, actually dependent on a "gauge" with respect to the Kähler transformations defined by

$$\begin{aligned} K(\hat{\Phi}^\dagger, \hat{\Phi}) &\to K(\hat{\Phi}^\dagger, \hat{\Phi}) + \Lambda(\hat{\Phi}) + \Lambda^\dagger(\hat{\Phi}^\dagger) \ , \\ W(\hat{\Phi}) &\to e^{-\Lambda(\hat{\Phi})} \, W(\hat{\Phi}) \ , \\ \Sigma(\hat{\Phi}) &\to e^{\Lambda(\hat{\Phi})/3} \, \Sigma(\hat{\Phi}) \ . \end{aligned} \tag{2.28}$$

Under the transformations, the Lagrangian (2.11) remains unchanged, while $F_\Sigma$ changes if $\Lambda$ includes some hidden-sector fields which acquire non-zero $F$-term VEVs. On the other hand, the transformations also alter the direct contribution from the hidden-sector fields in the Kähler potential, which compensates the change in $F_\Sigma$ and thus physical quantities are not affected [132]. For example, if we adopt a set of fixing conditions presented in Ref. [133], we obtain

$$F_\Sigma = m_{3/2} \ . \tag{2.29}$$

We will take this gauge and use this relation in the following discussion.

### 2.2.2 Anomaly-mediated SUSY breaking

Now we consider the case where all of the SUSY breaking superfields in the hidden sector are charged under certain symmetries. This scenario is motivated by the dynamical SUSY breaking, as mentioned before. Let $Z$ be such a SUSY breaking chiral field which is supposed to be the only source responsible for the MSSM soft terms, for the sake of brevity. Then, with a generic form of Kähler potential, scalar particles get their masses mainly from the following effective operators

$$\frac{c_m^{ij}}{M_*^2} \int d^4\theta |Z|^2 \Phi_i^\dagger \Phi_j \ , \tag{2.30}$$

where $c_m^{ij}$ are some $\mathcal{O}(1)$ coefficients. $M_*$ denotes the mass-scale at which the operators are generated. After the SUSY breaking, the $F$-component of the field $Z$ gets a non-zero VEV. Then, the above effective operators give rise to the soft-mass terms for the scalar particles, which are of the order of

$$M_S \sim \frac{F_Z}{M_*} \ , \tag{2.31}$$

with $F_Z$ the $F$-component VEV of the field $Z$. This result can be generally obtained in the gravity mediation. With $F_Z$, the gravitino mass is expressed by

$$m_{3/2} = \frac{F_Z}{\sqrt{3} M_P} \ , \tag{2.32}$$

as discussed above. It is of the same order as the scalar masses when $M_*$ is around the Planck scale $M_P$.

A specific feature for the present case can be found when one consider the gaugino masses. They are not generated by the dimension-five operator

$$\int d^2\theta \; Z\mathrm{Tr}[W^\alpha W_\alpha] \; , \tag{2.33}$$

since the symmetry under which the superfield $Z$ is charged prohibits such an operator. Similarly, the operator like

$$\frac{c_a^{ijk}}{M_*}\int d^2\theta Z\Phi_i\Phi_j\Phi_k \; , \tag{2.34}$$

is not allowed, and thus the $A$-terms are not generated from the operator.

Instead, these terms are induced by the *anomaly mediation* mechanism, which was first proposed in Refs. [46, 121]. In this mechanism, these soft terms are induced by quantum corrections. A characteristic feature here is that they are completely finite and calculable in the low-energy effective theory. The reason is following. If quantum corrections to the soft terms are divergent, one needs to prepare counter terms to cancel them. However, such local operators do not exist as we have seen above. Therefore, we expect that loop diagrams for the radiative corrections are finite.

To evaluate the soft terms, we use a technique described in Refs. [134, 135]. In the method, the $A$-terms associated to a chiral superfield $\Phi_i$ is incorporated into a running superfield wave function $\mathcal{Z}_i(\mu)$,

$$\int d^4\theta \mathcal{Z}_i(\mu)\Phi_i^\dagger\Phi_i \; , \tag{2.35}$$

as follows:

$$\mathcal{Z}_i(\mu) = Z_i(\mu)\bigl[1 + (a_i(\mu)\theta^2 + \mathrm{h.c.})\bigr] \; , \tag{2.36}$$

where $Z_i$ is the renormalized wave function. The coefficients $a_i$ contribute to the $A$-terms. For example, when the superpotential contains the Yukawa terms,

$$W_{\mathrm{Yukawa}} = y^{ijk}\Phi_i\Phi_j\Phi_k \; , \tag{2.37}$$

the corresponding $A$-terms,

$$\mathcal{L}_{\mathrm{soft}} = -a^{ijk}\phi_i\phi_j\phi_k \; , \tag{2.38}$$

are given as

$$a^{ijk} = (a_i + a_j + a_k)y^{ijk} \; , \tag{2.39}$$

where the sum with respect to the indices is not taken. In the SUSY limit, the wave function $Z_i(\mu)$ can be defined as the two-point one-particle-irreducible (1PI) vertex function for $\Phi_i^\dagger\Phi_i$ with the scale of momentum $\sim \mu$. Now remember that the d'Alembertian $\Box$ should be accompanied by the compensator as $\Box/\Sigma^\dagger\Sigma$. Similarly, the scale $\mu$ goes with the compensator as $\mu^2/\Sigma^\dagger\Sigma$ since the scale is defined by the flowing momentum $p$ of the 1PI vertex function and $\Box = -p^2$. Therefore, the SUSY breaking effects on the renormalization function are readily taken into account with the compensator as

$$\mathcal{Z}_i(\mu) = Z_i\biggl(\frac{\mu}{\sqrt{\Sigma^\dagger\Sigma}}\biggr) \; . \tag{2.40}$$

Then, we have

$$a_i = -\frac{1}{2}\frac{m_{3/2}}{Z_i(\mu)}\left(\mu\frac{d}{d\mu}Z_i(\mu)\right) = -\gamma_i\, m_{3/2}\ , \tag{2.41}$$

where

$$\gamma_i \equiv \frac{1}{2}\mu\frac{d}{d\mu}\ln Z_i(\mu) \tag{2.42}$$

is the anomalous dimension of the field $\Phi_i$.

We can apply a similar procedure to the case of gaugino masses. To that end, we first introduce a real superfield $R(\mu)$ with rewriting the gauge field-strength term as[6]

$$\mathcal{L} = \frac{1}{16}\int d^4\theta R(\mu) W^{A\alpha}\left(\frac{1}{4}\frac{D^2}{\Box}\right)W^A_\alpha + \text{h.c.}\ , \tag{2.43}$$

with

$$R(\mu) = \frac{1}{g(\mu)^2} - \left(\frac{m_\lambda(\mu)}{g^2(\mu)}\theta^2 + \text{h.c.}\right) + \dots\ , \tag{2.44}$$

where $m_\lambda$ is the gaugino mass. Ellipsis includes terms irrelevant to the present discussion [135]. Similarly to the previous case, $R(\mu)$ is expressed by means of the compensator as

$$R(\mu) = \left[g\left(\frac{\mu}{\sqrt{\Sigma^\dagger\Sigma}}\right)\right]^{-2}\ . \tag{2.45}$$

Then, we have

$$m_\lambda = -\frac{\beta(g)}{g}m_{3/2}\ , \tag{2.46}$$

where $\beta(g)$ is the beta function of the gauge coupling $g$ defined by

$$\beta(g) \equiv \mu\frac{dg(\mu)}{d\mu}\ . \tag{2.47}$$

The above results show that there is a strong relation between the radiatively induced soft parameters and the conformal anomaly. This results from the fact that the only source of the soft parameters is the auxiliary field of the superconformal compensator. Hence, the $m_{3/2}$ dependence appears in any stuff with mass dimensions that can be regarded as a chiral superfield spurion in an equivalent manner. It even includes regulators of quantum field theories. As a result, the SUSY breaking effects in the regulators induce the soft terms at loop level.

As we have seen, a distinctive feature of this framework is that there exists an one-loop hierarchy between the gaugino masses and the soft masses of matter fields. This feature is succeeded to the SUSY model which we will describe in the next section. In addition, notice that this framework is extremely simple: we need only a dynamical SUSY braking hidden sector, and then all of the SUSY breaking effects necessary to the SUSY SMs are obtained. We do not require any extra matters like in the case of the gauge mediation [136, 137, 138].[7] In the subsequent sections, we will see that such a simple SUSY SM is actually promising from a phenomenological point of view.

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[6]We summarize the properties of the operators $D_\alpha$ and $\overline{D}^{\dot{\alpha}}$ in Appendix A.2.

[7]For a review, see Ref. [139].

## 2.3 Phenomenological aspects

### 2.3.1 Model and mass spectrum

In this subsection, we discuss a concrete high-scale SUSY model which we mainly deal with throughout the thesis. We will see that though the model is unnatural in the sense discussed in Sec. 2.1.2, it is actually quite simple, and well-motivated from a phenomenological point of view.

This model is based on the scenario described in the previous section: *i.e.*, we only assume the presence of a hidden sector in which all of the SUSY breaking fields are charged under some symmetries. As we have seen, in this scenario, typical scale of soft masses is

$$M_S \sim \frac{F_Z}{M_*} , \tag{2.48}$$

where $F_Z$ is the $F$-component VEV of the field $Z$ and $M_*$ is the mass scale where the effective operators for the soft masses are generated. Thus, all the MSSM scalar particles except the lightest Higgs boson acquire the masses of the order $M_S$. The mass of the lightest Higgs boson is fine-tuned such that it has a mass of $\sim 126$ GeV. The gaugino masses are generated by the anomaly mediation mechanism,

$$M_a \simeq \frac{b_a g_a^2}{16\pi^2} m_{3/2} , \tag{2.49}$$

and thus suppressed by a loop factor compared with the gravitino mass. Here $b_a$ denote the one-loop beta-function coefficients of the gauge coupling constants, which are given by $(b_1, b_2, b_3) = (33/5, 1, -3)$. Therefore, wino is the lightest in the gaugino sector.

Next, we discuss the $\mu$-term. Since the term is supersymmetric, a typical scale of $\mu$ is around the cut-off scale like $M_P$ or $M_{\rm GUT}$. In the following discussion, however, we just assume that the term is fine-tuned to be zero at tree-level in the superpotential. In the minimal SUSY SU(5) GUT, one can indeed realize it by finely tuning the mass parameter of the $\mathbf{5}$ and $\bar{\mathbf{5}}$ Higgs multiplets, as we will see in Sec. 3.1. Once we make the term vanish at tree-level, to all orders in parturbation theory, the quantum corrections do not disturb it as long as SUSY is respected [116]. An alternative approach is to exploit the Peccei-Quinn symmetry [79] or the $R$-symmetry to forbid the $\mu$-term. Anyway, in such a case, the $\mu$-term is generated after the SUSY breaking so that its value is naturally of the order of the soft masses. The dominant contribution to the $\mu$-term comes from the following operator:

$$c \int d^4\theta \; \Sigma^\dagger \Sigma \left(\frac{H_u}{\Sigma}\right)\left(\frac{H_d}{\Sigma}\right) = c \int d^4\theta \; \frac{\Sigma^\dagger}{\Sigma} H_u H_d . \tag{2.50}$$

with $c$ an $\mathcal{O}(1)$ parameter. This is the so-called *Giudice-Masiero mechanism* [140].[8] Note that the operator becomes a total derivative in the case of global SUSY. It induces both the $\mu$- and $B\mu$-terms:

$$\mu = c \, m_{3/2} , \qquad B\mu = c \, m_{3/2}^2 . \tag{2.51}$$

For the $B\mu$-term, there is another operator:

$$\frac{c'}{M_P^2} \int d^4\theta Z^\dagger Z H_u H_d , \tag{2.52}$$

[8]See also Refs. [141, 142].

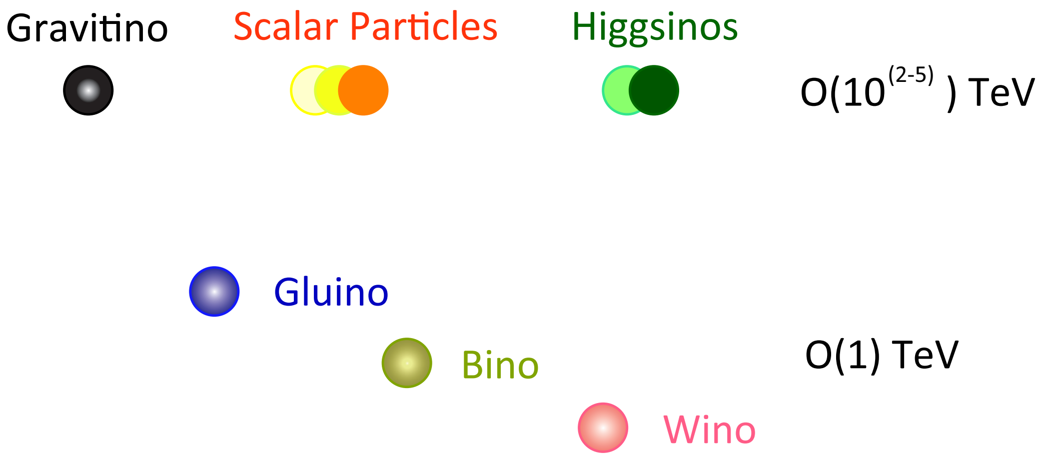


Figure 2.1: Typical mass spectrum in high-scale SUSY scenario.

which induces

$$B\mu = c' \frac{|F_Z|^2}{M_P^2} \ . \tag{2.53}$$

Notice, however, that the above operators still break the Peccei-Quinn symmetry, and thus these values can be much lower than the gravitino mass scale. We sometimes consider such possibilities in the following discussion.

When higgsinos are as heavy as gravitino, the gaugino masses receive sizable corrections at the higgsino threshold [46, 63]. The corrections are evaluated as

$$\begin{aligned}
\Delta M_1 &= \frac{3}{5}\frac{g_1^2}{16\pi^2} L \ , \\
\Delta M_2 &= \frac{g_2^2}{16\pi^2} L \ , \\
\Delta M_3 &= 0 \ ,
\end{aligned} \tag{2.54}$$

with

$$L = \mu \sin 2\beta \frac{m_A^2}{|\mu|^2 - m_A^2} \ln \frac{|\mu|^2}{m_A^2} \ . \tag{2.55}$$

As one can see, the corrections can be comparable to the anomaly mediated gaugino masses and may modify the original relation among them. In particular, bino or gluino can be the lightest gaugino in some parameter region. Therefore, in the following calculation, we will not insist on the anomaly mediation relation for the gaugino masses, and regard them as free parameters, though we expect wino is the lightest.

After all, a typical mass spectrum is as follows: sfermions and the heavy Higgs bosons have masses of the order of the gravitino mass or higher. Gauginos are lighter than gravitino by an one-loop factor. The higgsino mass lies around $m_{3/2}$, which can be much suppressed when there exist additional symmetries. The LSP is the neutral wino in general, though some effects such as the higgsino threshold corrections might change the consequence.[9] We illustrate such a mass

[9]For instance, extra particle in the high-energy region can affect the gaugino masses [143, 144, 145].

spectrum in Fig. 2.1.

From now on, we assume the sfermions are nearly degenerate in mass. The mass-scale is supposed to be around $M_S \simeq 10^2$–$10^5$ TeV, which is suggested by the 126 GeV Higgs mass and dark matter as we will see below. Models with such a mass spectrum and their phenomenology were first discussed in Refs. [46, 47, 48, 49, 50, 51]. After the earliest works, such kind of framework has been enthusiastically studied, especially after the Higgs discovery [54, 55, 143, 146, 147, 148, 149]. In this thesis, we refer to the framework as the *high-scale SUSY scenario*.[10]

### 2.3.2 126 GeV Higgs boson mass

First, we consider the predicted mass of Higgs boson in the framework, which is one of the motivations for the high-scale SUSY. From Eq. (1.30), one easily finds that the 126 GeV Higgs mass is achieved when the stop mass is sufficiently large compared with the mass of top quark. In the case of high-scale SUSY, however, we have another systematical way of calculating the Higgs mass based on the method of effective field theory. In the method, we carry out the calculation in an effective theory which consists of the SM particles, gauginos, and possibly higgsinos. The effective theory is matched with the MSSM at the SUSY breaking threshold. Then, by using the RGEs, we obtain the Higgs quartic coupling at the electroweak scale, from which one can read the mass of Higgs boson.

Let us briefly describe the computation. In the MSSM, the quadratic terms in the Higgs sector are given by (see Eq. (1.18))

$$\mathcal{L}^{(2)}_{\text{neutral}} = -(H_u^{*\alpha},\ \epsilon^{\alpha\beta}H_{d\beta}) \begin{pmatrix} |\mu|^2 + m_{H_u}^2 & B\mu \\ B\mu & |\mu|^2 + m_{H_d}^2 \end{pmatrix} \begin{pmatrix} H_{u\alpha} \\ \epsilon_{\alpha\beta}H_d^{*\beta} \end{pmatrix} , \tag{2.56}$$

where we use a convention of $\epsilon^{12} = \epsilon_{12}$ for the $\mathrm{SU}(2)_L$ indices. To achieve the correct electroweak symmetry breaking, a linear combination of the two Higgs fields,

$$H_\alpha = \sin\beta H_{u\alpha} + \cos\beta \epsilon_{\alpha\beta} H_d^{*\beta} , \tag{2.57}$$

is fine-tuned to have a mass eigenvalue of the order of the electroweak scale. The quartic coupling for the field is given by the gauge coupling constants in the case of the MSSM, as displayed in Eq. (1.18). In the effective theory below the SUSY breaking scale $M_S$, on the other hand, the Higgs potential is expressed by Eq. (1.3). The matching condition for the quartic coupling is then given as

$$\lambda_H(M_S) = \frac{1}{4}\left[g_2^2(M_S) + \frac{3}{5}g_1^2(M_S)\right]\cos^2 2\beta . \tag{2.58}$$

It corresponds to the first term in Eq. (1.30). Beyond the leading-order calculation, it is necessary to include threshold corrections at the SUSY breaking scale. The one-loop threshold effects on the Higgs quartic coupling are presented in Ref. [53]. The dominant contribution comes from the diagrams including the stop trilinear coupling:

$$\delta\lambda_H(M_S) \simeq -\frac{6y_t^4}{(4\pi)^2}\left[\frac{X_t^2}{m_{\tilde{t}_1}m_{\tilde{t}_2}}\left(1 - \frac{X_t^2}{12 m_{\tilde{t}_1}m_{\tilde{t}_2}}\right)\right] , \tag{2.59}$$

[10] Indeed, there are a variety of names which are given to this type of framework: *pure gravity mediation* [54, 55], *simply unnatural supersymmetry* [143], *spread supersymmetry* [146], *mini-split* [148], and so on.

where $X_t \equiv A_t - \mu \cot\beta$. The effects correspond to the last term in Eq. (1.30). From the relations, we see that the Higgs mass is essentially determined by only three parameters, $M_S$, $\mu$ and $\tan\beta$ (notice that in the high-scale SUSY scenario, $A$-terms are generated by the quantum effects, and thus negligible in the present calculation). Mass spectrum above and below the SUSY breaking scale affects the prediction only at the higher-order of perturbation throughout the RGEs and the threshold corrections, respectively. In this sense, one can make a robust prediction of the Higgs mass in terms of these three parameters. The two-loop RGEs and one-loop threshold corrections at the electroweak scale are also presented in Ref. [53].

A calculation of the Higgs mass with the method in the high-scale SUSY scenario is carried out in Refs. [53, 54, 55]. The authors in the references have found that the 126 GeV Higgs mass can be achieved with $M_S = \mathcal{O}(10^1 - 10^5)$ TeV and an $\mathcal{O}(1)$ value of $\tan\beta$. Motivated by this observation, we mainly consider such a parameter region in the following discussion.

In this set-up, the degree of fine-tuning given in Sec. 2.1.2 is at the level of $\Delta^{-1} = 10^4 - 10^{12}$. Nevertheless, we will not assess this meanings in the following discussion.

### 2.3.3 FCNC and CP violating phenomena

Although it is difficult to prove the high-scale SUSY scenario at collider experiments, the low-energy precision experiments might catch up the SUSY signature. Without any flavor symmetries, soft SUSY braking parameters in general give rise to extra sources of flavor and/or CP violation [52]. These effects, such as the *flavor-changing neutral currents (FCNC)* and the *electric dipole moments (EDMs)*, are suppressed by sfermion masses, so high-scale SUSY models do not conflict with the results from the current flavor and CP experiments. Interestingly, upcoming experiments for flavor and CP violating processes are likely to reach the $\mathcal{O}(10^2 - 10^3)$ TeV scale [150, 151, 152], which is favored from the viewpoint of the 126 GeV Higgs boson mass as explained above. Such experiments include the measurements of the electric dipole moments, the meson-antimeson oscillation and the charged lepton flavor violating (LFV) processes. In this section we study the current limits on the SUSY breaking scale from these measurements, and discuss the future prospects of these searches.

Among the low-energy precision observables, the EDMs provide a sensitive probe for the signature of the SUSY particles [150, 151]. Since the EDMs induced by the CP phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix are considerably below the sensitivities of the present and near future experiments [153, 154], the EDM-measurement is free from the SM background, thus provides a clean environment to detect a sign of high-energy physics beyond the SM.

The effects of SUSY particles which give rise to the EDMs are expressed in terms of the flavor-conserving CP-violating effective operators. Up to dimension five, they are written as

$$\begin{aligned}\mathcal{L}_{\text{CP}} = &- \sum_{f=u,d,s,e} m_f \bar{f} i\theta_f \gamma_5 f + \theta_G \frac{\alpha_s}{8\pi} G^A_{\mu\nu} \widetilde{G}^{A\mu\nu} \\ &- \frac{i}{2} \sum_{f=u,d,s,e} d_f \bar{f} \sigma^{\mu\nu} \gamma_5 f F_{\mu\nu} - \frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q \bar{q} g_s \sigma^{\mu\nu} \gamma_5 T^A q G^A_{\mu\nu} \ . \end{aligned} \tag{2.60}$$

Here, $m_f$ are the fermion masses, $g_s$ is the strong coupling constant ($\alpha_s = g_s^2/4\pi$), and $T^A$ are the generators of the $\mathrm{SU}(3)_C$. $F_{\mu\nu}$ and $G^A_{\mu\nu}$ are the field strength tensors of photon and gluon, and their dual fields are defined by, *e.g.*, $\widetilde{G}^A_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^{A\rho\sigma}$ with $\epsilon^{0123} = +1$. The

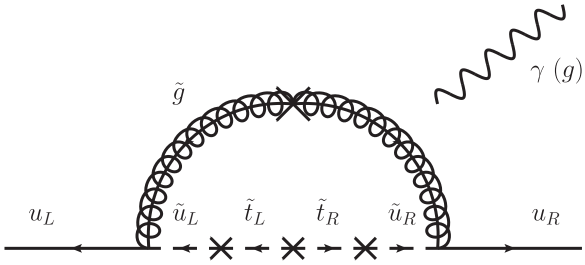


Figure 2.2: An example of the dominant diagram contributing to the EDMs and CEDMs of light quarks in the presence of the squark flavor mixing.

second term of the above expression is the effective QCD $\theta$ term, which is connected with the first term through the chiral rotation. These two terms are suppressed on the assumption of the Peccei-Quinn symmetry [79] to solve the strong CP problem, so we neglect them in the following discussion. The third and fourth terms represent the EDMs and the *chromoelectric dipole moments (CEDMs)* for light fermions, respectively. They are dimension-five operators, and thus quite sensitive to the TeV-scale physics beyond the SM.[11] Since these operators require a chirality flip, they are proportional to the corresponding fermion masses if they are induced by the flavor-conserving processes.

With a generic flavor structure in the sfermion mass matrices, on the other hand, the mass terms of the third generation fermions can flip the chirality [155, 156] accompanied by the flavor-violation. As it is possible for the flavor-violation to be sizable in the high-scale SUSY scenario [151], the dominant contribution to the EDMs and CEDMs of light fermions may originate from the flavor-violating processes. An example of such processes is illustrated in Fig. 2.2.

Let us evaluate the contribution. The EDM $d_u$ and CEDM $\tilde{d}_u$ of up quark are, for instance, approximately give as

$$\begin{aligned} d_u &\simeq -\frac{4}{3}\frac{\alpha_3}{4\pi}eQ_u\frac{m_t}{M_S^4}\mathrm{Im}\big[\mu M_3\cot\beta\,\delta_{13}^{\widetilde{Q}_L}\delta_{13}^{\widetilde{u}_R *}\big]\ , \\ \tilde{d}_u &\simeq 6\frac{\alpha_3}{4\pi}\frac{m_t}{M_S^4}\ln\left(\frac{M_s}{|M_3|}\right)\mathrm{Im}\big[\mu M_3\cot\beta\,\delta_{13}^{\widetilde{Q}_L}\delta_{13}^{\widetilde{u}_R *}\big]\ , \end{aligned} \tag{2.61}$$

with $eQ_u$ the charge of up quark. $\delta_{13}^{\widetilde{Q}_L}$ and $\delta_{13}^{\widetilde{u}_R *}$, which represent the flavor changing, are defined in Eq. (4.47). Similar expressions hold for other fermions. Notice that both the left-handed and right-handed squark mixings are required to generate the EDMs and CEDMs. In the case of the high-scale SUSY scenario, however, one needs to pay particular attention to the calculation of the diagrams like that in Fig. 2.2; since there exists a large difference between the mass scales of scalar and fermionic SUSY particles, large logarithmic factors appear and may spoil the perturbation theory. To evade the difficulties, we need to evaluate the effective operators by means of the renormalization-group equations (RGEs). The detail of the calculation is discussed in Ref. [158].

[11] In addition, the contribution of the dimension-six Weinberg operator [157] might be comparable to that of EDMs and CEDMs. In the present case, however, the operator is induced at $\mathcal{O}(\alpha_s^2)$, and thus can be neglected in the leading order calculation.

The EDMs and CEDMs of quarks are constrained by measurements of, *e.g.*, the neutron EDM $d_n$.[12] By using the method of the QCD sum rules, the authors in Ref. [159] estimate the neutron EDM as follows:[13]

$$d_n = 0.79 d_d - 0.20 d_u + e(0.30\tilde{d}_u + 0.59\tilde{d}_d) \ . \tag{2.62}$$

By using the relation, we find that the current experimental bound on the neutron EDM, $|d_n| < 2.9 \times 10^{-26}\ e \cdot \mathrm{cm}$ [161], has already excluded the squark mass scale well above 10 TeV in the presence of sizable flavor violation and $CP$-phases.[14] Future experiments of the neutron EDM, which aim at $d_n \sim 5 \times 10^{-28}\ e \cdot \mathrm{cm}$ [162], are expected to reach $\mathcal{O}(10^2)$ TeV. It covers a lot of the region favored from the high-scale SUSY scenario compatible with the 126 GeV Higgs mass and the existence of $\mathcal{O}(1)$ TeV gauginos. Hence, the EDM experiments are quite promising and expected to catch up the signature of SUSY particles.

For the electron EDM, only the bino exchanging process is enhanced by the mass of tau lepton. This is because winos do not couple to the right-handed electron so another chirality flip is required for the wino exchanging process to occur. Therefore, this process is suppressed by the electron mass. By evaluating the process, we find that the current experimental limit $d_e < 8.7 \times 10^{-29}\ e \cdot \mathrm{cm}$ [163] has already probed the slepton masses of well above $\mathcal{O}(10)$ TeV. If future experiments reach the level of $d_e \sim 10^{-30}\ e \cdot \mathrm{cm}$, the slepton masses of $\mathcal{O}(10^2)$ TeV may be probed [150, 151, 152].

When higgsinos are much lighter than the scalar particles, on the other hand, the dominant contribution to the fermion EDMs comes from the two-loop Barr-Zee type diagrams where the charged winos and higgsinos run in the loop [50, 164]. The contribution remains sizable even if sfermions are considerably heavy, as long as higgsinos are kept light. The current experimental limit $d_e < 8.7 \times 10^{-29}\ e \cdot \mathrm{cm}$ [163] leads to bound on the masses of winos and higgsinos of $\mathcal{O}(1)$ TeV [164].

Another important observable is the $\Delta F = 2$ meson mixings, which is a traditional probe of the new physics contribution. The dominant contribution to the $\Delta F = 2$ meson mixings come from the box diagram illustrated in Fig. 2.3. The contribution is expressed in terms of the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = \sum_{A=1}^{5} C_A \mathcal{O}_A + \sum_{A=1}^{3} \widetilde{C}_A \widetilde{\mathcal{O}}_A \ , \tag{2.63}$$

[12]EDMs of diamagnetic atoms, such as the EDM of mercury, also provide constraints on the squark flavor violation, which are comparable to those from the neutron EDM within the theoretical error.

[13] When one imposes the Peccei-Quinn symmetry, the strange CEDM contribution to the neutron EDM completely vanishes in the case of the sum-rule computation. This may indicate that the sum-rule calculation does not include the strange-quark contribution appropriately. In fact, the contribution is expected to be sizable from the estimation based on the chiral perturbation theory [160]. At this moment, both methods have large uncertainty and no consensus has been reached yet.

[14] In the case of the minimal flavor violation, on the other hand, the predicted neutron EDM lies around $d_n \simeq 10^{-30}\ e \cdot \mathrm{cm}$ for $M_S = 10^2$ TeV, which is much below the current experimental limit.

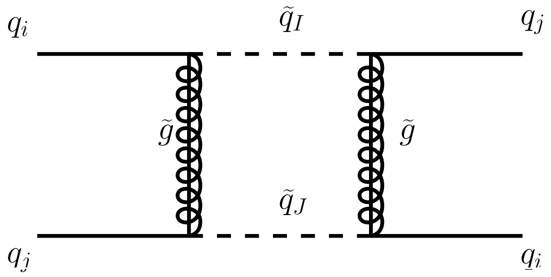


Figure 2.3: Dominant diagram contributing to the $\Delta F = 2$ meson mixings in the presence of the squark flavor mixing.

where the effective operators $\mathcal{O}_A$ and $\widetilde{\mathcal{O}}_A$ are defined by

$$\begin{aligned}
\mathcal{O}_1 &\equiv (\overline{q}_{Lia}\gamma_\mu q^a_{Lj})(\overline{q}_{Lib}\gamma^\mu q^b_{Lj}) \ , \\
\mathcal{O}_2 &\equiv (\overline{q}_{Ria} q^a_{Lj})(\overline{q}_{Rib} q^b_{Lj}) \ , \\
\mathcal{O}_3 &\equiv (\overline{q}_{Ria} q^b_{Lj})(\overline{q}_{Rib} q^a_{Lj}) \ , \\
\mathcal{O}_4 &\equiv (\overline{q}_{Ria} q^a_{Lj})(\overline{q}_{Lib} q^b_{Rj}) \ , \\
\mathcal{O}_5 &\equiv (\overline{q}_{Ria} q^b_{Lj})(\overline{q}_{Lib} q^a_{Rj}) \ ,
\end{aligned} \tag{2.64}$$

and $\widetilde{\mathcal{O}}_A$ are obtained by exchanging the left-(right-)handed quarks in $\mathcal{O}_A$ by the right-(left-)handed ones. When the squark masses are much heavier than the gluino mass, $M_S \gg M_3$, the Wilson coefficients of the effective operators are given as follows:

$$\begin{aligned}
C_1 &= -\frac{11\alpha_3^2}{36} H(m^2_{\widetilde{q}_{LI}}, m^2_{\widetilde{q}_{LJ}}) (R^\dagger_{\widetilde{q}_L})_{iI} (R_{\widetilde{q}_L})_{Ij} (R^\dagger_{\widetilde{q}_L})_{iJ} (R_{\widetilde{q}_L})_{Jj} \ , \\
C_4 &= +\frac{\alpha_3^2}{3} H(m^2_{\widetilde{q}_{RI}}, m^2_{\widetilde{q}_{LJ}}) (R^\dagger_{\widetilde{q}_R})_{iI} (R_{\widetilde{q}_L})_{Rj} (R^\dagger_{\widetilde{q}_L})_{iJ} (R_{\widetilde{q}_L})_{Jj} \ , \\
C_5 &= -\frac{5\alpha_3^2}{9} H(m^2_{\widetilde{q}_{RI}}, m^2_{\widetilde{q}_{LJ}}) (R^\dagger_{\widetilde{q}_R})_{iI} (R_{\widetilde{q}_L})_{Rj} (R^\dagger_{\widetilde{q}_L})_{iJ} (R_{\widetilde{q}_L})_{Jj} \ , \\
\widetilde{C}_1 &= -\frac{11\alpha_3^2}{36} H(m^2_{\widetilde{q}_{RI}}, m^2_{\widetilde{q}_{RJ}}) (R^\dagger_{\widetilde{q}_R})_{iI} (R_{\widetilde{q}_R})_{Ij} (R^\dagger_{\widetilde{q}_R})_{iJ} (R_{\widetilde{q}_R})_{Jj} \ ,
\end{aligned} \tag{2.65}$$

and the other coefficients are negligible in the high-scale SUSY scenario. Here, the mass function $H(m_1^2, m_2^2)$ is defined by

$$H(m_1^2, m_2^2) \equiv \frac{1}{m_1^2 - m_2^2} \ln\left(\frac{m_1^2}{m_2^2}\right) \ . \tag{2.66}$$

From the above expression, it turns out that the contribution does not depend on the gluino mass in the limit of $M_S \gg M_3$, which is expected in the high-scale SUSY scenario.

To obtain the constraints on the effective interactions from the low-energy experiments, we exploit the results in Refs. [165, 166, 167], where generic constraints on new-physics contributions to $\Delta F = 2$ processes are presented. As discussed in Refs. [150, 151, 152], the constraints from the $K^0$-$\bar{K}^0$ mixing parameters, especially $\epsilon_K$, in general provide the severest limits on the SUSY breaking scale. While the prediction quite depends on the flavor structure in squark mass matrices, the present constraints can reach almost $\mathcal{O}(10^3)$ TeV for $M_S$, especially when $\delta_{12}^{\widetilde{Q}_L}$ and $\delta_{12}^{\widetilde{d}_R}$ are sizable, which are again defined in Eq. (4.47). The sensitivity is, however, not expected to be improved because of the large uncertainty of the SM prediction. Constraints from other $\Delta F = 2$ processes, such as the $D^0$-$\bar{D}^0$ mixing, are lower than that from $K^0$-$\bar{K}^0$ by about an order of magnitude, though they are to be improved by an order of magnitude in future experiments.

Finally, we briefly comment on the charged lepton flavor violating processes. Such processes include $\mu \to e\gamma$, $\mu \to 3e$, the $\mu \to e$ conversion, and so on. For instance, the current bound on the $\mu \to e\gamma$ process from the MEG experiment, $\mathrm{BR}(\mu \to e\gamma) < 5.7 \times 10^{-13}$, gives a limit on $M_S$ of $\mathcal{O}(10)$ TeV with sizable lepton flavor violation. The most promising observable for the charged lepton flavor violation in future experiments is, on the other hand, the $\mu \to e$ conversion in Al, which may probe the SUSY scale of $\mathcal{O}(10^2)$ TeV.

In consequence, although the SUSY flavor and CP problems are avoided in the high-scale SUSY scenario at present, future low-energy precision experiments might catch a signal of SUSY. Therefore, it is important not only to improve the experimental sensitivities but also to reduce the theoretical uncertainty in the prediction for the precision observables.

### 2.3.4 Dark Matter

A quite attractive feature of the high-scale SUSY is that this scenario offers a possible candidate for dark matter in the Universe. As discussed above, in this case the neutral wino generally becomes the LSP and may explain the present dark matter density even if the scalar particles have masses much heavier than the electroweak scale. This is because the wino LSP can annihilate (and co-annihilate with the charged winos) very efficiently through the electroweak interactions and thus the masses of the sfermions are almost irrelevant to the annihilation cross sections. This is contrary to the case of the bino dark matter in, for instance, the CMSSM, where the bino annihilates via the sfermion exchanging processes and thus the annihilation cross section is suppressed when sfermions are quite heavy. In addition, the annihilation cross section is significantly enhanced by the non-perturbative effects called *Sommerfeld enhancement*, which was first pointed out in Refs. [168, 169]. Taking the effects into account, one finds that wino dark matter with a mass of $M_2 \simeq 2.7-3.1$ TeV explains the observed dark matter density [170]. Since the wino mass is suppressed by one-loop factor compared with the gravitino mass, a wino mass of $\sim 3$ TeV suggests $m_{3/2} \sim 10^3$ TeV, which is consistent with the scale favored from the Higgs mass. For relatively light wino dark matter, on the other hand, the non-thermal production via the late time decay of gravitino can be invoked to provide the correct abundance of dark matter [63, 64].

The wino dark matter can be searched both directly and indirectly. We first consider the direct detection of wino dark matter. In the dark matter direct detection experiments, one searches for the scattering signal of dark matter with nuclei on the earth. The experimental constraints are provided on the dark matter-nucleon scattering cross sections, which are induced by the effective interactions of dark matter with quarks and gluon. In the case of pure wino dark matter, dominant contribution comes from the one-loop wino-quark interactions (Fig. 2.4) and the two-loop wino-gluon interactions (Fig. 2.5). Note that the gluon contribution is comparable to the quark contribution even though it is generated in higher-loop processes [171, 172, 173, 174]. Briefly speaking, this enhancement originates from the large gluon contribution to the mass of nucleon. By evaluating these diagrams, we obtain the spin-independent elastic scattering cross cross sections of wino dark matter with a nucleon $\sigma_{\mathrm{SI}} \sim 10^{-47}$ cm$^2$ [171, 172, 173, 174]. This

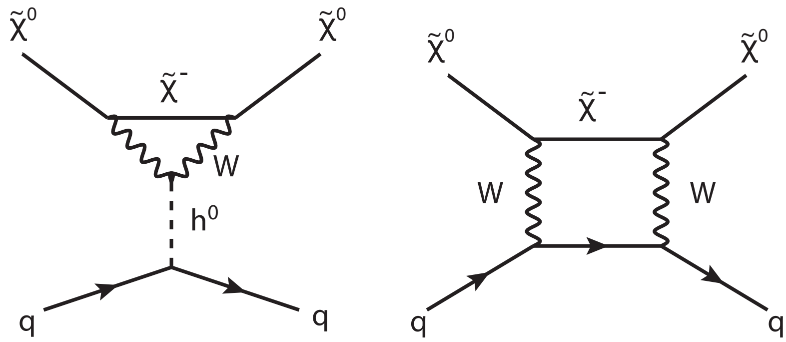


Figure 2.4: One-loop diagrams which induce wino-quark interactions.

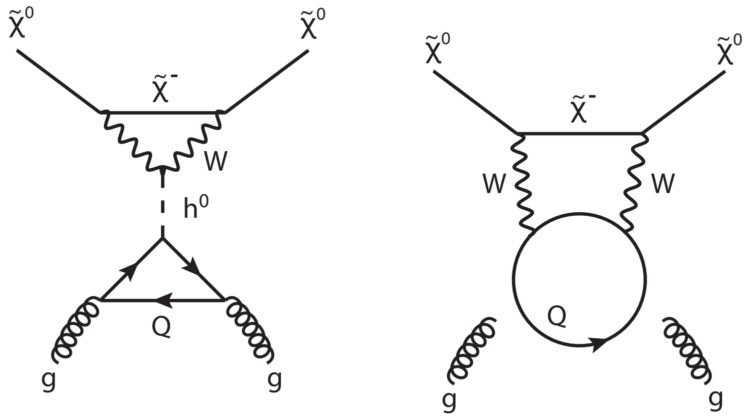


Figure 2.5: Two-loop diagrams which induce wino-gluon interactions.

value is far below the current experimental limits, $\sigma_{\rm SI} < 7.6 \times 10^{-46}$ cm$^2$ presented by the LUX experiment [175]. When higgsino mass $\mu$ is relatively small, the wino-like dark matter comes to couple with Higgs boson at tree-level, which may enhance the scattering cross sections [174].

The indirect search of wino dark matter is more promising since it has large annihilation cross sections as mentioned above. In fact, the dark matter is now being constrained by this kind of observations, with the strong bounds coming from the Fermi-LAT Collaboration [176] and the H.E.S.S. Collaboration [177]. The Fermi-LAT Collaboration is looking for the continuum $\gamma$-ray from the dwarf spheroidal satellite galaxies of the Milky Way, which is caused by the dark matter annihilation to the charged particles like $W^+W^-$. This search can give a constraint on the low mass region of wino dark matter. The H.E.S.S. Collaboration, on the other hand, searches for gamma-ray lines coming from the Galactic Center. By using the results, the authors in Refs. [178, 179] insist that a wide mass range of wino dark matter has been excluded. The consequence is, however, quite dependent on the dark matter density profile which one uses to derive the constraints. For example, if one uses the Burkert profile [180], not the NFW profile [181] used in the above references, the excluded region is relaxed to be $2.2 < M_2 < 2.6$ TeV. Future experiments such as the Cherenkov Telescope Array (CTA) experiment [182] are expected to settle the situation.

# Chapter 3

# SUSY GUT in high-scale SUSY

From now on, we study the high-scale SUSY scenario in the context of the grand unified theory.[1] Throughout the thesis, we mainly consider its simplest version, so-called the minimal SUSY SU(5) GUT, which is introduced in Sec. 3.1.

The grand unified theories introduce new superheavy particles lying around the unification scale. It is these particles that induce proton decay, and thus we need to estimate their masses to evaluate the proton decay rate. For this purpose, we use an indirect method proposed in Refs. [186, 187]. In this method, the masses of the GUT-scale particles are inferred from the threshold corrections required to realize the gauge coupling unification. The resultant prediction for the mass spectrum in the case of high-scale SUSY scenario turns out to be different from that in the low-scale one [188]; the results are given in Sec. 3.2. In particular, we will see that with heavy scalars the color-triplet Higgs multiplets, which are the SU(5) partners of the MSSM Higgs superfields, can be as heavy as $10^{16}$ GeV. Moreover, it turns out that the GUT scale tends to be slightly lower in the high-scale SUSY scenario than that in the low-scale one. The consequences of the observation on proton decay will be discussed in Chap. 4.

## 3.1 Minimal SUSY SU(5) GUT

To begin with, we review the *minimal SUSY SU(5) GUT* [32, 33] first formulated by N. Sakai, S. Dimopoulos and H. Georgi in 1981. Just like the Georgi-Glashow SU(5) model [13], the SM fermions, as well as their superpartners, are embedded in a $\bar{\mathbf{5}} \oplus \mathbf{10}$ representation of the SU(5) unified gauge group, which is found to be free from anomalies. The multiplets $\Phi$ and $\Psi$, which are the chiral superfields of $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations, respectively, contain the MSSM chiral superfields as

$$\Phi = \begin{pmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \\ E \\ -N \end{pmatrix} , \qquad \Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{U}_3 & -\bar{U}_2 & U^1 & D^1 \\ -\bar{U}_3 & 0 & \bar{U}_1 & U^2 & D^2 \\ \bar{U}_2 & -\bar{U}_1 & 0 & U^3 & D^3 \\ -U^1 & -U^2 & -U^3 & 0 & \bar{E} \\ -D^1 & -D^2 & -D^3 & -\bar{E} & 0 \end{pmatrix} , \tag{3.1}$$

[1] For a review of the GUTs, see Refs. [183, 184, 185].

with

$$L = \begin{pmatrix} N \\ E \end{pmatrix} , \qquad Q^a = \begin{pmatrix} U^a \\ D^a \end{pmatrix} . \tag{3.2}$$

Here, $a = 1, 2, 3$ denotes the color index. The MSSM Higgs superfields, on the other hand, are embedded into a pair of $\mathbf{5}$ and $\bar{\mathbf{5}}$ superfields accompanied with the new Higgs superfields $H_C^a$ and $\bar{H}_{Ca}$ called the *color-triplet Higgs multiplets*:

$$H = \begin{pmatrix} H_C^1 \\ H_C^2 \\ H_C^3 \\ H_u^+ \\ H_u^0 \end{pmatrix} , \qquad \bar{H} = \begin{pmatrix} \bar{H}_{C1} \\ \bar{H}_{C2} \\ \bar{H}_{C3} \\ H_d^- \\ -H_d^0 \end{pmatrix} , \tag{3.3}$$

where the last two components are corresponding to the MSSM Higgs superfields,

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} , \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} . \tag{3.4}$$

The color-triplet Higgs multiplets give rise to baryon- and lepton-number violating interactions and play a crucial role in proton decay, as we shall see below.

The SU(5) gauge theory contains the 24 gauge bosons and each of them corresponds to a component of a vector superfield, $\mathcal{V}^A$, with $A = 1, \dots, 24$ indicating the gauge index. By exploiting the fundamental representation of the SU(5) generators, $T^A$, we define a $5 \times 5$ matrix of the vector superfields: $\mathcal{V} \equiv \mathcal{V}^A T^A$. The components of the matrix are written as

$$\mathcal{V} = \frac{1}{\sqrt{2}} \begin{pmatrix} & & & X^{\dagger 1} & Y^{\dagger 1} \\ & G - \frac{2}{\sqrt{30}} B & & X^{\dagger 2} & Y^{\dagger 2} \\ & & & X^{\dagger 3} & Y^{\dagger 3} \\ X_1 & X_2 & X_3 & \frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{1}{\sqrt{2}} W^3 + \frac{3}{\sqrt{30}} B \end{pmatrix} , \tag{3.5}$$

where each component is expressed by the same symbol as that used for the corresponding gauge field. We collectively refer to $X_a$, $Y_a$, and their Hermitian conjugates as *$X$-bosons*, and use the following notation for them:

$$(X)_a^\alpha = \begin{pmatrix} X_a^1 \\ X_a^2 \end{pmatrix} \equiv \begin{pmatrix} X_a \\ Y_a \end{pmatrix} . \tag{3.6}$$

Here $\alpha, \beta, \dots$ denote the $\mathrm{SU}(2)_L$ indices. They can also induce the baryon- and lepton-number violation.

The unified gauge group SU(5) is spontaneously broken by the VEV of the adjoint Higgs multiplet $\Sigma^A$ $(A = 1, \dots, 24)$ to $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. Again, we define $\Sigma \equiv \Sigma^A T^A$ and write its components as

$$\Sigma = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(3^*,2)} & \Sigma_3 \end{pmatrix} + \frac{1}{2\sqrt{15}} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \Sigma_{24} . \tag{3.7}$$

The superpotential of the Higgs sector in the minimal SU(5) SUSY GUT is given by

$$W_{\rm Higgs} = \frac{1}{3}\lambda_\Sigma {\rm Tr}\Sigma^3 + \frac{1}{2}m_\Sigma {\rm Tr}\Sigma^2 + \lambda_H \bar{H}\Sigma H + m_H \bar{H}H\ . \tag{3.8}$$

Now suppose that the adjoint Higgs field gets the VEV in the direction to

$$\langle\Sigma\rangle = V\cdot {\rm diag}(2,2,2,-3,-3)\ , \tag{3.9}$$

so that the SU(5) gauge group is broken to the SM $\mathrm{SU}(3)_C\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ gauge groups. Further, we assume that the VEV does not break supersymmetry. To derive the condition for that, let us investigate the potential in the radial direction to the VEV, *i.e.*, $\Sigma = \sigma{\rm diag}(2,2,2,-3,-3)$, with which the superpotential $W_{\rm Higgs}$ is written as

$$W_{\rm Higgs}(\sigma) = -10\lambda_\Sigma\sigma^3 + 15m_\Sigma\sigma^2\ . \tag{3.10}$$

Then the above assumption reduces to $\partial W_{\rm Higgs}/\partial\sigma = 0$, which implies $V = \langle\sigma\rangle = m_\Sigma/\lambda_\Sigma$. In the broken phase, the superpotential is written as

$$\begin{aligned}
W_{\rm Higgs} =& \frac{1}{3}\lambda_\Sigma{\rm Tr}\big(\Sigma_8^3\big) + \frac{1}{3}\lambda_\Sigma{\rm Tr}\big(\Sigma_3^3\big) - \frac{1}{12\sqrt{15}}\lambda_\Sigma\Sigma_{24}^3 \\
&+ \frac{1}{\sqrt{15}}\lambda_\Sigma\Sigma_{24}{\rm Tr}\big(\Sigma_8^2\big) - \frac{3}{2\sqrt{15}}\lambda_\Sigma\Sigma_{24}{\rm Tr}\big(\Sigma_3^2\big) \\
&+ M_{\Sigma_8}{\rm Tr}\big(\Sigma_8^2\big) + M_{\Sigma_3}{\rm Tr}\big(\Sigma_3^2\big) + \frac{1}{2}M_{\Sigma_{24}}\Sigma_{24}^2 + M_{H_C}\bar{H}_{Ca}H_C^a\ ,
\end{aligned} \tag{3.11}$$

where

$$M_\Sigma \equiv M_{\Sigma_8} = M_{\Sigma_3} = \frac{5}{2}\lambda_\Sigma V\ , \qquad M_{\Sigma_{24}} = \frac{1}{2}\lambda_\Sigma V\ , \qquad M_{H_C} = 5\lambda_H V\ , \tag{3.12}$$

and the parameter $m_H$ is fine-tuned as $m_H = 3\lambda_H V$ in order to realize the doublet-triplet mass splitting in $H$ and $\bar{H}$. By looking into the gauge interactions of the adjoint Higgs fields, we also find that the $X$-boson mass is given by $M_X = 5\sqrt{2}g_5 V$ with $g_5$ the unified gauge coupling constant. The components $\Sigma_{(3^*,2)}$ and $\Sigma_{(3,2)}$ become the longitudinal component of the $X$-bosons, and thus do not show up as physical states.

On the assumption of the $R$-parity conservation[2], we have the following Yukawa terms described by the superpotential:

$$W_{\rm Yukawa} = \frac{1}{4}h^{ij}\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\Psi_i^{\hat{a}\hat{b}}\Psi_j^{\hat{c}\hat{d}}H^{\hat{e}} - \sqrt{2}f^{ij}\Psi_i^{\hat{a}\hat{b}}\Phi_{j\hat{a}}\bar{H}_{\hat{b}}\ , \tag{3.13}$$

where $i,j = 1,2,3$ and $\hat{a},\hat{b},\cdots = 1$–$5$ represent the generations and the SU(5) indices, respectively; $\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}$ is the totally antisymmetric tensor with $\epsilon_{12345} = 1$. The Yukawa couplings $h^{ij}$ and $f^{ij}$ in Eq. (3.13) have redundant degrees of freedom, most of which are eliminated by the field re-definition of $\Psi$ and $\Phi$. As $h^{ij}$ is a symmetric matrix, $h^{ij}$ and $f^{ij}$ have six and nine complex degrees of freedom, respectively. The field redefinition forms U(3)$\otimes$U(3) transformation group, and thus the physical degrees of freedom turn out to be $(12+18) - 9\times 2 = 12$. Among them, six is for quark mass eigenvalues and four is for the CKM matrix elements, so we have two additional phases [189].

[2] Especially, terms like $\Psi\Phi\Phi$ and $H\Phi$ are forbidden, which yield the $R$-parity violating terms in Eqs. (1.13) and (1.14).

To demonstrate the above discussion explicitly, we rewrite the matrices $h^{ij}$ and $f^{ij}$ by changing the matter basis. First, we diagonalize $f^{ij}$ using two unitary matrices $U_1$ and $U_2$ as

$$\hat{f} = U_1^\dagger f U_2 \ , \tag{3.14}$$

with $\hat{f}$ being diagonal and non-negative; $\hat{f}^{ij} = \hat{f}_i \delta_{ij}$. It can be realized by the field redefinition

$$\Psi = U_1^* \Psi', \qquad \Phi = U_2 \Phi' \ , \tag{3.15}$$

where the primed fields represent the new basis. This redefinition changes $h^{ij}$ into

$$h \to h' = U_1^\dagger h U_1^* \ . \tag{3.16}$$

One easily finds that $h'$ is symmetric, which follows from $h^T = h$. Therefore, it can be diagonalized by a unitary matrix $U_3$ as

$$\hat{h} = U_3^* h' U_3^\dagger \ , \tag{3.17}$$

where $\hat{h}^{ij} = \hat{h}_i \delta_{ij}$ is a diagonal and non-negative matrix. Now let us explicitly separate out the phase factors of the matrix elements of $U_3$ as

$$(U_3)_{ij} = e^{i\phi_{ij}} u_{ij} \ , \qquad (u_{ij} \in \mathbb{R}) \ . \tag{3.18}$$

Here we do not take summation over the indices $i, j$. Further, by using diagonal phase matrices defined by

$$U_4 = \begin{pmatrix} e^{i\phi_{11}} & & \\ & e^{i\phi_{21}} & \\ & & e^{i\phi_{31}} \end{pmatrix}, \qquad U_5 = \begin{pmatrix} 1 & & \\ & e^{i(\phi_{11}-\phi_{12})} & \\ & & e^{i(\phi_{11}-\phi_{13})} \end{pmatrix} , \tag{3.19}$$

we eliminate the phases from the first row and column of $U_3$ as

$$V = U_4^\dagger U_3 U_5 = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & * & * \\ u_{31} & * & * \end{pmatrix} . \tag{3.20}$$

Then, we have

$$h' = U_3^T \hat{h} U_3 = (U_4 V U_5^\dagger)^T \hat{h} (U_4 V U_5^\dagger) = U_5^* V^T U_4^2 \hat{h} V U_5^* \ . \tag{3.21}$$

Let us define a diagonal matrix $P$ by

$$U_4^2 = e^{2i\phi} P \ , \qquad \phi \equiv \sum_i \phi_{i1} \ . \tag{3.22}$$

Then, $\det P = 1$. The phase factor $e^{i\phi}$ as well as the phase matrix $U_5$ can be absorbed by the field redefinition

$$\Psi' = e^{-i\phi} U_5 \Psi'' \ , \qquad \Phi' = e^{i\phi} U_5^* \Phi'' \ , \tag{3.23}$$

without changing the diagonalized matrix $\hat{f}$ since $\hat{f}$ and $U_5$ are diagonal. After the process, $h'$ leads to $h' \to V^T P \hat{h} V$ and Eq. (3.13) is rewritten as follows:

$$W_{\text{Yukawa}} = \frac{1}{4} (V^T P \hat{h} V)^{ij} \epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}} \Psi_i^{\hat{a}\hat{b}} \Psi_j^{\hat{c}\hat{d}} H^{\hat{e}} - \sqrt{2} \hat{f}_i \Psi_i^{\hat{a}\hat{b}} \Phi_{i\hat{a}} \bar{H}_{\hat{b}} \ . \tag{3.24}$$

As we will see soon, the second term gives rise to the diagonalized Yukawa terms for down-type quarks and leptons with the same coupling constant $\hat{f}_i$. It indicates a specific prediction of the minimal SU(5) GUT, *i.e.*,

$$f_{d_i}(M_{\rm GUT}) = f_{e_i}(M_{\rm GUT}) \ , \tag{3.25}$$

with $M_{\rm GUT} \simeq 10^{16}$ GeV. Let us count the degrees of freedom of the matrices $V$ and $P$. Since $U_3$ is a $3 \times 3$ unitary matrix and we have removed five phases $U_4$ and $U_5$ from it, $V$ includes $9 - 5 = 4$ degrees of freedom. $P$ has two phases since $\det P = 1$. So we confirm the discussion presented in the previous paragraph.

For later use, we rotate the field $\Psi_i$ as $\Psi \to V^\dagger \Psi$ to diagonalize the coupling of the first term in $W_{\rm Yukawa}$. Then, it leads to

$$W_{\rm Yukawa} = \frac{1}{4}(P\hat{h})^{ij}\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\Psi_i^{\hat{a}\hat{b}}\Psi_j^{\hat{c}\hat{d}}H^{\hat{e}} - \sqrt{2}(V^*\hat{f})^{ij}\Psi_i^{\hat{a}\hat{b}}\Phi_{j\hat{a}}\bar{H}_{\hat{b}} \ . \tag{3.26}$$

Now we match the above superpotential to that in the MSSM. Generally speaking, the generation basis of the MSSM superfields is different from that of the SU(5) superfields $\Psi_i$ and $\Phi_i$. To take the difference into account, we write the relation between the SU(5) components and the MSSM superfields as

$$\begin{aligned}
&\Psi_i \ni \{Q_i,\ (V_{QU})_{ij}\overline{U}_j,\ (V_{QE})_{ij}\overline{E}_j\} \ , \\
&\Phi_i \ni \{\overline{D}_i,\ (V_{DL})_{ij}L_j\} \ ,
\end{aligned} \tag{3.27}$$

where $V_{QU}$, $V_{QE}$, and $V_{DL}$ are unitary matrices. Then, we express (3.26) in terms of the MSSM superfields:

$$\begin{aligned}
W_{\rm Yukawa} = {}& (P\hat{h})^{ij}(V_{QU})_{jk}(Q_i^a \cdot H_u)\overline{U}_{ka} - (V^*\hat{f})^{ij}(Q_i^a \cdot H_d)\overline{D}_{ja} \\
& - (V^*\hat{f})^{ij}(V_{QE})_{ik}(V_{DL})_{jl}\overline{E}_k(L_l \cdot H_d) \\
& - \frac{1}{2}(P\hat{h})^{ij}\epsilon_{abc}(Q_i^a \cdot Q_j^b)H_C^c + (V^*\hat{f})^{ij}(V_{DL})_{jk}(Q_i^a \cdot L_k)\overline{H}_{Ca} \\
& + (P\hat{h})^{ij}(V_{QU})_{ik}(V_{QE})_{jl}\overline{U}_{ka}\overline{E}_l H_C^a - (V^*\hat{f})^{ij}(V_{QU})_{ik}\epsilon^{abc}\overline{U}_{ka}\overline{D}_{jb}\overline{H}_{Cc} \ .
\end{aligned} \tag{3.28}$$

Let us take the generation basis for the MSSM superfields so that the Yukawa couplings of the up-type quarks and the charged leptons are diagonalized. In this case, the matching condition for the Yukawa couplings at the GUT scale is given as follows:

$$\begin{aligned}
\hat{h}PV_{QU} &= \hat{f}_u(M_{\rm GUT}) \ , \\
V^*\hat{f} &= V^*_{\rm CKM}(M_{\rm GUT})\hat{f}_d(M_{\rm GUT}) \ , \\
V_{QE}^T V^*\hat{f}V_{DL} &= \hat{f}_e(M_{\rm GUT}) = \hat{f}_d(M_{\rm GUT}) \ .
\end{aligned} \tag{3.29}$$

So, we can choose the unitary matrices $V_{QU}$, $V_{QE}$, and $V_{DL}$ as

$$V_{QU} = P^* \ , \qquad V_{QE} = V_{\rm CKM}(M_{\rm GUT}) \ , \qquad V_{DL} = \mathbb{1} \ , \tag{3.30}$$

and we have

$$\begin{aligned}
&\hat{h} = \hat{f}_u(M_{\rm GUT}) \ , \\
&\hat{f} = \hat{f}_d(M_{\rm GUT}) = \hat{f}_e(M_{\rm GUT}) \ , \\
&V = V_{\rm CKM}(M_{\rm GUT}) \ .
\end{aligned} \tag{3.31}$$

In this basis, the MSSM superfields are embedded into the SU(5) matter multiplets as

$$\Psi_i \ni \{Q_i,\ e^{-i\varphi_i}\overline{U}_i,\ V_{ij}\overline{E}_j\} \ , \qquad \Phi_i \ni \{\overline{D}_i,\ L_i\} \ , \tag{3.32}$$

where we write $P_{ij} = e^{i\varphi_i}\delta_{ij}$ with the condition $\sum_i \varphi_i = 0$. Then, Eq. (3.28) leads to

$$\begin{aligned} W_{\text{Yukawa}} = &\hat{h}_i (Q_i^a \cdot H_u)\overline{U}_{ia} - (V^*\hat{f})^{ij}(Q_i^a \cdot H_d)\overline{D}_{ja} - \hat{f}_i \overline{E}_i (L_i \cdot H_d) \\ &- \frac{1}{2}\hat{h}_i e^{i\varphi_i}\epsilon_{abc}(Q_i^a \cdot Q_i^b)H_C^c + (V^*\hat{f})^{ij}(Q_i^a \cdot L_j)\overline{H}_{Ca} \\ &+ (\hat{h}V)^{ij}\overline{U}_{ia}\overline{E}_j H_C^a - (V^*\hat{f})^{ij}e^{-i\varphi_i}\epsilon^{abc}\overline{U}_{ia}\overline{D}_{jb}\overline{H}_{Cc} \ . \end{aligned} \tag{3.33}$$

As one can see from this equation, the new phase factors only appear in the couplings of the color-triplet Higgs multiplets. So their effects would not show up in the low-energy physics. Nevertheless, they may play an important role in the prediction of the proton decay rate, as we will see later.

Next, we study the gauge interactions of the SU(5) matter fields. The interactions come from the Kähler potential

$$K = \left(\Psi^\dagger\right)_{\hat{a}\hat{b}}\left(e^{2g_5\mathcal{V}}\right)^{\hat{a}\hat{b}}{}_{\hat{c}\hat{d}}\Psi^{\hat{c}\hat{d}} + \left(\Phi^\dagger\right)^{\hat{a}}\left(e^{2g_5\mathcal{V}}\right)_{\hat{a}}{}^{\hat{b}}\Phi_{\hat{b}} \ . \tag{3.34}$$

For later use, let us expand the terms with respect to the gauge superfields, and explicitly write the terms proportional to $\mathcal{V}^A$. We have

$$\begin{aligned} K \ni\ &2g_5\left[Q_a^\dagger (G^A T^A)^a{}_b Q^b - \overline{U}_a (G^A T^A)^a{}_b \overline{U}^{\dagger b} - \overline{D}_a (G^A T^A)^a{}_b \overline{D}^{\dagger b}\right] \\ &+ 2g_5\left[Q_\alpha^\dagger (W^A t^A)^\alpha{}_\beta Q^\beta + L_\alpha^\dagger (W^A t^A)^\alpha{}_\beta L^\beta\right] \\ &+ 2g_5\sqrt{\frac{3}{5}}B\left[\frac{1}{6}Q^\dagger Q - \frac{1}{2}L^\dagger L - \frac{2}{3}\overline{U}^\dagger\overline{U} + \frac{1}{3}\overline{D}^\dagger\overline{D} + \overline{E}^\dagger\overline{E}\right] \\ &+ \sqrt{2}g_5\left(-\epsilon_{\alpha\beta}\overline{D}^{\dagger a}X_a^\alpha L^\beta + \epsilon^{abc}Q_{a\alpha}^\dagger X_b^\alpha P^\dagger \overline{U}_c + \epsilon_{\alpha\beta}\overline{E}^\dagger V^\dagger X_a^\alpha Q^{a\beta} + \text{h.c.}\right) \ . \end{aligned} \tag{3.35}$$

From the expression, we can conclude that at the GUT scale the gauge coupling constants of the SM gauge group are written in terms of the unified gauge coupling $g_5$ as

$$g_5(M_{\text{GUT}}) = g_s(M_{\text{GUT}}) = g(M_{\text{GUT}}) = \sqrt{\frac{5}{3}}g'(M_{\text{GUT}}) \ . \tag{3.36}$$

From now on, we also use the gauge coupling constants $g_a$ $(a = 1, 2, 3)$ defined by

$$g_1 \equiv \sqrt{\frac{5}{3}}g' \ , \qquad g_2 \equiv g \ , \qquad g_3 \equiv g_s \ , \tag{3.37}$$

which become same at the GUT scale. The prediction is in fact realized in the MSSM with the low-scale SUSY breaking as shown in Fig. 1.2. We will further see in the subsequent section that it is also accomplished in the case of high-scale SUSY scenario.

Soft SUSY breaking terms are also written in terms of the SU(5) multiplets. This implies that there exist some relations among the MSSM soft terms in the case of the SU(5) GUT. For instance, the soft masses of the $\bar{\mathbf{5}}$ and $\mathbf{10}$ matter fields are given as follows:

$$\mathcal{L}_{\text{soft mass}} = -\widetilde{\psi}^*_{i\hat{a}\hat{b}}\left(\widehat{m}^2_{\mathbf{10}}\right)_{ij}\widetilde{\psi}_j^{\hat{a}\hat{b}} - \widetilde{\phi}_i^{*\hat{a}}\left(\widehat{m}^2_{\bar{\mathbf{5}}}\right)_{ij}\widetilde{\phi}_{j\hat{a}} \ , \tag{3.38}$$

where $\widetilde{\psi}_i$ and $\widetilde{\phi}_i$ denote the scalar components of the $\Psi_i$ and $\Phi_i$ chiral superfields, respectively. This expression reads

$$\begin{aligned} &\widehat{m}^2_{\widetilde{Q}_L} = \widehat{m}^2_{\mathbf{10}} \ , \qquad \widehat{m}^2_{\widetilde{u}_R} = V^\dagger_{QU}(\widehat{m}^2_{\mathbf{10}})V_{QU} \ , \qquad \widehat{m}^2_{\widetilde{e}_R} = V^\dagger_{QE}(\widehat{m}^2_{\mathbf{10}})V_{QE} \ , \\ &\widehat{m}^2_{\widetilde{d}_R} = \widehat{m}^2_{\bar{\mathbf{5}}} \ , \qquad \widehat{m}^2_{\widetilde{L}_L} = V^\dagger_{DL}(\widehat{m}^2_{\bar{\mathbf{5}}})V_{DL} \ . \end{aligned} \tag{3.39}$$

Similar relations can be also obtained for the $A$-terms.

## 3.2 Grand unification in high-scale SUSY

Now let us estimate the masses of the superheavy particles appearing in the minimal SUSY SU(5) GUT by using the method presented in Refs. [186, 187]. This method is based on the assumption of the gauge coupling unification, and the use of the RGEs together with the threshold corrections for the gauge couplings allows us to evaluate the masses of the GUT scale particles. To illustrate the prescription, we first look into the matching condition for the gauge coupling constants at the GUT scale. We use the $\overline{\rm DR}$ scheme [190] in the following calculation. At the GUT scale, the gauge coupling constants in the $\mathrm{SU}(3)_C\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ gauge theories are equated to the unified coupling constant $g_5$ with the following threshold corrections at one-loop level [191, 192]:

$$\begin{aligned} \frac{1}{g_1^2(\mu)} &= \frac{1}{g_G^2(\mu)} + \frac{1}{8\pi^2}\left[\frac{2}{5}\ln\frac{\mu}{M_{H_C}} - 10\ln\frac{\mu}{M_X}\right] \ , \\ \frac{1}{g_2^2(\mu)} &= \frac{1}{g_G^2(\mu)} + \frac{1}{8\pi^2}\left[2\ln\frac{\mu}{M_\Sigma} - 6\ln\frac{\mu}{M_X}\right] \ , \\ \frac{1}{g_3^2(\mu)} &= \frac{1}{g_G^2(\mu)} + \frac{1}{8\pi^2}\left[\ln\frac{\mu}{M_{H_C}} + 3\ln\frac{\mu}{M_\Sigma} - 4\ln\frac{\mu}{M_X}\right] \ . \end{aligned} \tag{3.40}$$

Here, the conditions do not include constant (scale independent) terms since we use the $\overline{\rm DR}$ scheme for the renormalization. From the equations it immediately follows that

$$\begin{aligned} \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} - \frac{1}{g_1^2(\mu)} &= -\frac{3}{10\pi^2}\ln\frac{\mu}{M_{H_C}} \ , \\ \frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} &= -\frac{3}{2\pi^2}\ln\frac{\mu^3}{M_X^2 M_\Sigma} \ . \end{aligned} \tag{3.41}$$

The relations allow us to evaluate the masses of the heavy particles, $M_{H_C}$ and $M_X^2 M_\Sigma$, from the gauge coupling constants determined in the low-energy experiments through the RGEs [186, 187]. While the couplings are well measured with high precision, the estimation is quite dependent on the spectrum in the intermediate region, especially on the masses of gauginos and higgsinos. So let us discuss other thresholds in the intermediate region. For the SUSY breaking threshold, we just equate the gauge couplings above and below the threshold, and change the beta functions appropriately for each region. This approximation is valid since the particles appearing at the scale are assumed to be degenerate in mass. In the case of gauginos, on the other hand, we need to consider the threshold corrections since the mass difference among gauginos might be sizable.

The condition is

$$
\begin{aligned}
\frac{1}{g_1^2(\mu)_{\rm SM}} &= \frac{1}{g_1^2(\mu)_{\rm gaugino}} ~, \\
\frac{1}{g_2^2(\mu)_{\rm SM}} &= \frac{1}{g_2^2(\mu)_{\rm gaugino}} + \frac{1}{6\pi^2}\ln\frac{\mu}{M_2} ~, \\
\frac{1}{g_3^2(\mu)_{\rm SM}} &= \frac{1}{g_3^2(\mu)_{\rm gaugino}} + \frac{1}{4\pi^2}\ln\frac{\mu}{M_3} ~,
\end{aligned}
\tag{3.42}
$$

where $g_a(\mu)_{\rm SM}$ are the couplings in the SM while $g_a(\mu)_{\rm gaugino}$ are those above the gaugino threshold.

The Yukawa couplings are matched as usual, *i.e.*, at the SUSY breaking scale, the Yukawa couplings $y_i(\mu)$ below the SUSY breaking scale are matched with the supersymmetric ones, $f_i(\mu)$, as follows:

$$
\begin{aligned}
f_t(M_S) &= \frac{1}{\sin\beta} y_t(M_S) ~, \\
f_b(M_S) &= \frac{1}{\cos\beta} y_b(M_S) ~, \\
f_\tau(M_S) &= \frac{1}{\cos\beta} y_\tau(M_S) ~.
\end{aligned}
\tag{3.43}
$$

In order to clarify the relation among the masses of the superheavy particles and SUSY particles, we first solve the RGEs at one-loop level and, taking the threshold corrections into account, derive conditions on the masses and the gauge couplings. Such conditions reflect the dependence of $M_{H_C}$ and $M_X^2 M_\Sigma$ on the mass spectrum of the SUSY particles. By inserting to Eq. (3.41) the one-loop solutions of the RGEs for the gauge couplings, we have

$$
\frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi}\left[\frac{12}{5}\ln\left(\frac{M_{H_C}}{m_Z}\right) - 2\ln\left(\frac{M_S}{m_Z}\right) + 4\ln\left(\frac{M_3}{M_2}\right)\right] , \tag{3.44}
$$

$$
\frac{5}{\alpha_1(m_Z)} - \frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} = \frac{1}{2\pi}\left[12\ln\left(\frac{M_X^2 M_\Sigma}{m_Z^3}\right) + 4\ln\left(\frac{M_2}{m_Z}\right) + 4\ln\left(\frac{M_3}{m_Z}\right)\right] . \tag{3.45}
$$

From Eq. (3.44) we find that the mass of the color-triplet Higgs $M_{H_C}$ gets larger as the SUSY breaking scale $M_S$ is taken to be higher. This originates from the mass difference among the components of the fundamental Higgs multiplets, *i.e.*, the triplet-Higgs, higgsinos, heavy Higgs bosons, and the lightest Higgs boson. Therefore, the behavior of $M_{H_C}$ with respect to the SUSY breaking scale is universal in a sense. Further, $M_{H_C}$ depends only on the ratio of $M_2$ and $M_3$. $M_X^2 M_\Sigma$ is, on the other hand, independent of the SUSY braking scale $M_S$ while dependent on the scale of the gauginos, not their ratio. This is because the right-hand side of Eq. (3.45) results from the mass difference in the gauge vector multiplets and the adjoint Higgs multiplet, a part of which is included as the longitudinal mode of the gauge multiplets. It is also found that $M_X^2 M_\Sigma$ decreases when the gaugino masses are taken to be large values. This is owing to the opposite sign of the contribution of gauge fields to the gauge beta functions to those of matter fields. This feature is, therefore, again model-independent.

Next we carry out a similar analysis using the two-loop RGEs for the gauge couplings and taking into account the threshold corrections at one-loop level. The masses of sfermions, heavy

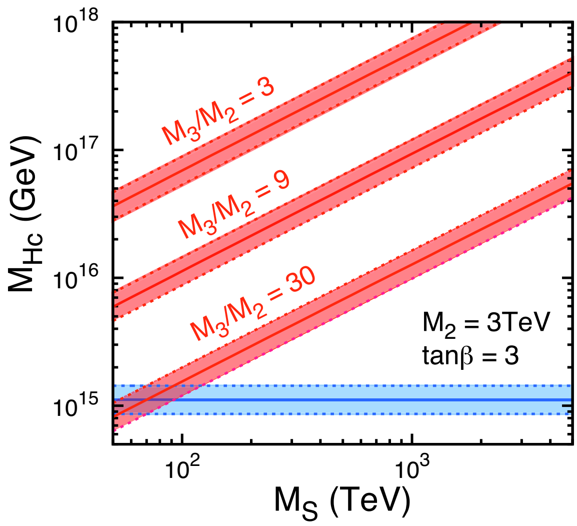


Figure 3.1: Predicted color-triplet Higgs mass $M_{H_C}$ as functions of the SUSY breaking scale $M_S$ (orange lines) [188]. Here, wino mass $M_2$ is fixed to be 3 TeV and $\tan\beta = 3$. Gluino-wino mass ratio, $M_3/M_2$, is set to be $M_3/M_2 = 3,\ 9$, and 30 from top to bottom, respectively. Theoretical errors coming from the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$ [18] are also shown. Horizontal blue line shows a result in the case of low-energy SUSY ($M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$).

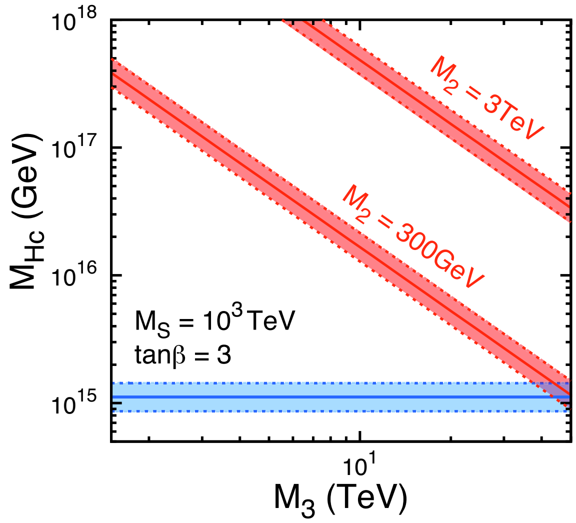


Figure 3.2: Color-triplet Higgs mass $M_{H_C}$ as functions of gluino mass $M_3$ (orange lines) [188]. Here, $\tan\beta = 3$ and $M_S = 10^3$ TeV. Upper and lower lines correspond to $M_2 = 3$ TeV and 300 GeV, respectively. Error bars indicate the input error of the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$ [18]. Horizontal blue line shows a result in the case of low-energy SUSY ($M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$).

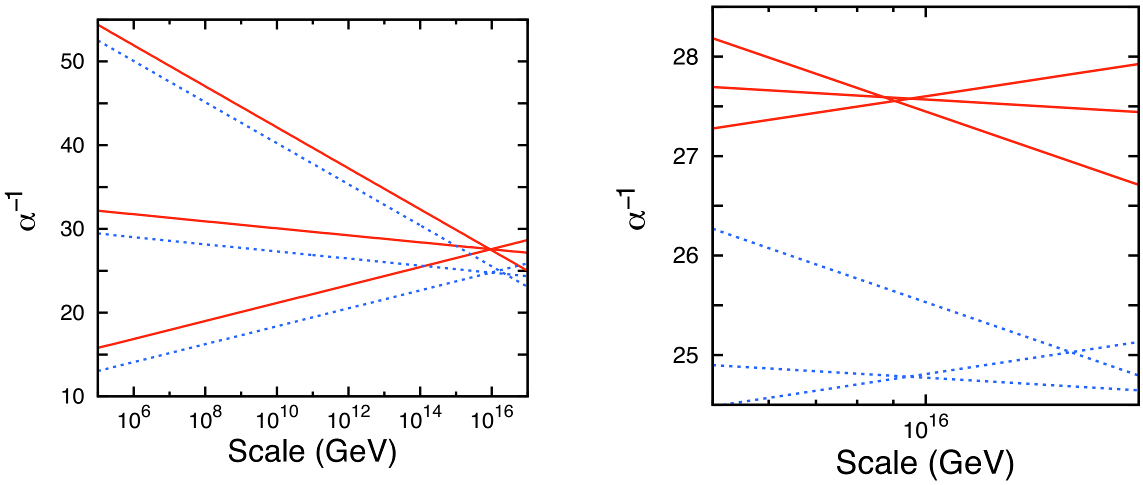


Figure 3.3: Comparison of the gauge coupling unification in the high- and low-scale SUSY GUTs. Orange solid and blue dotted lines correspond to the high- and low-SUSY cases, respectively. In the former case, we take $M_S = 100$ TeV, $M_2 = 3$ TeV, and $M_3/M_2 = 9$, while in the latter case $M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$.

Higgs bosons, and higgsinos are taken to be $M_S$ for brevity. Gaugino masses are assumed to be lighter than $M_S$ by one-loop factors.

First, we consider the color-triplet Higgs mass $M_{H_C}$. In Fig. 3.1, we plot the dependence of $M_{H_C}$ on the SUSY breaking scale $M_S$ in the orange lines. Here, the wino mass $M_2$ is fixed to be 3 TeV, which is favored from the thermal relic abundance of wino dark matter as discussed in Sec. 2.3.4, and we set $\tan\beta = 3$, which may explain the 126 GeV Higgs mass. The ratio of the gluino and wino masses, $M_3/M_2$, is set to be $M_3/M_2 = 3,\ 9$, and 30 from top to bottom, respectively. Further, we show the error of the calculation coming from that of the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$ [18]. The horizontal blue line shows a result in the case of low-energy SUSY ($M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$) as a reference. In this case, we have $8.6 \times 10^{14} \leq M_{H_C} \leq 1.4 \times 10^{15}$ GeV. This figure well illustrates the feature read from the approximated expression given in Eq. (3.44); $M_{H_C}$ increases as the SUSY breaking scale grows while it decreases when the ratio $M_3/M_2$ becomes large. To see the latter feature more clearly, we show its dependence on the gluino mass $M_3$. Again, we set $\tan\beta = 3$, and the SUSY breaking mass is fixed to be $M_S = 10^3$ TeV. The upper and lower lines correspond to $M_2 = 3$ TeV and 300 GeV, respectively. These two figures show that $M_{H_C}$ is strongly dependent on $M_S$ and $M_3/M_2$. Therefore, to predict the mass with high accuracy, precise determination of the masses of gauginos as well as the SUSY breaking scale is inevitable. Any way, in the high-scale SUSY scenario it is found to be possible for the mass of the color-triplet $M_{H_C}$ to be around $\sim 2 \times 10^{16}$ GeV, which is expected from the gauge coupling unification.

This observation indicates an interesting feature of the high-scale SUSY scenario. Namely, when the color-triplet Higgs multiplet is around the GUT scale, small threshold corrections are required to realize the gauge coupling unification. This feature is clearly seen if one looks into the running of gauge couplings. In Fig. 3.3, we compare the gauge coupling running in the high- and low-energy SUSY GUTs. The orange solid and the blue dotted lines correspond to the high- and low-SUSY cases, respectively. In the former case, we take $M_S = 100$ TeV, $M_2 = 3$ TeV, and

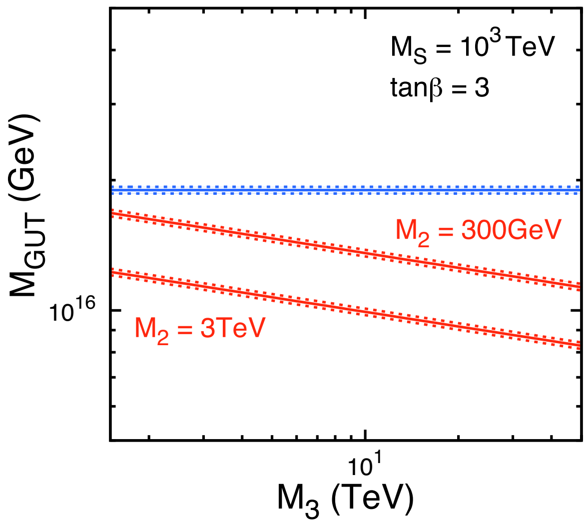


Figure 3.4: GUT scale $M_{\rm GUT} \equiv (M_X^2 M_\Sigma)^{1/3}$ as functions of gluino mass $M_3$ (orange lines) [188]. Here, $\tan\beta = 3$ and $M_S = 10^3$ TeV. Upper and lower lines correspond to $M_2 = 300$ GeV and 3 TeV, respectively. Error bars indicate the input error of the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$ [18]. Horizontal blue line shows a result in the case of low-energy SUSY ($M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$).

$M_3/M_2 = 9$, while in the latter case $M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$. As you can see, in both cases the gauge coupling unification is achieved with great accuracy. But when you zoom up around the GUT scale, you find that the unification is better in the high-scale SUSY scenario than that in the low-scale SUSY.

Next, we discuss constraints on $M_X^2 M_\Sigma$ derived from the relation (3.45). From now on we define

$$M_{\rm GUT} \equiv (M_X^2 M_\Sigma)^{1/3} , \tag{3.46}$$

and refer to it as the GUT scale. The equation (3.45) tells us that the GUT scale depends on only the gaugino masses at the leading order, so we express $M_{\rm GUT}$ as functions of the gaugino masses. In Fig. 3.4 we plot it as functions of gluino mass. Here again we fix $\tan\beta = 3$ and $M_S = 10^3$ TeV. The upper and lower lines correspond to $M_2 = 300$ GeV and 3 TeV, respectively. Again, the horizontal blue line shows a result in the case of low-energy SUSY with $M_S = 1$ TeV, $M_2 = 200$ GeV, and $M_3/M_2 = 3.5$, which gives $M_{\rm GUT} \simeq 1.9 \times 10^{16}$ GeV. The error bars indicate the input error of the strong coupling constant $\alpha_s(m_Z) = 0.1184(7)$ [18], though the effect is negligible. We see that the GUT scale has little dependence on the gaugino masses. In that sense, the prediction is robust compared with that for $M_{H_C}$. Note, however, that the GUT scale $M_{\rm GUT}$ gets lower when the gaugino masses become larger ($M_{\rm GUT} \propto (M_3 M_2)^{-1/9}$). This feature is quite interesting when one considers the proton decay via the $X$-boson exchange processes. Although the change in $M_{\rm GUT}$ is small, it might be significant since the proton decay lifetime scales as $\propto M_X^4$. The detailed discussions are given in the next chapter.

Finally, we briefly comment on the $\tan\beta$ dependence of the results. Although the one-loop computation is not related with $\tan\beta$, the two-loop results might be affected through the running of the Yukawa couplings. We have found, however, that the effects on the results are negligible.

At the end of the section, we discuss possible corrections to the results which we ignore in the above discussion.

In our calculation, we have assumed that all of the sfermions are degenerate in mass at $M_S$. This assumption is violated when one considers the renormalization effects; although the particles in an SU(5) multiplet have an identical soft-mass term at the GUT scale as shown in Eq. (3.39), their masses become different as they are evolved down to the low-energy regions. Then mass differences among the particles in the same SU(5) multiplet give rise to certain amount of threshold corrections at the SUSY breaking scale. To check the possibility, let us study the RGEs of the soft mass parameters. It readily turns out that the effects on the soft masses of the first two generations are negligible, since the gaugino masses are one-loop suppressed compared to the scalar masses in the high-scale SUSY scenario. Further, as long as one considers small $\tan\beta$, which is favored in the high-scale SUSY scenario to explain the observed 126 GeV Higgs mass, the contribution of the bottom and tau Yukawa couplings is also negligible. Thus, all we have to consider is the evolution of the stop mass parameters $\widetilde{m}^2_{\widetilde{Q}_{L3}}$ and $\widetilde{m}^2_{\widetilde{t}_{R3}}$. Their difference, however, results in small effects on the prediction of $M_{H_C}$ and $M_{\rm GUT}$ because of relatively small contribution of stops to the gauge coupling beta functions, compared with that of the GUT-scale particles. It turns out that the mass difference caused by the running effects may change the prediction of $M_{H_C}$ by at most $\mathcal{O}(10)$ %, while that of $M_{\rm GUT}$ is rarely changed. As a result, within the uncertainty coming from $\alpha_s$, we can safely neglect the RGE effects.

Another possibility is the contribution of the higher-dimensional operators suppressed by the Planck scale [193, 194, 195]. For instance, one can add the following terms to the superpotential which induce the dimension-five effective operators:

$$W_{\rm eff} = \frac{a}{M_P}({\rm Tr}\Sigma^2)^2 + \frac{b}{M_P}{\rm Tr}\Sigma^4 \ . \tag{3.47}$$

In this case, the mass of $\Sigma_8$ becomes different from the mass of $\Sigma_3$, which induces the threshold corrections at the GUT scale. We find that the corrections may raise the color-triplet Higgs mass by a factor of $\mathcal{O}(10)$.

Moreover, we have another dimension-five operator

$$\int d^2\theta \frac{c}{M_P}{\rm Tr}\left[\Sigma W_5^\alpha W_{5\alpha}\right] \ , \tag{3.48}$$

which modifies the wave-function renormalization factor of each gauge superfield. In this case, the gauge couplings $\alpha_i$ no longer unify at the GUT scale; instead, they receive extra corrections of the order of $c\alpha_i V/M_P$ [196]. The corrections can reduce the color-triplet Higgs mass by more than an order of magnitude.

As we have seen, the threshold corrections from the Planck-suppressed operators might significantly affect $M_{H_C}$ and $M_{\rm GUT}$, which makes it difficult to predict their values. For this reason, we just assume them to be around the GUT scale in the following calculation.

## 3.3 Yukawa unification in high-scale SUSY

As expressed in Eq. (3.25), the minimal SUSY SU(5) GUT predicts the unification of the Yukawa couplings of down-type quarks and charged leptons.[3] Indeed, Fig. 1.1 shows that these Yukawa

[3]In the SO(10) GUTs, the up-type Yukawa couplings may be also unified with the other Yukawa couplings.

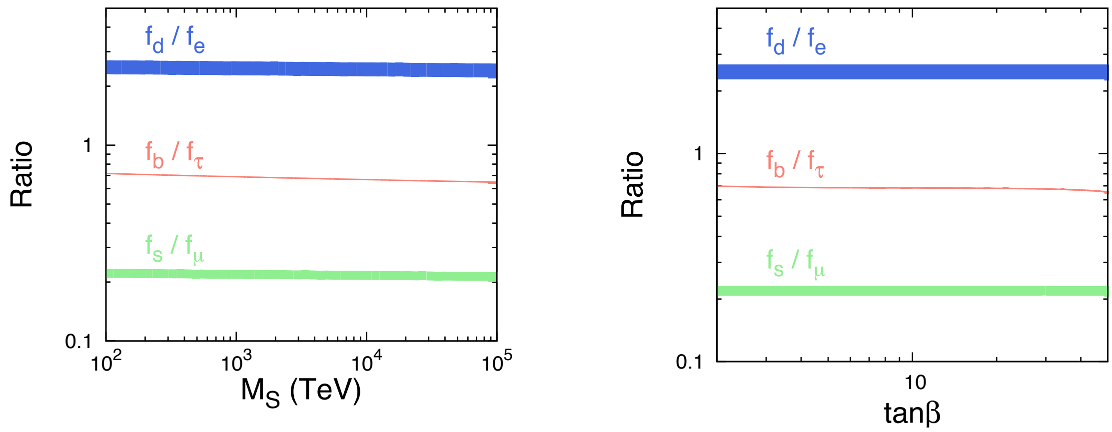


Figure 3.5: Ratios $f_{d_i}/f_{e_i}$ $(i = 1, 2, 3)$ as functions of $M_S$ (left) and $\tan\beta$ (right). Top blue, middle red, and bottom green lines correspond to the first, third, and second generations, respectively. In the left panel, we take $M_2 = 3$ TeV, $M_1 = 6$ TeV, $M_3 = 20$ TeV, $\tan\beta = 3$, and $M_{\rm GUT} = 2.0 \times 10^{16}$ GeV, while in the right panel $M_S = 10^3$ TeV, and other parameters are similar to the left ones. Bands represent the error of input quark masses.

couplings at the $m_Z$ scale have similar values, and we expect that they come close to each other in high-energy regions. In this section, we study such possibilities. It turns out, however, difficult to realize the unification within the minimal SUSY GUT in the case of high-scale SUSY scenario. Possible modifications to realize the Yukawa unification are also discussed shortly. For a recent discussion in the low-scale SUSY, see Ref. [197].

The difficulty of the Yukawa unification becomes apparent when one considers the unification relations of the Yukawa couplings of the first two generation; $f_d(M_{\rm GUT}) = f_e(M_{\rm GUT})$ and $f_s(M_{\rm GUT}) = f_\mu(M_{\rm GUT})$. Since these couplings are small, the renormalization effects on $f_d$ $(f_e)$ are similar to those on $f_s$ $(f_\mu)$. Hence, the relation

$$\frac{f_s}{f_d} = \frac{f_\mu}{f_e} \ , \tag{3.49}$$

which follows from the unification relations at the GUT scale, is a renormalization group invariant up to the leading order calculation. Consequently, the relation should hold at any scale. On the other hand, we have

$$\frac{m_s}{m_d}(m_Z) \simeq 20 \ , \qquad \frac{m_\mu}{m_e}(m_Z) \simeq 200 \ , \tag{3.50}$$

which disagree with the above relation.[4]

The unification of $f_b$ and $f_\tau$ is relatively good, though there remains sizable difference in a wide range of parameter region. To illustrate the features more clearly, in Fig. 3.5, we plot the ratios $f_{d_i}/f_{e_i}$ $(i = 1, 2, 3)$ as functions of $M_S$ (left) and $\tan\beta$. The top blue, middle red, and bottom green lines correspond to the first, third, and second generations, respectively. In the left panel, we take $M_2 = 3$ TeV, $M_1 = 6$ TeV, $M_3 = 20$ TeV, $\tan\beta = 3$, and $M_{\rm GUT} = 2.0\times 10^{16}$ GeV, while in the right panel $M_S = 10^3$ TeV, and other parameters are similar to the left ones.

[4]The *Georgi-Jarlskog mass relation* [198], which assumes $f_e = f_d/3$, $f_\mu = 3m_s$, and $f_b = f_\tau$, may explain the result.

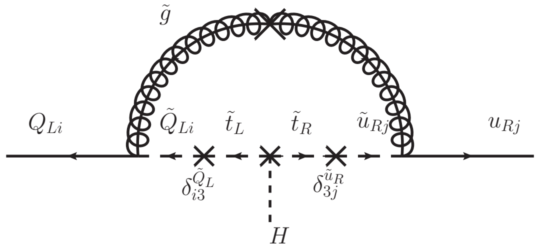


Figure 3.6: Threshold corrections to the up-type Yukawa couplings.

Bands represent the error of input quark masses. The one-loop RGEs are exploited in the calculation, and the threshold corrections are neglected. The figures show that the results have little dependence on the SUSY breaking scale and the values of $\tan\beta$.

From the figures, we find that all of the ratios $f_{d_i}/f_{e_i}$ deviate from unity. To realize the Yukawa coupling unification, therefore, one needs additional corrections to the couplings. It is found that the two-loop contribution to the RGEs is quite small and negligible in the present argument. Then, the remaining possibilities are roughly divided into two types as long as one persists in the particle contents of the minimal SUSY SU(5) GUT;[5] one is the threshold corrections at the SUSY breaking scale and the other is the contribution at or above the GUT scale. The latter type includes not only the threshold corrections by the superheavy particles at the GUT scale, but also the higher-dimensional operators induced at the Planck scale.

Let us first discuss the threshold corrections at the SUSY breaking scale. Dominant contributions to the corrections are given by the gluino-squark and higgsino-squark loop diagrams. In the high-scale SUSY scenario, these contributions are insignificant because of the loop-suppressed gluino mass and $A$-terms, as well as a small value of $\tan\beta$.[6] Even in such a case, the Yukawa couplings of the first two generations may receive significant corrections if there exists sizable flavor violation in the sfermion masses. Notice however that in the presence of such contribution, the quark EDMs and CEDMs are generally also induced, which are severely constrained as discussed in Sec. 2.3.3. To see the correlation, let us consider the threshold corrections to the up-type Yukawa couplings, which are to be compared with the up-quark EDM and CEDM presented in Eq. (2.61). The dominant contribution is given by the diagram displayed in Fig. 3.6. By evaluating it, we obtain

$$\delta f_u^{ij} \simeq \frac{9}{8}\frac{f_t\alpha_3}{4\pi}\frac{\mu^* M_3^*}{M_S^2}\cot\beta\ \delta_{i3}^{\widetilde{Q}_L}\delta_{j3}^{\widetilde{u}_R *}\ . \tag{3.51}$$

In particular, the correction to the Yukawa coupling of up-quark ($\sim f_u^{11}$) has similar form to those in Eq. (2.61). A similar relation can be also found in the case of the down-type Yukawa couplings. Therefore, we expect these threshold effects to be small as long as we consider the parameter region which evades the current limits from the EDM experiments.

[5]The Yukawa unification is easily achieved when one includes extra particles. For example, extra Higgs multiplets in the **45** representation can modify the Yukawa couplings and thus may provide the Yukawa unification, as in the model proposed in Ref. [198].

[6]In Ref. [199], large $A$-terms with the high-scale SUSY breaking are considered to explain the Yukawa unification, though it seems difficult to reproduce the soft parameters exploited in the reference with some simple SUSY breaking mechanism.

Next, we consider the corrections at the GUT scale. Particularly important contribution comes from the higher dimensional operators including the adjoint Higgs multiplet suppressed by the Planck scale [193, 194, 195]. For the down-type Yukawa couplings, the following two operators of dimension-five are relevant:

$$W_{\rm eff} = \sqrt{2}\frac{c_1^{ij}}{M_P}\Psi_i^{\hat{a}\hat{b}}\Phi_{j\hat{a}}\Sigma^{\hat{c}}_{\hat{b}}\bar{H}_{\hat{c}} + \sqrt{2}\frac{c_2^{ij}}{M_P}\Phi_{j\hat{a}}\Sigma^{\hat{a}}_{\hat{b}}\Psi_i^{\hat{b}\hat{c}}\bar{H}_{\hat{c}} \ , \tag{3.52}$$

which generate the down-type Yukawa terms after the SU(5) breaking as

$$-3V\frac{c_1^{ij}}{M_P}\big(Q_i^a\cdot H_d\big)\overline{D}_{ja} - 3V\frac{c_1^{ij}}{M_P}\overline{E}_i\big(L_j\cdot H_d\big) + 2V\frac{c_2^{ij}}{M_P}\big(Q_i^a\cdot H_d\big)\overline{D}_{ja} - 3V\frac{c_2^{ij}}{M_P}\overline{E}_i\big(L_j\cdot H_d\big) \ . \tag{3.53}$$

Therefore, terms proportional to $c_2^{ij}$ may account for the discrepancy between the down-type quark and charged-lepton Yukawa couplings. The contribution is $\lesssim \mathcal{O}(V/M_P)$, while $|f_b - f_\tau| \lesssim \mathcal{O}(10^{-2})$. Therefore, it can well provide the threshold corrections required to realize the Yukawa unification.

After all, we expect at least $\mathcal{O}(f_d - f_e)$ uncertainty for the evaluation of the Yukawa couplings at the GUT scale. It may significantly affect the prediction of the proton decay rate, as we will see in the following chapter.

# Chapter 4

# Proton decay in high-scale SUSY

In this chapter, we discuss proton decay in the minimal SUSY GUT with high-scale SUSY breaking. In the minimal SUSY GUT, proton decay is induced by the exchanges of the color-triplet Higgs multiplets and the $X$-bosons, and their effects are described in terms of the dimension-five and -six effective operators, respectively. Because of the lower-mass dimension of the effective operators, the former process usually yields the dominant decay modes, such as $p \to K^+\bar{\nu}$ [65, 66]. The lifetime of the channel is estimated as $\tau(p \to K^+\bar{\nu}) \lesssim 10^{30}$ yrs [67, 68], with the SUSY particles, in particular those of the third generation, assumed to have masses of around the electroweak scale. On the other hand, the Super-Kamiokande experiment gives stringent limits on the channels: $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs [69, 70]. This contradiction makes it widely believed that the minimal SUSY SU(5) GUT has been already excluded [68] and, therefore, needs some extensions so that the dimension-five proton decay is significantly suppressed.

The dimension-five operators contain squarks and/or sleptons in their external lines. These fields are to be integrated out below the SUSY scale through the gauginos and higgsinos exchanging processes, and then the four-Fermi operators, which are suppressed by the sfermion masses, are induced. Hence, their effects are expected to be considerably reduced when the SUSY scale is much higher than the electroweak scale, and the experimental constraints on proton decay rate may be evaded.

In Sec. 4.1, we study such a possibility in the context of the high-scale SUSY scenario. We will find that the minimal SUSY SU(5) GUT actually evades the constraints from the proton decay experiments with the SUSY braking scales which naturally explain the 126 GeV Higgs boson and the existence of dark matter in the Universe [200]. In addition, we study the case where sfermion mass matrices have sizable flavor violation [201]. We will find a smoking-gun signature for the sfermion flavor violation. The resultant proton lifetime lies in the regions which may be reached in the future proton decay experiments. Therefore, although the high-scale SUSY scenario is hard to be probed in the collider experiments, the proton decay searches may give us a chance to verify the scenario as well as the existence of supersymmetry and the grand unification.

Next, we discuss the dimension-six proton decay in the subsequent section. As seen in the previous chapter, the GUT scale tends to be lower in the high-scale SUSY scenario than the low-scale SUSY one. In such a case, the $p \to e^+\pi^0$ mode via the $X$-boson exchanging process is expected to be enhanced. We will see that the channel actually may be searched in future experiments.

Finally, in Sec. 4.3, we give a brief review on the current and future proton decay experiments.

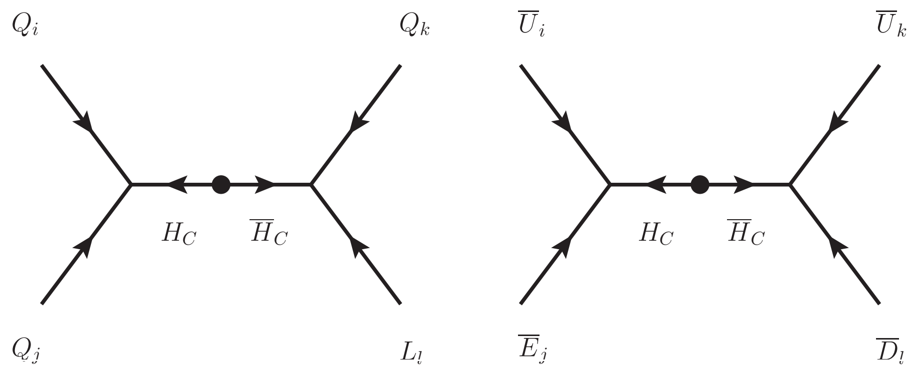


Figure 4.1: Supergraphs for color-triplet Higgs exchanging processes where dimension-five effective operators for proton decay are induced. Bullets indicate color-triplet Higgs mass term.

## 4.1 Dimension-5 proton decay

In this section, we discuss the proton decay via the color-triplet Higgs exchange. We first give a set of formulae which are required to evaluate the proton decay rate in Sec. 4.1.1. Then, we compute the proton decay rate on the assumption of the minimal flavor violation in Sec. 4.1.2. We will see that the current experimental bound can be evaded in the high-scale SUSY scenario. In Sec. 4.1.3, we study a role of flavor structure of sfermion mass matrices in the dimension-five proton decay rate. We will also briefly discuss the effects of soft SUSY breaking baryon-number violating operators in the subsequent section. Finally, in Sec. 4.1.5, the proton decay via the Planck suppressed dimension-five operators is briefly discussed.

### 4.1.1 Dimension-5 proton decay

The Yukawa interactions of color-triplet Higgs multiplets, which are displayed in Eq. (3.33), give rise to the dimension-five proton decay operators. The diagrams which induce the operators are illustrated in Fig. 4.1. By integrating out the color-triplet Higgs multiplets, we obtain the effective Lagrangian

$$\mathcal{L}_5^{\rm eff} = C_{5L}^{ijkl}\mathcal{O}^{5L}_{ijkl} + C_{5R}^{ijkl}\mathcal{O}^{5R}_{ijkl} \quad + \quad \text{h.c.} \ , \tag{4.1}$$

where the effective operators $\mathcal{O}^{5L}_{ijkl}$ and $\mathcal{O}^{5R}_{ijkl}$ are defined by

$$\begin{aligned} \mathcal{O}^{5L}_{ijkl} &\equiv \int d^2\theta \ \frac{1}{2}\epsilon_{abc}(Q^a_i \cdot Q^b_j)(Q^c_k \cdot L_l) \ , \\ \mathcal{O}^{5R}_{ijkl} &\equiv \int d^2\theta \ \epsilon^{abc}\overline{U}_{ia}\overline{E}_j\overline{U}_{kb}\overline{D}_{lc} \ , \end{aligned} \tag{4.2}$$

and the Wilson coefficients $C_{5L}^{ijkl}$ and $C_{5R}^{ijkl}$ are given by

$$\begin{aligned} C_{5L}^{ijkl}(M_{\rm GUT}) &= +\frac{1}{M_{H_C}}(P\hat{h})^{ij}(V^*\hat{f})^{kl} = +\frac{1}{M_{H_C}}\hat{h}_i e^{i\varphi_i}\delta^{ij}(V^*\hat{f})^{kl} \ , \\ C_{5R}^{ijkl}(M_{\rm GUT}) &= +\frac{1}{M_{H_C}}(\hat{h}V)^{ij}(P^*V^*\hat{f})^{kl} = +\frac{1}{M_{H_C}}(\hat{h}V)^{ij}(V^*\hat{f})^{kl}e^{-i\varphi_k} \ . \end{aligned} \tag{4.3}$$

Note that because of the totally antisymmetric tensor in the operators $\mathcal{O}^{5L}_{ijkl}$ and $\mathcal{O}^{5R}_{ijkl}$ they must include at least two generations of quarks. For this reason, the dominant mode of proton decay induced by the operators is accompanied by a strange quark;[1] like the $p \to K^+\bar{\nu}$ mode. The above operators are rewritten in terms of component fields as

$$\begin{aligned}
\mathcal{O}^{5L}_{ijkl} = -\frac{1}{2}\epsilon_{abc}\big[&(u^a_{Li}d^b_{Lj})\widetilde{u}^c_{Lk}\widetilde{e}_{Ll} + (u^a_{Li}u^c_{Lk})\widetilde{d}^b_{Lj}\widetilde{e}_{Ll} + (u^a_{Li}e_{Ll})\widetilde{d}^b_{Lj}\widetilde{u}^c_{Lk} \\
&+ (d^b_{Lj}u^c_{Lk})\widetilde{u}^a_{Li}\widetilde{e}_{Ll} + (d^b_{Lj}e_{Ll})\widetilde{u}^a_{Li}\widetilde{u}^c_{Lk} + (u^c_{Lk}e_{Ll})\widetilde{u}^a_{Li}\widetilde{d}^b_{Lj} \\
&- (d^a_{Li}u^b_{Lj})\widetilde{u}^c_{Lk}\widetilde{e}_{Ll} - (d^a_{Li}u^c_{Lk})\widetilde{u}^b_{Lj}\widetilde{e}_{Ll} - (d^a_{Li}e_{Ll})\widetilde{u}^b_{Lj}\widetilde{u}^c_{Lk} \\
&- (u^b_{Lj}u^c_{Lk})\widetilde{d}^a_{Li}\widetilde{e}_{Ll} - (u^b_{Lj}e_{Ll})\widetilde{d}^a_{Li}\widetilde{u}^c_{Lk} - (u^c_{Lk}e_{Ll})\widetilde{d}^a_{Li}\widetilde{u}^b_{Lj} \\
&- (u^a_{Li}d^b_{Lj})\widetilde{d}^c_{Lk}\widetilde{\nu}_{Ll} - (u^a_{Li}d^c_{Lk})\widetilde{d}^b_{Lj}\widetilde{\nu}_{Ll} - (u^a_{Li}\nu_{Ll})\widetilde{d}^b_{Lj}\widetilde{d}^c_{Lk} \\
&- (d^b_{Lj}d^c_{Lk})\widetilde{u}^a_{Li}\widetilde{\nu}_{Ll} - (d^b_{Lj}\nu_{Ll})\widetilde{u}^a_{Li}\widetilde{d}^c_{Lk} - (d^c_{Lk}\nu_{Ll})\widetilde{u}^a_{Li}\widetilde{d}^b_{Lj} \\
&+ (d^a_{Li}u^b_{Lj})\widetilde{d}^c_{Lk}\widetilde{\nu}_{Ll} + (d^a_{Li}d^c_{Lk})\widetilde{u}^b_{Lj}\widetilde{\nu}_{Ll} + (d^a_{Li}\nu_{Ll})\widetilde{u}^b_{Lj}\widetilde{d}^c_{Lk} \\
&+ (u^b_{Lj}d^c_{Lk})\widetilde{d}^a_{Li}\widetilde{\nu}_{Ll} + (u^b_{Lj}\nu_{Ll})\widetilde{d}^a_{Li}\widetilde{d}^c_{Lk} + (d^c_{Lk}\nu_{Ll})\widetilde{d}^a_{Li}\widetilde{u}^b_{Lj}\big] ,
\end{aligned} \tag{4.4}$$

and

$$\begin{aligned}
\mathcal{O}^{5R}_{ijkl} = -\epsilon^{abc}\big[&(u^\dagger_{Ria}e^\dagger_{Rj})\widetilde{u}^*_{Rkb}\widetilde{d}^*_{Rlc} + (u^\dagger_{Ria}u^\dagger_{Rkb})\widetilde{e}^*_{Rj}\widetilde{d}^*_{Rlc} + (u^\dagger_{Ria}d^\dagger_{Rlc})\widetilde{e}^*_{Rj}\widetilde{u}^*_{Rkb} \\
&+ (e^\dagger_{Rj}u^\dagger_{Rkb})\widetilde{u}^*_{Ria}\widetilde{d}^*_{Rlc} + (e^\dagger_{Rj}d^\dagger_{Rlc})\widetilde{u}^*_{Ria}\widetilde{u}^*_{Rkb} + (u^\dagger_{Rkb}d^\dagger_{Rlc})\widetilde{u}^*_{Ria}\widetilde{e}^*_{Rj}\big] ,
\end{aligned} \tag{4.5}$$

where the operators are written in the interaction basis.

The Wilson coefficients in Eq. (4.3) are determined at the GUT scale. To evaluate the proton decay rate, we need to evolve them down to low-energy regions by using the RGEs. The RGEs are derived in Appendix. C.3 as

$$\begin{aligned}
\mu\frac{d}{d\mu}C^{ijkl}_{5L}(\mu) = \frac{1}{16\pi^2}\bigg[&\Big(-\frac{2}{5}g_1^2 - 6g_2^2 - 8g_3^2\Big)C^{ijkl}_{5L} + (f_uf_u^\dagger + f_df_d^\dagger)^i{}_{i'}C^{i'jkl}_{5L} \\
&+ (f_uf_u^\dagger + f_df_d^\dagger)^j{}_{j'}C^{ij'kl}_{5L} + (f_uf_u^\dagger + f_df_d^\dagger)^k{}_{k'}C^{ijk'l}_{5L} + (f_ef_e^\dagger)^l{}_{l'}C^{ijkl'}_{5L}\bigg] , \\
\mu\frac{d}{d\mu}C^{ijkl}_{5R}(\mu) = \frac{1}{16\pi^2}\bigg[&\Big(-\frac{12}{5}g_1^2 - 8g_3^2\Big)C^{ijkl}_{5R} + C^{i'jkl}_{5R}(2f_u^\dagger f_u)_{i'}{}^i \\
&+ C^{ij'kl}_{5R}(2f_e^\dagger f_e)_{j'}{}^j + C^{ijk'l}_{5R}(2f_u^\dagger f_u)_{k'}{}^k + C^{ijkl'}_{5R}(2f_d^\dagger f_d)_{l'}{}^l\bigg] .
\end{aligned} \tag{4.6}$$

The dimension-five operators contain sfermions in their external lines. At the SUSY breaking scale $M_S$, SUSY particles decouple from the theory, and the dimension-five operators reduce to the dimension-six four-Fermi operators via the exchange of gauginos and higgsinos. In Fig. 4.2, an one-loop diagram which yields the four-Fermi operators is illustrated. Here, the gray dot indicates the dimension-five effective interactions and the black dot represents the mass term of exchanged particles. The four-Fermi operators induced here are written in an invariant form under the $\mathrm{SU}(3)_C\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ symmetry. A set of such operators is summarized in

[1] A charm quark is heavier than a proton, and thus the effective operators including charm quarks do not induce proton decay.

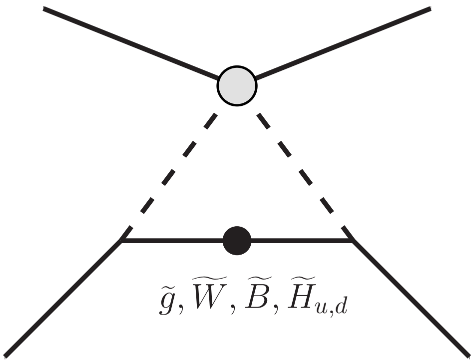


Figure 4.2: One-loop diagram which gives rise to the baryon-number violating four-Fermi operators. Gray dot indicates the dimension-five effective interactions and black dot represents the mass term of exchanged particles; gauginos or higgsinos.

Refs [202, 203, 204][2] as follows:

$$\begin{aligned}
\mathcal{O}^{(1)}_{ijkl} &= \epsilon_{abc}(u^a_{Ri} d^b_{Rj})(Q^c_{Lk} \cdot L_{Ll}) \ , \\
\mathcal{O}^{(2)}_{ijkl} &= \epsilon_{abc}(Q^a_{Li} \cdot Q^b_{Lj})(u^c_{Rk} e_{Rl}) \ , \\
\mathcal{O}^{(3)}_{ijkl} &= \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q^a_{Li\alpha} Q^b_{Lj\gamma})(Q^c_{Lk\delta} L_{Ll\beta}) \ , \\
\mathcal{O}^{(4)}_{ijkl} &= \epsilon_{abc}(u^a_{Ri} d^b_{Rj})(u^c_{Rk} e_{Rl}) \ .
\end{aligned} \tag{4.7}$$

Here we explicitly write the way of contracting the $\mathrm{SU}(2)_L$ indices for $\mathcal{O}^{(3)}_{ijkl}$. Let us express their Wilson coefficients by $C^{ijkl}_{(I)}$ for $\mathcal{O}^{(I)}_{ijkl}$ $(I = 1, 2, 3, 4)$. Then, they are matched with $C^{ijkl}_{5L}$ and $C^{ijkl}_{5R}$ at the SUSY breaking scale. We separately discuss the contribution of each gaugino and higgsino in the subsequent several paragraphs.

---

**Gluino contribution**

Before computation, let us consider the structure of sfermion mass matrices. As long as $M_S \gg m_Z$, the theory at the SUSY breaking scale can be regarded as the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ symmetric. Therefore, the left-right mixing terms $m^2_{\widetilde{q}_{LR}}$ and $m^2_{\widetilde{e}_{LR}}$ in sfermion mass matrices can be neglected and the mass matrices of squarks and charged sleptons have forms like

$$\mathcal{M}^2_{\widetilde{u}} \simeq \begin{pmatrix} \widetilde{m}^2_{\widetilde{Q}_L} & 0 \\ 0 & \widetilde{m}^2_{\widetilde{u}_R} \end{pmatrix} \ , \quad \mathcal{M}^2_{\widetilde{d}} \simeq \begin{pmatrix} V_S^\dagger \, \widetilde{m}^2_{\widetilde{Q}_L} V_S & 0 \\ 0 & \widetilde{m}^2_{\widetilde{d}_R} \end{pmatrix} \ , \quad \mathcal{M}^2_{\widetilde{e}} \simeq \begin{pmatrix} \widetilde{m}^2_{\widetilde{L}_L} & 0 \\ 0 & \widetilde{m}^2_{\widetilde{e}_R} \end{pmatrix} \ , \tag{4.8}$$

and thus the rotation matrices $R_u$ and $R_d$ lead to

$$R_u \simeq \begin{pmatrix} R_Q & 0 \\ 0 & R_U \end{pmatrix} \ , \quad R_d \simeq \begin{pmatrix} R_Q V_S & 0 \\ 0 & R_D \end{pmatrix} \ , \quad R_e \simeq \begin{pmatrix} R_L & 0 \\ 0 & R_E \end{pmatrix} \ , \tag{4.9}$$

[2] We have slightly changed the labels of the operators as well as the order of fermions from those presented in Ref. [204].

with $V_S \equiv V_{\rm CKM}(M_S)$. Then, the upper three fermions ($\widetilde{f}_I$ with $I = 1,2,3$) are nearly left-handed while the lower three ones ($I = 4,5,6$) are almost right-handed. Moreover one can always choose the basis so that $m_{\widetilde{u}_I} = m_{\widetilde{d}_I} \equiv m_{\widetilde{Q}_I}$ and $m_{\widetilde{e}_I} = m_{\widetilde{\nu}_I} \equiv m_{\widetilde{L}_I}$ for $I = 1,2,3$. We further introduce $m_{\widetilde{F}_I} \equiv m_{\widetilde{f}_{I+3}}$ with $I = 1,2,3$ for later use.

By evaluating the gluino exchanging diagrams, we obtain the Wilson coefficients for the four-Fermi operators as

$$\begin{aligned}
C^{ijkl}_{(3)}|_{\widetilde{g}} =& -\frac{4}{3}\frac{\alpha_3}{4\pi}(C^{i'j'kl}_{5L} - C^{kj'i'l}_{5L})F(M_3, m^2_{\widetilde{Q}_I}, m^2_{\widetilde{Q}_J})\{(R^\dagger_Q)_{i'I}(R_Q)_{Ii}(R^\dagger_Q)_{j'J}(R_Q)_{Jj}\} ~, \\
C^{ijkl}_{(4)}|_{\widetilde{g}} =& -\frac{4}{3}\frac{\alpha_3}{4\pi}\big[(C^{*i'lkj'}_{5R} - C^{*kli'j'}_{5R})F(M^*_3, m^2_{\widetilde{U}_I}, m^2_{\widetilde{D}_J})\{(R^\dagger_U)_{i'I}(R_U)_{Ii}(R^\dagger_D)_{j'J}(R_D)_{Jj}\} \\
&- (C^{*i'lk'j}_{5R} - C^{*k'li'j}_{5R})F(M^*_3, m^2_{\widetilde{U}_I}, m^2_{\widetilde{U}_K})\{(R^\dagger_U)_{i'I}(R_U)_{Ii}(R^\dagger_U)_{k'K}(R_U)_{Kk}\}\big] ~. \qquad (4.10)
\end{aligned}$$

Here, $F(M, m_1^2, m_2^2)$ is a loop-function defined as

$$\begin{aligned}
F(M, m_1^2, m_2^2) &\equiv \int \frac{d^4q}{\pi^2}\frac{i}{(\not{q} - M)(q^2 - m_1^2)(q^2 - m_2^2)} ~, \\
&= \frac{M}{m_1^2 - m_2^2}\left[\frac{m_1^2}{m_1^2 - M^2}\ln\left(\frac{m_1^2}{M^2}\right) - \frac{m_2^2}{m_2^2 - M^2}\ln\left(\frac{m_2^2}{M^2}\right)\right] ~. \qquad (4.11)
\end{aligned}$$

From this expression, it is found that in the limit of degenerate squark masses or no flavor-violation, the coefficients $C^{ijkl}_{(3)}|_{\widetilde{g}}$ vanish; they become proportional to $(C^{ijkl}_{5L} - C^{kjil}_{5L})$, so

$$C^{ijkl}_{(3)}|_{\widetilde{g}}\mathcal{O}^{(3)}_{ijkl} \propto (C^{ijkl}_{5L} - C^{kjil}_{5L})\mathcal{O}^{(3)}_{ijkl} = \frac{1}{2}C^{ijkl}_{5L}\{\mathcal{O}^{(3)}_{ijkl} + \mathcal{O}^{(3)}_{jikl} - \mathcal{O}^{(3)}_{kijl} - \mathcal{O}^{(3)}_{kjil}\} = 0 ~. \qquad (4.12)$$

The last equality immediately follows from the identity[3]

$$\epsilon^{\alpha\beta}\epsilon^{\gamma\delta} - \epsilon^{\gamma\beta}\epsilon^{\alpha\delta} + \epsilon^{\alpha\gamma}\epsilon^{\delta\beta} = 0 ~, \qquad (4.13)$$

and the Fierz identities.

In the case of $C^{ijkl}_{(4)}|_{\widetilde{g}}$, they again vanish in the degenerate mass limit. On the other hand, they may not vanish when there is no flavor-mixing in squark mass matrices; in this case,

$$C^{ijkl}_{(4)}|_{\widetilde{g}} \propto (C^{*ilkj}_{5R} - C^{*klij}_{5R})[F(M^*_3, m^2_{\widetilde{U}_i}, m^2_{\widetilde{D}_j}) - F(M^*_3, m^2_{\widetilde{U}_i}, m^2_{\widetilde{U}_k})] ~, \qquad (4.14)$$

and thus they can remain sizable when there exists mass difference among right-handed squarks. Their contribution to the proton decay rate turns out to be negligible, though. Since charm quark is heavier than proton, all we have to consider is the $i = k = 1$ components, which prove to be zero as one can see from the above expression.

The absence of the gluino contribution to the proton decay rate in the case of flavor conservation can also be understood in terms of the $\mathrm{SU}(3)_L$ flavor symmetry of light quarks. As long as we consider the flavor-conserving interactions, the effective operators that give rise to proton decay are generated only from the $\int d^2\theta UDSN_l$ operator. Since superfields are bosonic, the antisymmetry with respect to color indices implies that they must also be totally antisymmetric

[3] This can be proved by noting that the left-hand side of the equation is completely antisymmetric while the indices take only two values [204].

in the $\mathrm{SU}(3)_L$ flavor indices, *i.e.*, a flavor singlet.[4] Therefore, as the gluino exchanges conserve the $\mathrm{SU}(3)_L$ flavor symmetry, the resultant four-Fermi operators also must be singlet under the symmetry. Thus, the operators must be written in a form like

$$\epsilon_{abc}\epsilon^{\lambda\kappa\tau} q^a_{L\lambda\alpha} q^b_{L\kappa\beta}\ q^c_{L\tau\gamma}\nu_\delta\ , \tag{4.15}$$

where $\lambda, \kappa, \tau$ denote the $\mathrm{SU}(3)_F$ indices and $\alpha, \beta, \dots$ represent the spinor indices. The spinor indices must be contracted appropriately to make the operator singlet under the Lorentz transformations. Now the color and flavor structure of the operator requires that the quark fields should be totally antisymmetric in the spin indices, but it is impossible for three spin-1/2 fields. Consequently, the operator should vanish. Similar arguments can be applied to the case of the bino and neutral-wino contributions.

**Bino contribution**

The bino contribution is given as

$$\begin{aligned}
C^{ijkl}_{(3)}|_{\widetilde{B}} =& \frac{6}{5}\frac{\alpha_1}{4\pi}[Y_{Q_L}Y_{L_L}(C^{ijk'l'}_{5L} - C^{k'jil'}_{5L})F(M_1, m^2_{\widetilde{Q}_K}, m^2_{\widetilde{L}_L})\big\{(R^\dagger_Q)_{k'K}(R_Q)_{Kk}(R^\dagger_L)_{l'L}(R_L)_{Ll}\big\} \\
&+ Y^2_{Q_L}(C^{i'j'kl}_{5L} - C^{kj'i'l}_{5L})F(M_1, m^2_{\widetilde{Q}_I}, m^2_{\widetilde{Q}_J})\big\{(R^\dagger_Q)_{i'I}(R_Q)_{Ii}(R^\dagger_Q)_{j'J}(R_Q)_{Jj}\big\}]\ , \\
C^{ijkl}_{(4)}|_{\widetilde{B}} =& -\frac{6}{5}\frac{\alpha_1}{4\pi}[Y_{u_R}Y_{d_R}(C^{*kli'j'}_{5R} - C^{*i'lkj'}_{5R})F(M^*_1, m^2_{\widetilde{U}_I}, m^2_{\widetilde{D}_J})\big\{(R^\dagger_U)_{i'I}(R_U)_{Ii}(R^\dagger_D)_{j'J}(R_D)_{Jj}\big\} \\
&+ Y^2_{u_R}(C^{*i'lk'j}_{5R} - C^{*k'li'j}_{5R})F(M^*_1, m^2_{\widetilde{U}_I}, m^2_{\widetilde{U}_K})\big\{(R^\dagger_U)_{i'I}(R_U)_{Ii}(R^\dagger_U)_{k'K}(R_U)_{Kk}\big\} \\
&+ Y_{d_R}Y_{e_R}(C^{*il'kj'}_{5R} - C^{*kl'ij'}_{5R})F(M^*_1, m^2_{\widetilde{D}_J}, m^2_{\widetilde{E}_L})\big\{(R^\dagger_D)_{j'J}(R_D)_{Jj}(R^\dagger_E)_{l'L}(R_E)_{Ll}\big\} \\
&+ Y_{u_R}Y_{e_R}(C^{*k'l'ij}_{5R} - C^{*il'k'j}_{5R})F(M^*_1, m^2_{\widetilde{U}_K}, m^2_{\widetilde{E}_L})\big\{(R^\dagger_U)_{k'K}(R_U)_{Kk}(R^\dagger_E)_{l'L}(R_E)_{Ll}\big\}]\ .
\end{aligned} \tag{4.16}$$

Here again, the contribution to proton decay vanishes in the limit of degenerate squark mass or no flavor-violation.

**Wino contribution**

We evaluate the wino contribution as follows:

$$\begin{aligned}
&C^{ijkl}_{(3)}|_{\widetilde{W}} \\
=&\frac{\alpha_2}{4\pi}F(M_2, m^2_{\widetilde{Q}_I}, m^2_{\widetilde{Q}_J})\big\{(R^\dagger_Q)_{i'I}(R_Q)_{Ii}(R^\dagger_Q)_{j'J}(R_Q)_{Jj}\big\}[(C^{i'kj'l}_{5L} - C^{i'j'kl}_{5L}) + \frac{1}{2}(C^{kj'i'l}_{5L} - C^{i'j'kl}_{5L})] \\
+&\frac{\alpha_2}{4\pi}F(M_2, m^2_{\widetilde{Q}_K}, m^2_{\widetilde{L}_L})\big\{(R^\dagger_Q)_{k'K}(R_Q)_{Kk}(R^\dagger_L)_{l'L}(R_L)_{Ll}\big\}[(C^{ik'jl'}_{5L} - C^{ijk'l'}_{5L}) + \frac{1}{2}(C^{k'jil'}_{5L} - C^{ijk'l'}_{5L})]\ .
\end{aligned} \tag{4.17}$$

In this case, the above coefficient does not vanish in the limit of degenerate squark mass nor no flavor-violation. One can find that it is the charged wino contribution that does remain in this limit.

[4] In fact, the operator is invariant under the $SU(3)_L \otimes SU(3)_R$ symmetry.

#### Higgsino contribution

The higgsino-dressing diagrams yield

$$
\begin{aligned}
C^{ijkl}_{(1)}|_{\widetilde{H}} =& \frac{1}{(4\pi)^2}(2C^{i'j'kl}_{5L} - C^{ki'j'l}_{5L} - C^{kj'i'l}_{5L})F(\mu^*, m^2_{\widetilde{Q}_I}, m^2_{\widetilde{Q}_J})\{(R^\dagger_Q)_{i'I}(R_Q f^*_u)_{Ii}(R^\dagger_Q)_{j'J}(R_Q f^*_d)_{Jj}\} \\
&+ \frac{1}{(4\pi)^2}(C^{*k'l'ij}_{5R} - C^{*il'k'j}_{5R})F(\mu, m^2_{\widetilde{U}_K}, m^2_{\widetilde{E}_L})\{(R^\dagger_U)_{k'K}(R_U f^T_u)_{Kk}(R^\dagger_E)_{l'L}(R_E f^T_e)_{Ll}\} \ , \\
C^{ijkl}_{(2)}|_{\widetilde{H}} =& \frac{1}{(4\pi)^2}(C^{ijk'l'}_{5L} - C^{k'jil'}_{5L})F(\mu^*, m^2_{\widetilde{Q}_K}, m^2_{\widetilde{L}_L})\{(R^\dagger_Q)_{k'K}(R_Q f^*_u)_{Kk}(R^\dagger_L)_{l'L}(R_L f^*_e)_{Ll}\} \\
&+ \frac{1}{(4\pi)^2}(C^{*kli'j'}_{5R} - C^{*i'lkj'}_{5R})F(\mu, m^2_{\widetilde{U}_I}, m^2_{\widetilde{D}_J})\{(R^\dagger_U)_{i'I}(R_U f^T_u)_{Ii}(R^\dagger_D)_{j'J}(R_D f^T_d)_{Jj}\} \ .
\end{aligned}
\tag{4.18}
$$

The contribution is found to be sizable, as we will see below.

---

As a result, we obtain the Wilson coefficients for the operators (4.7) as

$$
\begin{aligned}
C^{ijkl}_{(1)}(M_S) &= C^{ijkl}_{(1)}|_{\widetilde{H}} \ , \\
C^{ijkl}_{(2)}(M_S) &= C^{ijkl}_{(2)}|_{\widetilde{H}} \ , \\
C^{ijkl}_{(3)}(M_S) &= C^{ijkl}_{(3)}|_{\widetilde{g}} + C^{ijkl}_{(3)}|_{\widetilde{W}} + C^{ijkl}_{(3)}|_{\widetilde{B}} \ , \\
C^{ijkl}_{(4)}(M_S) &= C^{ijkl}_{(4)}|_{\widetilde{g}} + C^{ijkl}_{(4)}|_{\widetilde{B}} \ .
\end{aligned}
\tag{4.19}
$$

Again, they are evolved down to the electroweak scale according to the RGEs. The RGEs below the SUSY breaking scale are given as (see Appendix C.2)

$$
\begin{aligned}
\mu\frac{d}{d\mu}C^{ijkl}_{(1)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{11}{10}\right) + \frac{\alpha_2}{4\pi}\left(-\frac{9}{2}\right) + \frac{\alpha_3}{4\pi}(-4)\right]C^{ijkl}_{(1)} \ , \\
\mu\frac{d}{d\mu}C^{ijkl}_{(2)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{23}{10}\right) + \frac{\alpha_2}{4\pi}\left(-\frac{9}{2}\right) + \frac{\alpha_3}{4\pi}(-4)\right]C^{ijkl}_{(2)} \ , \\
\mu\frac{d}{d\mu}C^{ijkl}_{(3)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{1}{5}\right) + \frac{\alpha_2}{4\pi}(-3) + \frac{\alpha_3}{4\pi}(-4)\right]C^{ijkl}_{(3)} + \frac{\alpha_2}{4\pi}(-4)\big(C^{jikl}_{(3)} + C^{kjil}_{(3)} + C^{ikjl}_{(3)}\big) \ , \\
\mu\frac{d}{d\mu}C^{ijkl}_{(4)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{6}{5}\right) + \frac{\alpha_3}{4\pi}(-4)\right]C^{ijkl}_{(4)} + \frac{\alpha_1}{4\pi}(-4)C^{kjil}_{(4)} \ .
\end{aligned}
\tag{4.20}
$$

The contribution to the RGEs of the gauge interactions is consistent with that presented in Ref. [204].

Below the electroweak scale $\mu = m_Z$, the effective operators are no longer invariant under the $\mathrm{SU}(3)_C\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$ symmetry; instead, they must respect the $\mathrm{SU}(3)_C\otimes\mathrm{U}(1)_{\rm em}$, and all of the fields in the operators are to be written in the mass basis. As we shall see below, the experimental constraint is most stringent on the $p \to K^+\bar{\nu}$ mode. The effective Lagrangian which yields the decay mode is written down as follows:

$$\begin{aligned}\mathcal{L}(p \to K^+\bar{\nu}_i) =& C_{RL}(dsu\nu_i)\big[\epsilon_{abc}(d_R^a s_R^b)(u_L^c \nu_i)\big] + C_{RL}(usd\nu_i)\big[\epsilon_{abc}(u_R^a s_R^b)(d_L^c \nu_i)\big] \\ &+ C_{RL}(uds\nu_i)\big[\epsilon_{abc}(u_R^a d_R^b)(s_L^c \nu_i)\big] + C_{LL}(dsu\nu_i)\big[\epsilon_{abc}(d_L^a s_L^b)(u_L^c \nu_i)\big] \\ &+ C_{LL}(usd\nu_i)\big[\epsilon_{abc}(u_L^a s_L^b)(d_L^c \nu_i)\big] + C_{LL}(uds\nu_i)\big[\epsilon_{abc}(u_L^a d_L^b)(s_L^c \nu_i)\big] \ . \end{aligned} \tag{4.21}$$

Here, all of the fermions are written in terms of the mass eigenstates. The matching condition for the Wilson coefficients $C_{RL}$ and $C_{LL}$ at the electroweak scale is

$$\begin{aligned} C_{RL}(dsu\nu_i) &= 0 \ , \\ C_{RL}(usd\nu_i) &= -(V_{\rm CKM})_{j1} C_{(1)}^{12ji}(m_Z) \ , \\ C_{RL}(uds\nu_i) &= -(V_{\rm CKM})_{j2} C_{(1)}^{11ji}(m_Z) \ , \\ C_{LL}(dsu\nu_i) &= (V_{\rm CKM})_{j1}(V_{\rm CKM})_{k2}[C_{(3)}^{jk1i}(m_Z) - C_{(3)}^{kj1i}(m_Z)] \ , \\ C_{LL}(usd\nu_i) &= (V_{\rm CKM})_{j1}(V_{\rm CKM})_{k2} C_{(3)}^{k1ji}(m_Z) \ , \\ C_{LL}(uds\nu_i) &= (V_{\rm CKM})_{j1}(V_{\rm CKM})_{k2} C_{(3)}^{j1ki}(m_Z) \ . \end{aligned} \tag{4.22}$$

From the equations, it is found that only the operators $\mathcal{O}^{(1)}_{ijkl}$ and $\mathcal{O}^{(3)}_{ijkl}$ contribute to the $p \to K^+\bar{\nu}$ mode.

The Wilson coefficients are taken down to the hadronic scale $\mu = 2$ GeV, where the matrix elements of the effective operators are evaluated. The QCD corrections induced in the step are calculated at two-loop level in Ref. [205]; see Appendix C.1 for the detail discussion. The Wilson coefficients $C(\mu)$ satisfy the following RGE at two-loop level:

$$\mu \frac{d}{d\mu} C(\mu) = -\left[4\frac{\alpha_s}{4\pi} + \left(\frac{14}{3} + \frac{4}{9}N_f + \Delta\right)\frac{\alpha_s^2}{(4\pi)^2}\right] C(\mu) \ , \tag{4.23}$$

where $N_f$ denotes the number of quark flavors, and $\Delta = 0$ ($\Delta = -10/3$) for $C_{LL}$ ($C_{RL}$).

For the hadron matrix elements of the effective operators, we use the results presented by the lattice QCD calculation [206]. They are discussed in Appendix D.2. By using the results, we can eventually obtain the partial decay width of the $p \to K^+\bar{\nu}_i$ mode as

$$\Gamma(p \to K^+\bar{\nu}_i) = \frac{m_p}{32\pi}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 |\mathcal{A}(p \to K^+\bar{\nu}_i)|^2 \ , \tag{4.24}$$

where $m_p$ and $m_K$ are the masses of proton and kaon, respectively. One factor of $1 - m_K^2/m_p^2$ comes from the phase space, and the other is from the energy of the final-state anti-neutrino. The amplitude $\mathcal{A}(p \to K^+\bar{\nu}_i)$ is the sum of the Wilson coefficients times hadron matrix elements:

$$\begin{aligned}\mathcal{A}(p \to K^+\bar{\nu}_i) =& C_{RL}(dsu\nu_i)\langle K^+|(ds)_R u_L|p\rangle + C_{RL}(usd\nu_i)\langle K^+|(us)_R d_L|p\rangle \\ &+ C_{RL}(uds\nu_i)\langle K^+|(ud)_R s_L|p\rangle + C_{LL}(dsu\nu_i)\langle K^+|(ds)_L u_L|p\rangle \\ &+ C_{LL}(usd\nu_i)\langle K^+|(us)_L d_L|p\rangle + C_{LL}(uds\nu_i)\langle K^+|(ud)_L s_L|p\rangle \ . \end{aligned} \tag{4.25}$$

The definition of the matrix elements as well as their values is presented in Appendix D.2.

Now we have obtained the formulae which we need to evaluate the proton decay rate induced by the color-triplet Higgs exchange. In the following sections, we apply them to evaluate proton lifetime in the case of high-scale SUSY scenario. We will see that in the scenario the resultant lifetime can well evade the current experimental limits.

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Before concluding the subsection, we also summarize the results for other decay modes.

**Kaon and charged lepton**

The effective Lagrangian which induces the $p \to K^0 l_i^+$ ($l_i^+ = e^+, \mu^+$) mode is given as

$$\begin{aligned}\mathcal{L}(p \to K^0 l_i^+) &= C_{RL}(usul_i)\big[\epsilon_{abc}(u_R^a s_R^b)(u_L^c l_{Li})\big] + C_{LL}(usul_i)\big[\epsilon_{abc}(u_L^a s_L^b)(u_L^c l_{Li})\big] \\ &\quad + C_{LR}(usul_i)\big[\epsilon_{abc}(u_L^a s_L^b)(u_R^c l_{Ri})\big] + C_{RR}(usul_i)\big[\epsilon_{abc}(u_R^a s_R^b)(u_R^c l_{Ri})\big] \ . \end{aligned} \tag{4.26}$$

The matching condition for the Wilson coefficients is

$$\begin{aligned} C_{RL}(usul_i) &= C_{(1)}^{121i}(m_Z) \ , \\ C_{LR}(usul_i) &= (V_{\rm CKM})_{j2}\big[C_{(2)}^{1j1i}(m_Z) + C_{(2)}^{j11i}(m_Z)\big] \ , \\ C_{LL}(usul_i) &= -(V_{\rm CKM})_{j2} C_{(3)}^{1j1i}(m_Z) \ , \\ C_{RR}(usul_i) &= C_{(4)}^{121i}(m_Z) \ . \end{aligned} \tag{4.27}$$

Then, we obtain the partial decay width as

$$\Gamma(p \to K^0 l_i^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_K^2}{m_p^2}\right)^2 \big[|\mathcal{A}_L(p \to K^0 l_i^+)|^2 + |\mathcal{A}_R(p \to K^0 l_i^+)|^2\big] \ , \tag{4.28}$$

where

$$\begin{aligned} \mathcal{A}_L(p \to K^0 l_i^+) &= C_{RL}(usul_i)\langle K^0|(us)_R u_L|p\rangle + C_{LL}(usul_i)\langle K^0|(us)_L u_L|p\rangle \ , \\ \mathcal{A}_R(p \to K^0 l_i^+) &= C_{LR}(usul_i)\langle K^0|(us)_R u_L|p\rangle + C_{RR}(usul_i)\langle K^0|(us)_L u_L|p\rangle \ . \end{aligned} \tag{4.29}$$

Notice that we have used the parity transformation to obtain the hadron matrix elements for $\mathcal{A}_R$ (see Appendix D.2).

**Pion and anti-neutrino**

For the $p \to \pi^+ \bar{\nu}_i$ modes, the effective Lagrangian is given as

$$\mathcal{L}(p \to \pi^+ \bar{\nu}_i) = C_{RL}(udd\nu_i)\big[\epsilon_{abc}(u_R^a d_R^b)(d_L^c \nu_{Li})\big] + C_{LL}(udd\nu_i)\big[\epsilon_{abc}(u_L^a d_L^b)(d_L^c \nu_{Li})\big] \ , \tag{4.30}$$

and the matching condition for the Wilson coefficients is

$$\begin{aligned} C_{RL}(udd\nu_i) &= -(V_{\rm CKM})_{j1} C_{(1)}^{11ji} \ , \\ C_{LL}(udd\nu_i) &= (V_{\rm CKM})_{j1}(V_{\rm CKM})_{k1} C_{(3)}^{j1ki} \ . \end{aligned} \tag{4.31}$$

The partial decay width is then computed as

$$\Gamma(p \to \pi^+ \bar{\nu}_i) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 |\mathcal{A}(p \to \pi^+ \bar{\nu}_i)|^2 \ , \tag{4.32}$$

where

$$\mathcal{A}_L(p \to \pi^+ \bar{\nu}_i) = C_{RL}(udd\nu_i)\langle \pi^+|(ud)_R d_L|p\rangle + C_{LL}(udd\nu_i)\langle \pi^+|(ud)_L d_L|p\rangle \ . \tag{4.33}$$

**Pion/eta and charged lepton**

The effective Lagrangian for the $p \to \pi^0 l_i^+$ is

$$\begin{aligned}\mathcal{L}(p \to \pi^0 l_i^+) &= C_{RL}(udul_i)\big[\epsilon_{abc}(u_R^a d_R^b)(u_L^c l_{Li})\big] + C_{LL}(udul_i)\big[\epsilon_{abc}(u_L^a d_L^b)(u_L^c l_{Li})\big] \\ &+ C_{LR}(udul_i)\big[\epsilon_{abc}(u_L^a d_L^b)(u_R^c l_{Ri})\big] + C_{RR}(udul_i)\big[\epsilon_{abc}(u_R^a d_R^b)(u_R^c l_{Ri})\big] ~. \end{aligned} \tag{4.34}$$

We have the matching condition for the Wilson coefficients at the electroweak scale as

$$\begin{aligned} C_{RL}(udul_i) &= C_{(1)}^{111i}(m_Z) ~, \\ C_{LR}(udul_i) &= (V_{\rm CKM})_{j1}\big[C_{(2)}^{1j1i}(m_Z) + C_{(2)}^{j11i}(m_Z)\big] ~, \\ C_{LL}(udul_i) &= -(V_{\rm CKM})_{j1} C_{(3)}^{1j1i}(m_Z) ~, \\ C_{RR}(udul_i) &= C_{(4)}^{111i}(m_Z) ~. \end{aligned} \tag{4.35}$$

With the coefficients, the partial decay width is expressed as

$$\Gamma(p \to \pi^0 l_i^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\pi^2}{m_p^2}\right)^2 \big[|\mathcal{A}_L(p \to \pi^0 l_i^+)|^2 + |\mathcal{A}_R(p \to \pi^0 l_i^+)|^2\big] ~, \tag{4.36}$$

where

$$\begin{aligned} \mathcal{A}_L(p \to \pi^0 l_i^+) &= C_{RL}(udul_i)\langle \pi^0|(ud)_R u_L|p\rangle + C_{LL}(udul_i)\langle \pi^0|(ud)_L u_L|p\rangle ~, \\ \mathcal{A}_R(p \to \pi^0 l_i^+) &= C_{LR}(udul_i)\langle \pi^0|(ud)_R u_L|p\rangle + C_{RR}(udul_i)\langle \pi^0|(ud)_L u_L|p\rangle ~. \end{aligned} \tag{4.37}$$

While the decay modes are generally suppressed in the case of the dimension-five proton decay, they become dominant for the dimension-six proton decay discussed in the next section.

The same interaction also induces the $p \to \eta^0 l_i^+$ modes. In this case we have

$$\Gamma(p \to \eta^0 l_i^+) = \frac{m_p}{32\pi}\left(1 - \frac{m_\eta^2}{m_p^2}\right)^2 \big[|\mathcal{A}_L(p \to \eta^0 l_i^+)|^2 + |\mathcal{A}_R(p \to \eta^0 l_i^+)|^2\big] ~, \tag{4.38}$$

with

$$\begin{aligned} \mathcal{A}_L(p \to \eta^0 l_i^+) &= C_{RL}(udul_i)\langle \eta^0|(ud)_R u_L|p\rangle + C_{LL}(udul_i)\langle \eta^0|(ud)_L u_L|p\rangle ~, \\ \mathcal{A}_R(p \to \eta^0 l_i^+) &= C_{LR}(udul_i)\langle \eta^0|(ud)_R u_L|p\rangle + C_{RR}(udul_i)\langle \eta^0|(ud)_L u_L|p\rangle ~. \end{aligned} \tag{4.39}$$

### 4.1.2 Decoupling can revive the minimal SUSY GUT

First, we consider the so-called minimal flavor violation [207, 208] case, namely, we assume that the CKM matrix is the only source for all of the flavor-violating terms in the MSSM. It implies $R_Q = R_U = R_D = R_L = R_E = \mathbb{1}$. Further, we take all of the sfermion masses equal to $M_S$, for brevity. In this case, only the charged winos and higgsinos give rise to sizable contribution to proton decay, as we have seen in the previous section. The resultant Wilson coefficients are evaluated as

$$
\begin{aligned}
C_{LL}(usd\nu_e) &= C_{LL}(uds\nu_e) \\
&\simeq \frac{2\alpha_2^2}{\sin 2\beta}\frac{m_d V_{ud}^*}{m_W^2 M_{H_C}} f(M_2, M_S^2)\left[e^{i\varphi_2} m_c V_{cd} V_{cs} + e^{i\varphi_3} m_t V_{td} V_{ts}\right] , \\
C_{LL}(usd\nu_\mu) &= C_{LL}(uds\nu_\mu) \\
&\simeq \frac{2\alpha_2^2}{\sin 2\beta}\frac{m_s V_{us}^*}{m_W^2 M_{H_C}} f(M_2, M_S^2)\left[e^{i\varphi_2} m_c V_{cd} V_{cs} + e^{i\varphi_3} m_t V_{td} V_{ts}\right] , \\
C_{LL}(usd\nu_\tau) &= C_{LL}(uds\nu_\tau) \\
&\simeq \frac{2\alpha_2^2}{\sin 2\beta}\frac{m_b V_{ub}^*}{m_W^2 M_{H_C}} f(M_2, M_S^2)\left[e^{i\varphi_2} m_c V_{cd} V_{cs} + e^{i\varphi_3} m_t V_{td} V_{ts}\right] ,
\end{aligned} \tag{4.40}
$$

and

$$
\begin{aligned}
C_{RL}(usd\nu_\tau) &\simeq -\frac{\alpha_2^2}{\sin^2 2\beta}\frac{m_t^2 m_s m_\tau}{m_W^4 M_{H_C}} f(\mu, M_S^2) e^{i\varphi_1} V_{us} V_{td} V_{tb}^* , \\
C_{RL}(uds\nu_\tau) &\simeq -\frac{\alpha_2^2}{\sin^2 2\beta}\frac{m_t^2 m_d m_\tau}{m_W^4 M_{H_C}} f(\mu, M_S^2) e^{i\varphi_1} V_{ud} V_{ts} V_{tb}^* ,
\end{aligned} \tag{4.41}
$$

where we ignore the sub-leading terms additionally suppressed by light-quark masses or the CKM matrix elements. Moreover, we neglect the renormalization factors for simplicity. The loop function $f(M, m^2)$ is defined by

$$
\begin{aligned}
f(M, m^2) &\equiv \lim_{m_0^2 \to m^2} F(M, m^2, m_0^2) \\
&= M\left[\frac{1}{m^2 - M^2} - \frac{M^2}{(m^2 - M^2)^2}\ln\left(\frac{m^2}{M^2}\right)\right] .
\end{aligned} \tag{4.42}
$$

The terms in Eqs. (4.40) and (4.41) are induced by the wino- and higgsino exchanging processes, respectively. The processes are illustrated in Fig. 4.3. Here, the gray and black dots indicate the dimension-five effective interactions and the mass terms for wino or higgsino, respectively. Notice that although the contribution of the diagram (b) is suppressed by the CKM matrix elements as it is generated in the flavor changing process, it turns out to be sizable because of the large Yukawa couplings of the third generation fermions [67, 209, 210]. Let us further consider the significance of the contributions in the case of the high-scale SUSY model. In the limit of $M \ll M_S$, the function leads to

$$
f(M, M_S^2) \to \frac{M}{M_S^2}, \qquad (M \ll M_S) , \tag{4.43}
$$

while in the limit of $M \to M_S$, it follows that

$$
f(M, M_S^2) \to \frac{1}{2M_S}, \qquad (M \to M_S) . \tag{4.44}
$$

Note that the function is proportional to $M$, when $M \lesssim M_S$. For this reason, the contribution of the diagrams in Fig. 4.3 is enhanced when the masses of the exchanged particles are large. In particular, in the case of $\mu \gg M_2$, the higgsino exchange contribution (the diagram (b) in

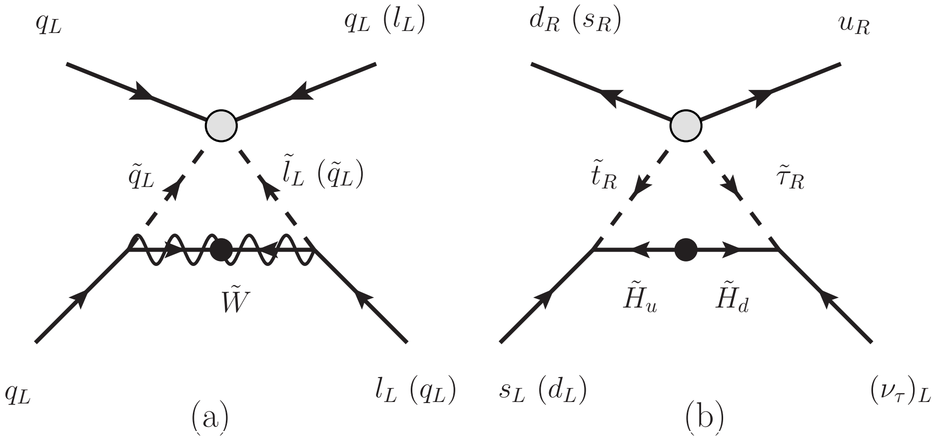


Figure 4.3: One-loop diagrams which yield the baryon-number violating four-Fermi operators. Diagrams (a) and (b) are generated by charged wino and higgsino exchanging processes, respectively. Gray dots indicate dimension-five effective interactions, while black dots represent wino or higgsino mass terms.

Fig. 4.3) dominates the wino exchange one. We also find from the behavior of the loop function that the transition amplitude is considerably suppressed when the sfermions have sufficiently large masses. Thus, we expect that the experimental constraints on the proton decay rate may be avoided in the high-scale SUSY scenario.

Now we show numerical results of the proton decay lifetime. From now on, we define $\hat{f}$ by $\hat{f} \equiv \hat{f}_d(M_{\rm GUT})$. Uncertainty coming from the definition is briefly discussed below. First, we consider the case where the higgsino mass is of the order of the sfermion masses, $M_S$. In this case, the higgsino exchange contribution (the diagram (b) in Fig. 4.3) dominates the wino exchange one. For this reason, the lifetime has little dependence on the GUT phases $\varphi_i$ as well as the wino mass. Thus, it is possible to make a robust prediction for the proton decay lifetime. As the right-handed stop and stau run in the loop in the higgsino exchanging diagram, $M_S$ should be regarded as their masses, which we assume to be degenerate for brevity. For $M_S = \mu$, the proton lifetime $\tau_p$ is approximately given as

$$\tau_p \simeq 4 \times 10^{35} \times \sin^4 2\beta \left(\frac{M_S}{10^2 \text{ TeV}}\right)^2 \left(\frac{M_{H_C}}{10^{16} \text{ GeV}}\right)^2 \text{ yrs} , \tag{4.45}$$

with small dependence on the renormalization factors being ignored. As you can see from this formula, the proton decay rate can be well above the current experimental limits from Super-Kamiokande [69, 70],

$$\tau(p \to K^+ \bar{\nu}) > 5.9 \times 10^{33} \text{ yrs} , \tag{4.46}$$

with the SUSY scale being much higher than the electroweak scale. As discussed in the previous chapter, $M_{H_C}$ can be evaluated by means of the RGE analysis, which suggests that it may be around the GUT scale in the case of high-scale SUSY [188]. The prediction is, however, quite sensitive to the mass spectrum below the GUT scale, especially to the masses of higgsinos and gauginos. For this reason, we just fix it to be around the GUT scale in the following calculation.

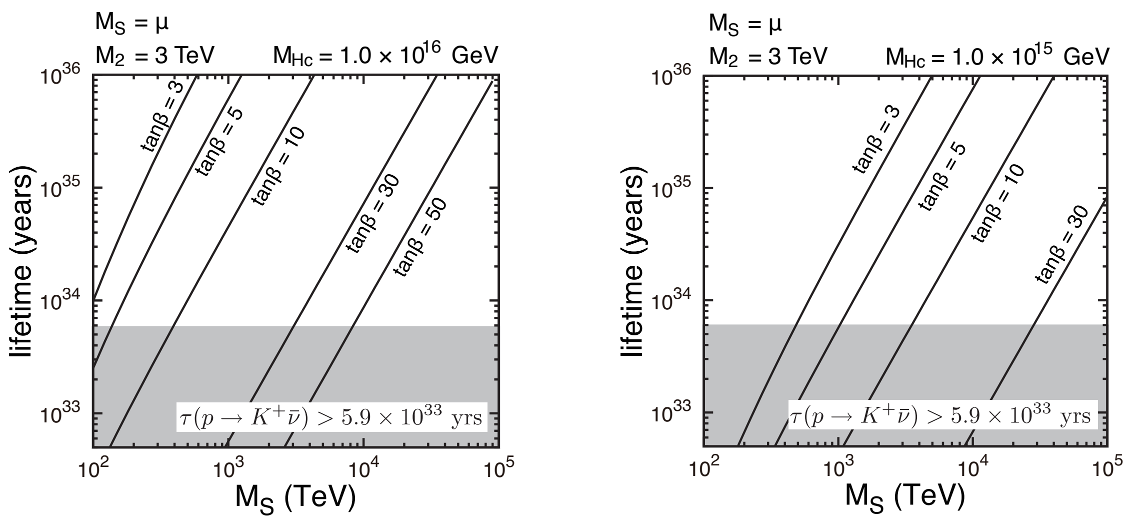


Figure 4.4: Lifetime of $p \to K^+\bar{\nu}$ mode as functions of $M_S = \mu$. Wino mass is set to be 3 TeV and $M_{H_C} = 1.0 \times 10^{16}$ GeV (left) and $1.0 \times 10^{15}$ GeV (right). Solid lines correspond to $\tan\beta = 3, 5, 10, 30$, and 50 from left-top to right-bottom, respectively. Shaded region is excluded by the current experimental bound, $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs [69, 70].

One easily obtains proton lifetimes corresponding to other values of $M_{H_C}$ by using the power law given in Eq. (4.45).

In Fig. 4.4, we present the lifetime of the $p \to K^+\bar{\nu}$ mode as functions of $M_S = \mu$. Here, the wino mass is set to be 3 TeV, while the result rarely depends on the mass as long as $M_2 \ll M_S$. The color-triplet Higgs mass is fixed to $M_{H_C} = 1.0 \times 10^{16}$ GeV ($1.0 \times 10^{15}$ GeV) in the left (right) panel. The solid lines are for $\tan\beta = 3, 5, 10, 30$, and 50 from left-top to right-bottom, respectively. The shaded region is excluded by the current experimental bound, $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs [69, 70]. In the calculation, we use the one-loop RGEs for both the couplings and the effective operators, and the hadron matrix elements are evaluated with the indirect method. The figure illustrates the behavior presented in Eq. (4.45), and shows that the proton decay lifetime in the high-scale SUSY scenario can evade the experimental constraints, especially for small $\tan\beta$ and high SUSY breaking scales. Moreover, we see that the relation between the results presented in the left and right panels is well explained by the simple power law in Eq. (4.45), though the renormalization factors may also be changed with different values of $M_{H_C}$. For this reason, we just fix $M_{H_C} = 1.0 \times 10^{16}$ GeV in the following analysis.

Next, we consider the case where the higgsinos are lighter than the sfermions. In this case, the lifetime depends on the extra phases appearing in the GUT Yukawa couplings. Here, we take the phases so that they yield the maximal amplitude for the proton decay rate. This requirement together with the constraint $\sum_i \varphi_i = 0$ uniquely determines all of the phases $\varphi_i$. Since the choice of phases gives the maximal proton decay rate, we are to obtain the most stringent limit on the parameters. In addition, we assume that both the higgsino and wino mass parameters are real and positive. However, as long as one chooses the phases constructively, the results would not change since it is always possible to include the extra phases of the higgsino and wino masses into the redefinition of the phases $\varphi_i$.

In Fig. 4.5, we plot the proton lifetime as functions of the higgsino mass. Here, the wino

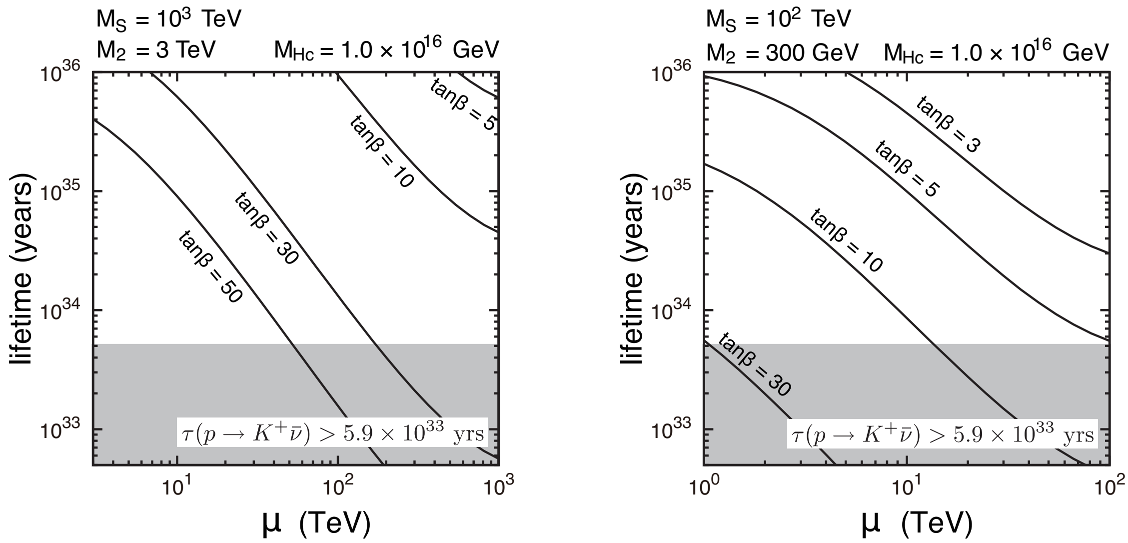


Figure 4.5: Lifetime of $p \to K^+\bar{\nu}$ mode as functions of higgsino mass $\mu$. Color-triplet Higgs mass is set to be $M_{H_C} = 1.0 \times 10^{16}$ GeV. Left panel shows the case of $M_S = 10^3$ TeV and $M_2 = 3$ TeV, while right shows $M_S = 10^2$ TeV and $M_2 = 300$ GeV. Solid lines correspond to $\tan\beta = 5, 10, 30$, and 50 from right-top to left-bottom, respectively. Shaded region is excluded by the current experimental bound, $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs [69, 70].

and sfermion[5] masses are set to be $M_2 = 3$ TeV and $M_S = 10^3$ TeV ($M_2 = 300$ GeV and $M_S = 10^2$ TeV) in the left (right) panel, respectively, and the color-triplet Higgs mass is taken to be $M_{H_C} = 1.0 \times 10^{16}$ GeV. The solid lines correspond to $\tan\beta = 3, 5, 10, 30$, and 50 from right-top to left-bottom, respectively. Again the shaded region is excluded by the current experimental bound, $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs . It is found that the lifetime considerably depends on the mass of higgsino as well as the value of $\tan\beta$. It illustrates that the higgsino contribution is dominant in a wide range of parameter region. Indeed, the contribution gets more significant as the higgsino mass is raised up. Anyway, we have found that in the high-scale SUSY scenario, the minimal SUSY SU(5) GUT is still alive without any conspiracy of suppressing the dimension-five operators.

Before concluding the section, let us briefly discuss uncertainties of the computation of the proton decay rate. As for the uncertainty of input parameters, the error of the hadron matrix elements dominates other factors. To estimate the effects on the resultant decay rate, we use the matrix elements evaluated with the indirect method, and include the uncertainties of $\alpha_p$ and $\beta_p$ presented in Eq. (D.28). On the other hand, the theoretical error of the calculation mainly originates from the unknown threshold corrections to the Yukawa couplings at the GUT scale. As discussed in Sec. 3.3, such corrections are expected to make up for the failure of the Yukawa unification. The significance of the contribution is thus expected to be $\mathcal{O}(f_d - f_e)$ at least. In the present analysis, we estimate its effects by varying the down-type Yukawa couplings $\hat{f}_i$ at the GUT scale by a factor of $(\hat{f}_{d_i}/\hat{f}_{e_i})^{1/2}$. In Fig. 4.6, we show the resultant

[5] To be concrete, we regard $M_S$ as the stop mass, and all of the other sfermion masses are assumed to be degenerate with $M_S$. Generally speaking, stops are lighter than other sfermions, especially those of the first and second generations. So, even though one relaxes the degeneration assumption, one ends up obtaining a smaller proton decay rate.

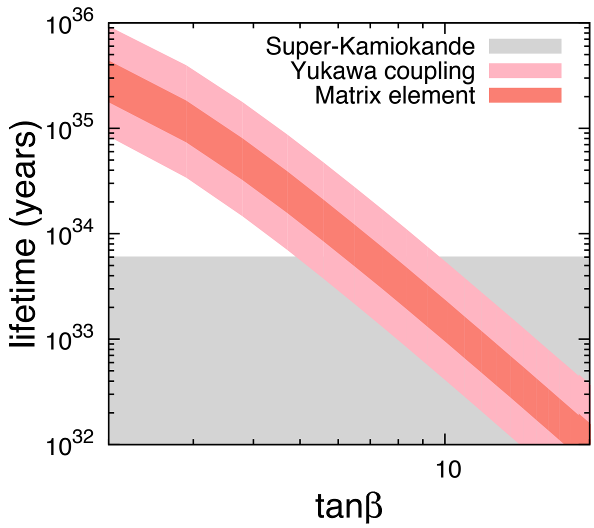


Figure 4.6: Lifetime of the $p \to K^+\bar{\nu}$ mode as a function of $\tan\beta$. Dark and light pink regions represent the uncertainty coming from the input error of the matrix elements and the unknown threshold corrections to the Yukawa couplings at the GUT scale, respectively. We take $M_S =$ 100 TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, and $M_{H_C} = 10^{16}$ GeV. Additional phases in the GUT Yukawa couplings, $\varphi_i$, are set to be $\varphi_i = 0$. Shaded region is excluded by the current experimental bound, $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs [69, 70].

uncertainty of the lifetime of the $p \to K^+\bar{\nu}$ mode as a function of $\tan\beta$. The dark and light pink regions represent the uncertainty coming from the input error of the matrix elements and the unknown threshold corrections to the Yukawa couplings at the GUT scale, respectively. Here, we take $M_S = 100$ TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, and $M_{H_C} = 10^{16}$ GeV. Additional phases in the GUT Yukawa couplings, $\varphi_i$, are set to be $\varphi_i = 0$, though the uncertainty from them is much smaller than the above ones. The shaded region again shows the Super-Kamiokande limit. From this figure, it is found that the uncertainties are so large that they may change the decay rate by an $\mathcal{O}(1)$ factor. Hence, it is difficult to determine the SUSY parameters precisely by using the results from the proton decay experiments even when a signal of the proton decay mode is actually detected in the future experiment.

### 4.1.3 Proton decay in high-scale SUSY with sfermion flavor violation

Next, we discuss the case where the sfermion sector contains sizable flavor violation [201]. In this case, not only the charged wino and higgsino exchanging processes, but also the neutral gaugino and higgsino exchange can contribute to the proton decay.

To begin with, we parametrize the structure of the sfermion mass matrices as follows:

$$\widetilde{m}^2_{\tilde{f}} = M_S^2 \begin{pmatrix} 1+\Delta_1^{\tilde{f}} & \delta_{12}^{\tilde{f}} & \delta_{13}^{\tilde{f}} \\ \delta_{12}^{\tilde{f}*} & 1+\Delta_2^{\tilde{f}} & \delta_{23}^{\tilde{f}} \\ \delta_{13}^{\tilde{f}*} & \delta_{23}^{\tilde{f}*} & 1+\Delta_3^{\tilde{f}} \end{pmatrix} , \tag{4.47}$$

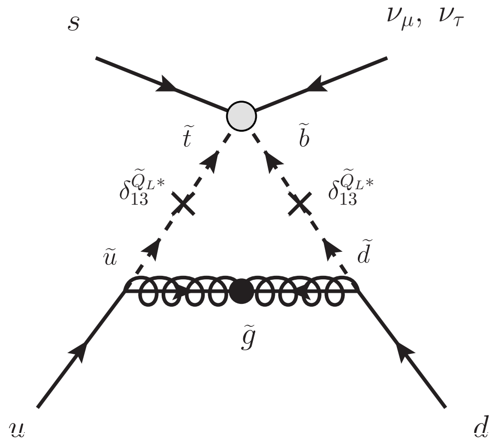


Figure 4.7: Diagram which induces the dominant contribution in the presence of the $\delta_{13}^{\widetilde{Q}_L}$ flavor mixing, which is denoted by x-mark.

with $\tilde{f} = \widetilde{Q}_L, \widetilde{u}_R, \widetilde{d}_R, \widetilde{e}_R, \widetilde{L}_L$. In the minimal SU(5) GUT, there are relations among the sfermion mass-squared matrices at the GUT scale as in Eq. (3.39):

$$\widetilde{m}^2_{\tilde{Q}_L} = V_{QU}(\widetilde{m}^2_{\tilde{u}_R})^T V^\dagger_{QU} = V_{QE}(\widetilde{m}^2_{\tilde{e}_R})^T V^\dagger_{QE} \ , \quad \widetilde{m}^2_{\tilde{d}_R} = V^*_{DL}(\widetilde{m}^2_{\tilde{L}_L})^T V^T_{DL} \ , \tag{4.48}$$

where $V_{QU}$, $V_{QE}$ and $V_{DL}$ are given in Sec. 3.1. The relations are violated in the low-energy regions due to the renormalization effects. In the following analysis, we regard all of the components in Eq. (4.47) as just free parameters for brevity.

In the presence of sizable flavor violation, the gluino contribution becomes significant because of the large coupling of $\alpha_3$. Since only the $C^{ijkl}_{(3)}|_{\tilde{g}}$ in Eq. (4.10) contributes to the $p \to K^+\bar{\nu}$ proton decay, the flavor mixing in the mass matrix of $\widetilde{Q}_L$ is most important; in particular $\delta_{13}^{\widetilde{Q}_L}$ gives rise to the biggest effects. Let us estimate the significance. The dominant contribution to the $p \to K^+\bar{\nu}$ mode is induced by the diagram in Fig. 4.7. Here, the cross-mark indicates the flavor mixing. When the flavor violation is small but sizable, *e.g.*, $\delta_{13}^{\widetilde{Q}_L} \sim 0.1$, the contribution is evaluated as

$$\begin{aligned} C_{LL}(uds\nu_\mu) &\simeq -\frac{4}{3}\frac{\alpha_2\alpha_3}{\sin 2\beta}\frac{m_t m_s}{M_{H_C} m_W^2} f(M_3, M_S^2)\ e^{i\varphi_3}(V_{ud}V_{cs}V^*_{cs})\big(\delta_{13}^{\widetilde{Q}_L *}\big)^2 \ , \\ C_{LL}(uds\nu_\tau) &\simeq -\frac{4}{3}\frac{\alpha_2\alpha_3}{\sin 2\beta}\frac{m_t m_b}{M_{H_C} m_W^2} f(M_3, M_S^2)\ e^{i\varphi_3}(V_{ud}V_{cs}V^*_{cb})\big(\delta_{13}^{\widetilde{Q}_L *}\big)^2 \ , \end{aligned} \tag{4.49}$$

and other Wilson coefficients are found to be sub-dominant. By comparing the results to the higgsino contribution (4.41) in the minimal flavor violation case, we can see that the gluino contribution becomes dominant when

$$\big|\delta_{13}^{\widetilde{Q}_L}\big| \ \gtrsim\ 2\times 10^{-3} \times \left(\frac{1}{\sin 2\beta}\left|\frac{\mu}{M_3}\right|\right)^{\frac{1}{2}} \ . \tag{4.50}$$

Before showing the results for the full computation, we briefly comment on the features of other contributions. The wino and bino contributions are in general suppressed by the relatively

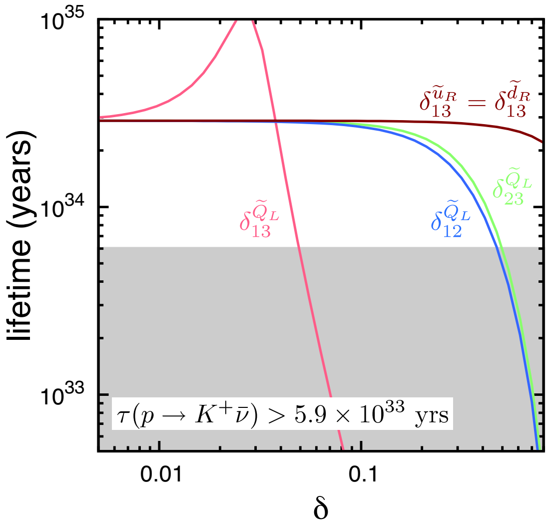


Figure 4.8: Lifetime of the $p \to K^+\bar{\nu}$ mode as functions of flavor mixing parameters $\delta$. We take $M_S = 100$ TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, $M_{H_C} = 10^{16}$ GeV, and $\tan\beta = 5$. Pink, blue, green, brown lines correspond to $\delta_{13}^{\tilde{Q}_L}$, $\delta_{12}^{\tilde{Q}_L}$, $\delta_{23}^{\tilde{Q}_L}$, $\delta_{13}^{\tilde{u}_R} = \delta_{13}^{\tilde{d}_R}$, respectively. Additional phases in the GUT Yukawa couplings, $\varphi_i$, are set to be $\varphi_i = 0$. Shaded region is excluded by the current experimental bound, $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs [69, 70].

small gauge couplings compared with the gluino contribution. The higgsino contribution has already exploited the flavor changing in the Yukawa couplings, which is required to make the best of the third generation Yukawa couplings. Therefore, the flavor mixing in sfermion masses does not result in the enhancement of the contribution.

As we will see below, the effects of the other mixing parameters are generally sub-dominant. In particular, when the flavor violation occurs only in the slepton sector, the proton decay rate is rarely changed. This is because the gluino exchange process does not contribute to the proton decay in such a case. In addition, when only the right-handed squarks feel the flavor violation, the $p \to K^+\bar{\nu}$ mode is not enhanced because of the same reason. In such a case, on the other hand, the decay modes including a charged lepton in their final states, such as the $p \to K^0\mu^+$ mode, are considerably enhanced.

Now we show the results. In Fig. 4.8, we plot the lifetime of the $p \to K^+\bar{\nu}$ mode as functions of flavor mixing parameters $\delta$. Here, we take $M_S = 100$ TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, $M_{H_C} = 10^{16}$ GeV, and $\tan\beta = 5$. The pink, blue, green, brown lines correspond to $\delta_{13}^{\tilde{Q}_L}$, $\delta_{12}^{\tilde{Q}_L}$, $\delta_{23}^{\tilde{Q}_L}$, $\delta_{13}^{\tilde{u}_R} = \delta_{13}^{\tilde{d}_R}$, respectively. The dependence on other parameters, such as $\delta_{13}^{\tilde{L}_L}$, is found to be negligible, and thus we do not show it. The extra phases in the GUT Yukawa couplings, $\varphi_i$, are set to be $\varphi_i = 0$ in this figure. Shaded region is excluded by the current experimental bound, $\tau(p \to K^+\bar{\nu}) > 5.9 \times 10^{33}$ yrs [69, 70]. In the computation, we use the hadron matrix elements obtained by the direct method. From this figure, we can find that the proton decay rate is significantly enhanced when $\delta_{13}^{\tilde{Q}_L}$ is sizable. $\delta_{12}^{\tilde{Q}_L}$ and $\delta_{23}^{\tilde{Q}_L}$ also shorten the proton lifetime. This observation shows that the gluino contribution is important in the presence of flavor violation.

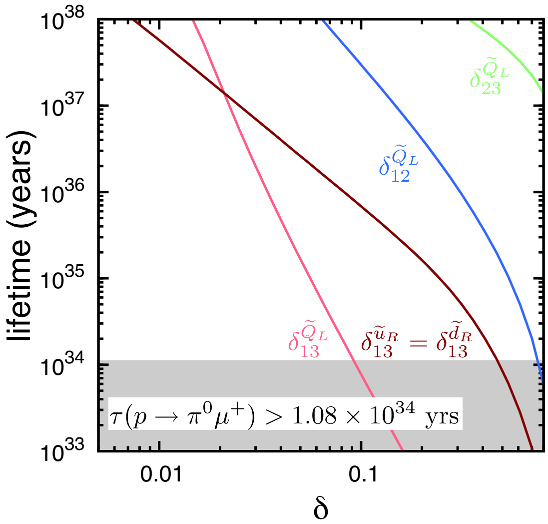


Figure 4.9: Lifetime of the $p \to \pi^0 \mu^+$ mode as functions of flavor mixing parameters $\delta$. We take $M_S = 100$ TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, $M_{H_C} = 10^{16}$ GeV, and $\tan\beta = 5$. Pink, blue, green, brown lines correspond to $\delta_{13}^{\widetilde{Q}_L}$, $\delta_{12}^{\widetilde{Q}_L}$, $\delta_{23}^{\widetilde{Q}_L}$, $\delta_{13}^{\widetilde{u}_R} = \delta_{13}^{\widetilde{d}_R}$, respectively. Additional phases in the GUT Yukawa couplings, $\varphi_i$, are set to be $\varphi_i = 0$. Shaded region is excluded by the current experimental bound, $\tau(p \to \pi^0 \mu^+) > 1.08 \times 10^{34}$ yrs [211].

We further conduct a similar analysis for the $p \to \pi^0 \mu^+$ decay mode, and plot its lifetime as functions of the flavor mixing parameters in Fig. 4.9. Here again, we take $M_S = 100$ TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, $M_{H_C} = 10^{16}$ GeV, and $\tan\beta = 5$. The pink, blue, green, brown lines correspond to $\delta_{13}^{\widetilde{Q}_L}$, $\delta_{12}^{\widetilde{Q}_L}$, $\delta_{23}^{\widetilde{Q}_L}$, $\delta_{13}^{\widetilde{u}_R} = \delta_{13}^{\widetilde{d}_R}$, respectively. We take $\varphi_i = 0$ in this figure. Shaded region is excluded by the current experimental bound, $\tau(p \to \pi^0 \mu^+) > 1.08 \times 10^{34}$ yrs [211]. This figure again shows that the lifetime highly depends on $\delta_{13}^{\widetilde{Q}_L}$. A characteristic feature in this case is that the right-handed squark flavor violation, such as $\delta_{13}^{\widetilde{u}_R}$ and $\delta_{13}^{\widetilde{d}_R}$, is also important. This is because when the final state of proton decay includes a charged lepton—$\mu^+$ in the present case—not only the operators $\mathcal{O}_{ijkl}^{(1)}$ and $\mathcal{O}_{ijkl}^{(3)}$ but also $\mathcal{O}_{ijkl}^{(2)}$ and $\mathcal{O}_{ijkl}^{(4)}$ can contribute to the decay rate. Notice that in the gluino exchange process the right-handed squark flavor violation can only contribute to the operator $\mathcal{O}_{ijkl}^{(4)}$. For this reason, although $\delta_{13}^{\widetilde{u}_R}$ and $\delta_{13}^{\widetilde{d}_R}$ scarcely affect the neutral lepton decay modes such as $p \to K^+ \bar{\nu}$, they can enhance the charged lepton modes through the operator $\mathcal{O}_{ijkl}^{(4)}$.

Finally, we show the results for other decay modes. They are summarized in Fig. 4.10. Here, the left and right panels show the cases of the minimal flavor violation and $\delta_{13}^{\widetilde{Q}_L} = 0.1$, respectively. We take a similar set of parameters to those used in the previous figures. It is found that the flavor violation considerably accelerates all of the decay modes. Now let us look for a specific feature of the proton decay associated with sfermion flavor violation. As one can see from Fig. 4.10, in the minimal flavor violation case, only the neutral lepton decay modes, $p \to K^+ \bar{\nu}$ and $p \to \pi^+ \bar{\nu}$, have sizable decay rates. Therefore we should focus on the charged

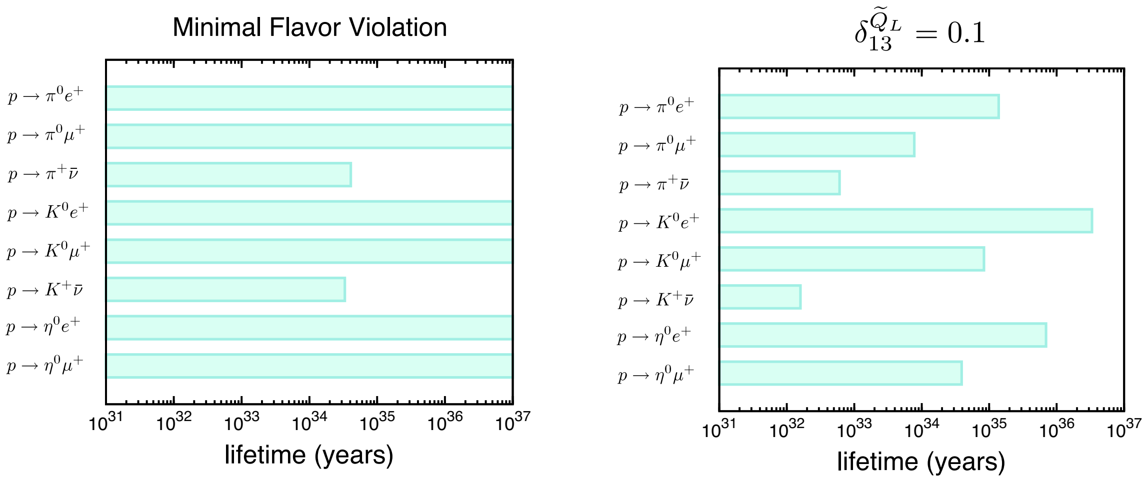


Figure 4.10: Bar chart for lifetime of each decay mode in the case of the minimal flavor violation (left) and $\delta_{13}^{\widetilde{Q}_L} = 0.1$. Again, we take $M_S = 100$ TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, $M_{H_C} = 10^{16}$ GeV, $\tan\beta = 5$, and $\varphi_i = 0$.

lepton decay modes. Charged leptonic decay is also generated via the $X$-boson exchanging process. Since the process is induced by the gauge interactions, the CKM matrix is the only source for the flavor violation. Thus, in the $X$-boson exchange contribution, the decay modes including different generations in their final states, such as $p \to \pi^0\mu^+$ and $p \to K^0 e^+$, suffer from the CKM suppression. We will see this feature in the next section. Hence, such decay modes can be regarded as characteristic of new flavor violation. Among them, the experimental constraint on the $p \to \pi^0\mu^+$ mode is the severest, and thus it may offer a good prove for the sfermion flavor violation. If the decay process as well as the $p \to K^+\bar{\nu}$ decay is detected in future experiments, it may suggest the existence of sizable flavor violation in the sfermion sector.

Throughout the calculation, we fix the extra phases in the GUT Yukawa couplings to be $\varphi_i = 0$. Let us briefly comment on the uncertainty coming from our ignorance of the phases. We have checked that when the flavor violation is moderate, a significant cancellation can occur between the ordinary (minimal flavor) contribution and that induced by the sfermion flavor violation. In such region, resultant decay lifetime may suffer from the theoretical uncertainty of more than an order of magnitude. When the flavor violating contribution dominates the minimal flavor one, on the other hand, the decay rate rarely depends on the phases, thus the prediction becomes robust.

Of course, large amount of sfermion flavor violation is constrained by the low-energy precision measurements, even in the case of high-scale SUSY. It is important to study the allowed region for sfermion flavor violation in terms of both the proton decay experiments and the precision experiments, especially the measurements of EDMs, the meson mixing, the charged lepton flavor violating processes, and so on. A systematic analysis in the direction will be done elsewhere.

After all, in the presence of sfermion flavor violation, which can naturally be sizable in the high-scale SUSY scenario, a variety of proton decay modes may lie in a region which can be probed in future proton decay experiments. In consequence, proton decay experiments might shed light on SUSY even though it is broken at a relatively high-scale, and provide a way of investigating the structure of sfermion sector.

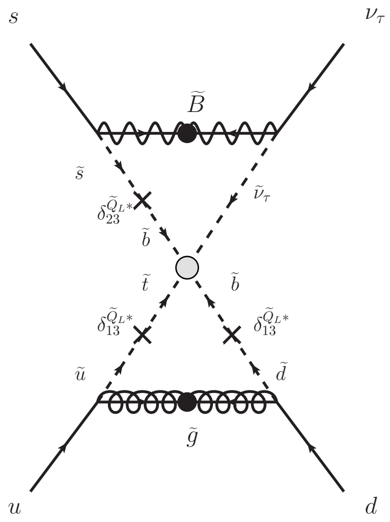


Figure 4.11: Contribution of the soft terms for the dimension-five effective operators to proton decay. As discussed in the text, the contribution turns out to vanish.

### 4.1.4 Contribution of soft SUSY breaking operator

Up to now, we only consider the dimension-five effective operators which are exactly supersymmetric. However, these operators also give rise to the dimension-four soft SUSY breaking operators through the SUGRA effects [212, 213, 214]. As explained in Sec. 2.2.1, the dimension-five operators should be accompanied by the compensator as

$$\int d^2\theta \, \frac{\langle\Sigma\rangle}{\Sigma}\left[C_{5L}^{ijkl}\mathcal{O}_{ijkl}^{5L} + C_{5R}^{ijkl}\mathcal{O}_{ijkl}^{5R}\right] , \tag{4.51}$$

where we normalize the above equation by $\langle\Sigma\rangle$ to make it have appropriate mass dimensions. Then, the dimension-four soft-terms are readily obtained:

$$\mathcal{L}_4^{\text{soft}} = -m_{3/2}\left[C_{5L}^{ijkl}\widetilde{\mathcal{O}}_{ijkl}^{5L} + C_{5R}^{ijkl}\widetilde{\mathcal{O}}_{ijkl}^{5R}\right] , \tag{4.52}$$

where $\widetilde{\mathcal{O}}$ imply that all of the superfields in the operator should be replaced by their scalar components. The interactions induce the proton decay four-Fermi operators through the two-loop diagrams with the exchange of gauginos and higgsinos. This contribution is suppressed by additional factor $g^2/(16\pi^2)(M_{\tilde{g}}/M_S)$, compared to the usual one loop contribution. This effectively results in a two-loop suppression factor in the case of anomaly-mediation. However it is not trivial whether the soft-term contribution is really suppressed in the presence of large flavor violation, since additional enhancement of the third generation Yukawa couplings can be exploited via the flavor violation. Such an example for the process is shown in Fig. 4.11. To make the most of the enhancement, all the fields included in the effective interaction vertex, which is illustrated by a gray dot in Fig. 4.11, should be of the third generation. However, such a vertex is forbidden by the antisymmetry of the color indices, and therefore the diagram presented in Fig. 4.11 actually vanishes. After all, the contribution of the soft terms could not use additional enhancement by the third generation Yukawa couplings, and thus can be safely neglected in the present calculation.

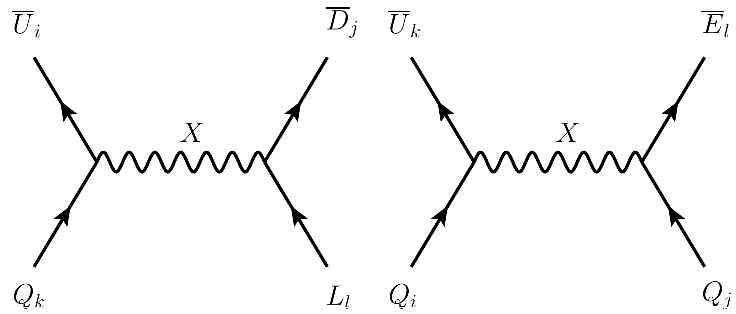


Figure 4.12: Supergraphs which induce the dimension-six proton decay operators.

### 4.1.5 Proton decay induced by the Planck suppressed operators

As already mentioned above, one expects that there is no explicit continuous global symmetry in nature [215, 216]. So, in an effective theory below the fundamental scale, *e.g.*, the Planck scale, we would expect that any baryon and lepton number violating operators allowed by the symmetry in the effective theory should be present with their coefficients suppressed by the Planck mass $M_{\rm Pl}$. In this subsection, therefore, we briefly comment on possible effects of such operators on the proton decay lifetime. For recent work on this matter, see Ref. [217].

Let us assume that there exist the dimension-five proton decay operators with coefficients of the order $1/M_{\rm Pl}$. Note that we do not assume the operators to be suppressed by the Yukawa couplings as in the case of those induced by the color-triplet Higgs exchanges. In this case, the $p \to K^+\bar{\nu}$ decay can be induced by the neutral gaugino exchanging processes even if the flavor-changing effects by the sfermion mass matrices are negligible. Then it turns out that the resultant proton decay rate is well above those evaluated in the previous sections and the current experimental limits on the proton decay lifetime give stronger limits on the SUSY breaking scale; for instance, with gauginos lighter than sfermions by a loop factor, the scalar masses must be at least $10^6$ TeV [217].

Usually, one expects that there exists some underlying flavor symmetry, which may be responsible for the structure of the Yukawa couplings, and the symmetry gives additional suppression on the operators. Such a symmetry may also suppress the flavor violation in the sfermion mass matrices. The significance of the suppression is, however, completely model-dependent, and to study the possibilities of the flavor symmetry is beyond the scope of this thesis.

## 4.2 Dimension-6 proton decay

Next, we discuss the proton decay induced by the SU(5) gauge boson exchange. In this case, the effective Lagrangian is expressed in terms of the dimension-six effective operators:

$$\mathcal{L}_6^{\rm eff} = C_{6(1)}^{ijkl}\mathcal{O}_{ijkl}^{6(1)} + C_{6(2)}^{ijkl}\mathcal{O}_{ijkl}^{6(2)} ~, \tag{4.53}$$

where

$$\mathcal{O}^{6(1)}_{ijkl} = \int d^2\theta d^2\bar{\theta}\ \epsilon_{abc}\epsilon_{\alpha\beta}\big(\overline{U}^\dagger_i\big)^a\big(\overline{D}^\dagger_j\big)^b e^{-\frac{2}{3}g'B}\big(e^{2g_3G}Q^\alpha_k\big)^c L^\beta_l\ , \tag{4.54}$$

$$\mathcal{O}^{6(2)}_{ijkl} = \int d^2\theta d^2\bar{\theta}\epsilon_{abc}\epsilon_{\alpha\beta}\ Q^{a\alpha}_i Q^{b\beta}_j e^{\frac{2}{3}g'B}\big(e^{-2g_3G}\overline{U}^\dagger_k\big)^c\overline{E}^\dagger_l\ . \tag{4.55}$$

By evaluating the supergraphs illustrated in Fig. 4.12 with the gauge interactions in Eq. (3.35), we obtain the Wilson coefficients as

$$\begin{aligned} C^{ijkl}_{6(1)} &= -\frac{g_5^2}{M_X^2}e^{i\varphi_i}\delta^{ik}\delta^{jl}\ , \\ C^{ijkl}_{6(2)} &= -\frac{g_5^2}{M_X^2}e^{i\varphi_i}\delta^{ik}(V^*)^{jl}\ . \end{aligned} \tag{4.56}$$

Note that the results do not suffer from the model-dependence, such as the structure of the soft SUSY breaking terms. In this sense, the SU(5) gauge interactions provide a robust prediction for the proton decay rate. Moreover, it is found that the resultant amplitude does not depend on the new phases appearing the GUT Yukawa couplings, since the factors only affect the overall phase.

The coefficients are evolved down according to the one-loop RGEs,

$$\begin{aligned} \mu\frac{d}{d\mu}C^{ijkl}_{6(1)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{11}{15}\right)+\frac{\alpha_2}{4\pi}(-3)+\frac{\alpha_3}{4\pi}\left(-\frac{8}{3}\right)\right]C^{ijkl}_{6(1)}\ , \\ \mu\frac{d}{d\mu}C^{ijkl}_{6(2)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{23}{15}\right)+\frac{\alpha_2}{4\pi}(-3)+\frac{\alpha_3}{4\pi}\left(-\frac{8}{3}\right)\right]C^{ijkl}_{6(2)}\ , \end{aligned} \tag{4.57}$$

which are derived in Appendix C.4. We have numerically checked that the two-loop contribution yields negligible effects. At the SUSY breaking scale, the coefficients are matched with those of the four-Fermi operators as

$$\begin{aligned} C^{ijkl}_{(1)}(M_S) &= C^{ijkl}_{6(1)}(M_S)\ , \\ C^{ijkl}_{(2)}(M_S) &= C^{ijkl}_{6(2)}(M_S)\ . \end{aligned} \tag{4.58}$$

The rest of the calculation is same as that carried out in the previous section.

In the case of the dimension-six proton decay, the $p \to e^+\pi^0$ mode is most severely restricted from the experiments. The current bound on the lifetime of the mode is [69, 70]

$$\tau(p \to e^+\pi^0) > 1.4 \times 10^{34}\ \text{years}\ , \tag{4.59}$$

which is given by Super-Kamiokande. On the other hand, we evaluate the lifetime as

$$\tau(p \to e^+\pi^0) \simeq 3 \times 10^{35} \times \left(\frac{M_X}{1.0 \times 10^{16}\ \text{GeV}}\right)^4 \text{years}\ . \tag{4.60}$$

Here, we take $M_S = 10^2$ TeV, though the result hardly depends on the SUSY scale. In the computation, we use the hadronic matrix elements calculated with the direct method. From these equations we find that the predicted lifetime is well above the current experimental limits.

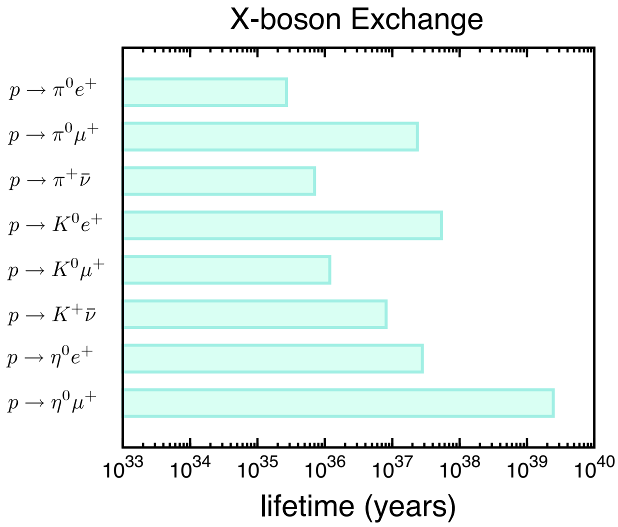


Figure 4.13: Lifetime of each decay mode induced by the $X$-boson exchange. We take $M_S = 100$ TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, $M_X = 10^{16}$ GeV, and $\tan\beta = 5$.

We also evaluate the decay lifetime for other modes, which are summarized in the bar chart in Fig. 4.13. Here, we set the $X$-boson mass to be $M_X = 10^{16}$ GeV, and other parameters are taken as follows: $M_S = 100$ TeV, $M_1 = 600$ GeV, $M_2 = 300$ GeV, $M_3 = -2$ TeV, $\mu = M_S$, and $\tan\beta = 5$. From the figure, we see that the decay rates of the modes that contain different generations in their final states are considerably suppressed, as mentioned above. This is because in the $X$-boson exchanging process the CKM is the only source of the flavor violation, which can be seen from Eq. (4.56). Further, there is no room for the flavor mixing effects in the sfermion mass matrices to modify the decay rates. In this sense, the prediction given here is robust.

As discussed in the previous chapter, in the high-scale SUSY scenario, the GUT scale $M_{\rm GUT}$ tends to be lower than the low-scale SUSY one. Although the change in $M_{\rm GUT}$ is small, it might be significant since the lifetime of proton decay caused by the $X$-boson exchange process scales as $\propto M_X^4$. A problem here is that we could not directly determine the mass of $X$-boson from the GUT scale evaluated in Sec. 3.2; since $M_{\rm GUT} = (M_X^2 M_\Sigma)^{1/3}$, $M_X$ depends on the relative size with respect to $M_\Sigma$. Recall that $M_X$ and $M_\Sigma$ are expressed in terms of the VEV of the adjoint Higgs boson as $M_X = 5\sqrt{2} g_5 V$ and $M_\Sigma = \frac{5}{2}\lambda_\Sigma V$, respectively. Thus, we obtain

$$M_X = \sqrt{2}\left(\frac{g_5}{\lambda_\Sigma}\right)^{\frac{1}{3}} M_{\rm GUT} \ . \tag{4.61}$$

Namely, we have an unknown parameter $\lambda_\Sigma$. From the equation, we find that as $\lambda_\Sigma$ grows the mass of $X$-boson decreases. Of course, too large a value of $\lambda_\Sigma$ breaks perturbativity. Requirement of $\lambda_\Sigma < 4\pi$ leads to a lower bound on $M_X$:

$$M_X > 6 \times 10^{15} \text{ GeV}. \tag{4.62}$$

Keeping the argument in mind, we estimate the lifetime of the $p \to \pi^0 e^+$ mode as a function gluino mass. In the figure, we take $M_2 = 3$ TeV and $M_S = 10^3$ TeV. The mass of $X$-boson is

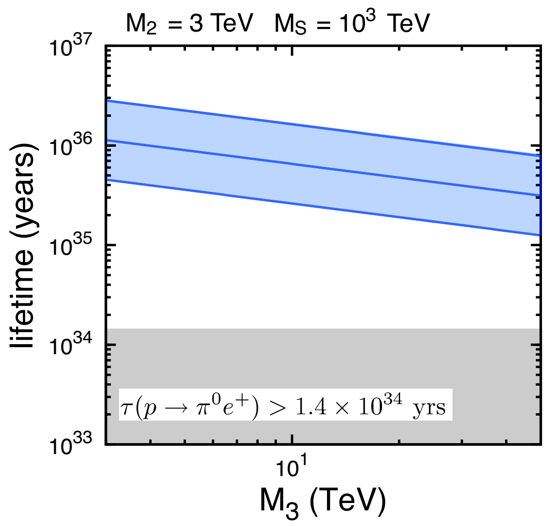


Figure 4.14: Lifetime of $p \to \pi^0 e^+$ induced by the $X$-boson exchange as a function of gluino mass $M_3$. We take $M_2 = 3$ TeV and $M_S = 10^3$ TeV. Mass of $X$-boson is estimated from the RGE analysis described in Sec. 3.2 and Eq. (4.61). Band represents $1/2 < \lambda_\Sigma < 2$. Shaded region corresponds to the current experimental bound from Super-Kamiokande [69, 70].

estimated from the RGE analysis described in Sec. 3.2 and Eq. (4.61). The blue band corresponds to the region of $1/2 < \lambda_\Sigma < 2$. Shaded region represents the current experimental bound from Super-Kamiokande [69, 70]. As can be seen from the plot, the lifetime decreases as the gluino mass, in fact both the gluino and wino masses as shown in Sec. 3.2, is taken to be larger. Future proton decay experiments such as Hyper-Kamiokande may reach the lifetime region.

The above consequence might be altered if there exist extra particles in the intermediate scale. With such particles belonging to a representation of the grand unified group, the gauge coupling unification is still achieved, while its value at the unified scale turns out to be enhanced. Then, the proton lifetime is considerably reduced due to the large gauge coupling [218]. Indeed, existence of such extra particles are motivated in some models within the context of high-scale SUSY scenario [143, 144, 145]. The resultant proton lifetime is, however, dependent on models, in particular on the number of particles and its mass scale So we do not further discuss the subject in this thesis.

## 4.3 Proton decay experiments

Before concluding this chapter, we give a brief review of the present status of proton decay experiments. The current experimental limits on proton decay mainly come from the Super-Kamiokande experiment. The experiment began in 1996, and until now 260 kiloton·years of exposure has been recorded. The fiducial mass of the water Cherenkov detector is 22500 ton, which corresponds to $7.5 \times 10^{33}$ protons and $6.0 \times 10^{33}$ neutrons.

The detection mechanism is based on the *Cherenkov effect*; relativistic charged particles emit Cherenkov radiations when they pass through the water in the detector. The production angle of the radiated photons with respect to the direction of motion of the charged particle is given by

$$\cos\theta_c = 1/n\beta \; , \tag{4.63}$$

where $\beta$ is the velocity of the particle and $n$ is the refractive index of the medium. In the case of water, $n = 1.33$, and thus $\theta_c = 42^\circ$ for $\beta = 1$. One can also distinguish species of particles by analyzing the ring image of the Cherenkov radiation. Compared with the images of radiations caused by $\mu^\pm$, those generated by $e^\pm$ and photons are distorted since they induce electromagnetic showers in the medium.

Let us discuss the two important decay channels of proton: $p \to K^+\bar{\nu}$ and $p \to \pi^0 e^+$. As for the $p \to \pi^0 e^+$ mode, the strategy is quite simple since all of the momenta in the final state can be reconstructed. One searches for the back-to-back signal with the total energy being the mass of proton. Some additional effects such as nuclear effects are also taken into account.

In the case of $p \to K^+\bar{\nu}$, on the other hand, one cannot use a similar strategy, since the final state contains a missing particle (an antineutrino) and $K^+$ coming from the proton does not exceed the Cherenkov threshold. In this case, one basically searches for the decay of $K^+$ at rest. For $K^+ \to \pi^+\pi^0$ branching (21%), one can use a similar method to that for the $p \to \pi^0 e^+$ search. In the case of $K^+ \to \mu^+\nu$ (64%), tagging of a low-energy $\gamma$ from the de-excitation of $^{15}\mathrm{N}^*$, which is a leftover of proton decay in $^{16}\mathrm{O}$, is used to reduce background. The antimuon of course emits the Cherenkov radiation, and decays within the detector emitting a positron. All of the information is used for the event discrimination.

A variety of other decay modes are also analyzed. In Fig. 4.15, we show the summary of lifetime limits presented by the Super-Kamiokande experiment [69, 70]. We use these values for the experimental limits in this thesis.

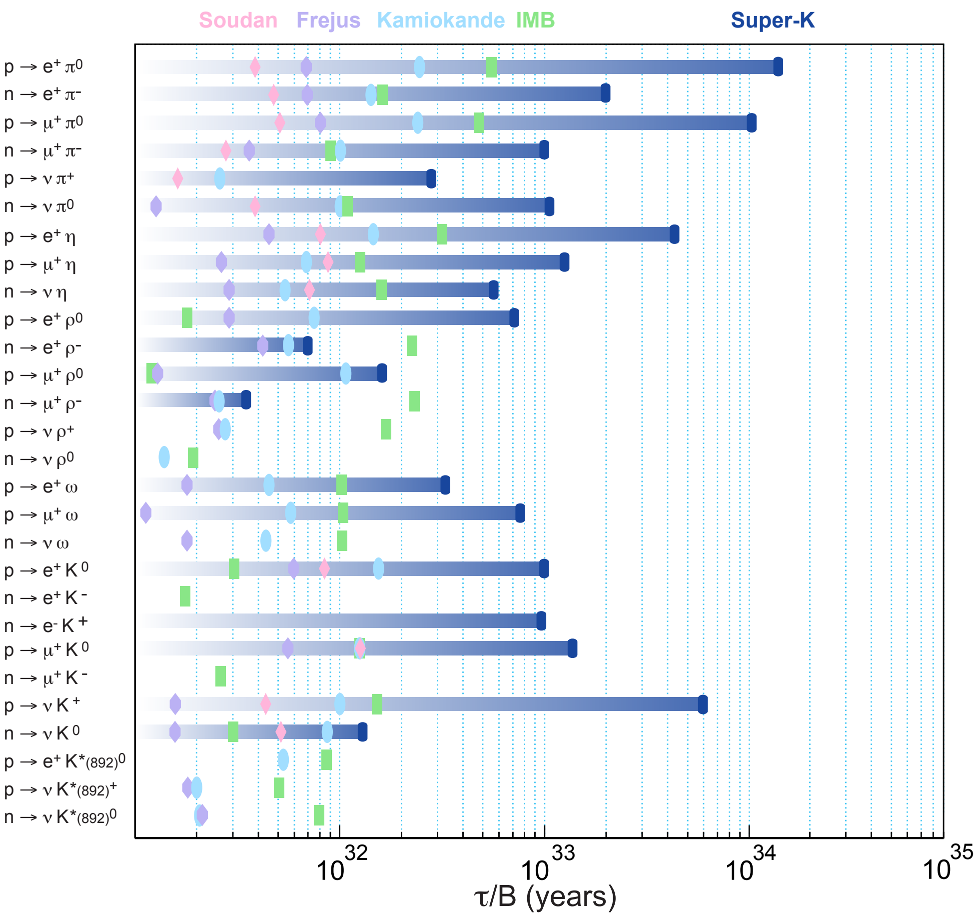


Figure 4.15: Summary of lifetime limits presented by the Super-Kamiokande experiment [69, 70].

# Chapter 5

# Conclusions and discussion

In this thesis, we revisit the minimal SUSY SU(5) GUT and discuss proton decay in the theory. In Chap. 3.2, we have presented constraints on the masses of the GUT scale particles from the gauge coupling unification in the case of the high-scale SUSY scenario. To that end, we have used the two-loop RGEs for the gauge couplings with one-loop threshold corrections considered. As a result, the mass of the color-triplet Higgs multiplets $M_{H_C}$ turns out to be considerably large compared with previous results in the traditional low-energy SUSY scenario, while the GUT scale is found to be slightly lower. These are generic features resulting from the mass difference among the components of the same supermultiplet of the SU(5) gauge group. Interestingly, all of the superheavy particles might be around $10^{16}$ GeV in the high-scale SUSY models.

The mass spectrum of the GUT scale particles predicted here stimulates us to reconsider the proton decay in the case of high-scale supersymmetry. In Chap. 4.1, we have evaluated the proton decay lifetime via the dimension-five operators in the high-scale SUSY scenario. It is found that the higgsino exchanging diagram gives rise to the dominant contribution in the case of minimal flavor violation. As a result, we have revealed that the proton lifetime can evade the current experimental limit and, thus, the minimal SUSY SU(5) GUT is not excluded in the high-scale SUSY scenario. We have also computed the rates of various proton decay channels in the presence of sizable flavor violation in the sfermion mass matrices, which can be allowed in the case of high-scale SUSY breaking. In such a case, the decay rates can be significantly increased, and the charged lepton modes such as $p \to \pi^0 \mu^+$ may be smoking-gun signature of sfermion flavor violation.

While the dimension-five proton decay is suppressed by the heavy sfermion masses, the dimension-six one through the $X$-boson exchange does not suffer from such a suppression. As referred to above, the GUT scale in the high-scale SUSY is slightly lower than the ordinary one. Since the dimension-six proton decay lifetime scales as $\propto M_X^4$ with $M_X$ the mass of $X$-boson, it may be significantly enhanced even by a small change in the GUT scale. Although the size of enhancement depends on not only gaugino mass spectrum but also the GUT scale potential, we expect that the dimension-six proton decay might be searched in future experiments.

As we have discussed, proton decay induced by both the dimension-five and -six operators may be accessible in future experiments. For instance, the expected sensitivities of the Hyper-Kamiokande with ten years exposure [69, 70] are $1.3 \times 10^{35}$ and $2.5 \times 10^{34}$ years at 90 % confidence level for the $p \to e^+\pi^0$ and $p \to \bar{\nu} K^+$ modes,[1] respectively, which enable us to explore a wide

[1] Recent improvements in the analysis of the $K^+ \to \pi^+\pi^0$ decay channel may provide a better sensitivity for

range of parameter region in high-scale SUSY models. After all, the minimal SUSY SU(5) GUT is still quite promising, and the proton decay experiments may reveal the existence of supersymmetry as well as the grand unification.

the $p \to \bar{\nu} K^+$ mode: $3.2 \times 10^{34}$ years at 90 % confidence level [69].

## Acknowledgments

First of all, I am very grateful to Junji Hisano for supervising my work, for instructive suggestions, and for his continuous encouragement. I would also like to thank another supervisor of mine, Takeo Moroi, for helping me throughout the doctoral course. Further, I wish to thank Shigeki Matsumoto for always encouraging me and my work.

Thanks to my collaborators of this work and other related topics; Kaori Fuyuto, Koji Ishiwata, Daiki Kobayashi, Takumi Kuwahara, Yu Muramatsu, Satoshi Shirai, Tomohiro Takesako, and Koji Tsumura.

I am also grateful to Hitoshi Murayama, Koichi Hamaguchi, Masahiro Ibe, Masato Shiozawa, and Yoichiro Suzuki for valuable advice on the thesis.

I am indebted to the E-ken (ex-)members in Nagoya University for a pleasant atmosphere. Especially, I would like to thank Tomohiro Abe for helping me in my first year at the Nagoya University, Hidetoshi Kawase for a lot of interesting arguments, Nobuhiro Maekawa for discussions on naturalness and grand unification, Masaharu Tanabashi for stimulating me with his incentive comments, and Kazuhiro Tobe for helpful discussions.

I also thank fellows, Manami Hashi, Kyosuke Hirochi, Yuto Ito, Io Kawaguchi, Yohei Kikuta, Hikaru Matsuo, Rio Saito, and Kohsaku Tobioka for no particular reason.

This work is supported by Research Fellowships of the Japan Society for the Promotion of Science for Young Scientists.

# Appendix A

# Notations

## A.1 Conventions

We use "natural units" ($c = \hbar = 1$), where $c$ is the speed of light in vacuum and $\hbar \equiv h/(2\pi)$ is the Planck constant. The metric of the four-dimensional Minkowski spacetime is given by

$$\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1). \tag{A.1}$$

Greek indices ($\mu, \nu, \dots$) denote the spacetime components, and run over (0, 1, 2, 3), with $x^0$ the time coordinate. The "Levi-Civita tensor" $\epsilon^{\mu\nu\rho\sigma}$ is defined as the totally antisymmetric tensor with $\epsilon^{0123} = +1$.

In this thesis, we often use the two-component spinor notations. They are irreducible representations of the Lorentz transformation. So let us begin with reviewing the irreducible representations of the Lorentz transformation. The Lorentz transformation are linear transformations acting on four-vectors

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} \ , \tag{A.2}$$

where $\Lambda$ is an element of the homogeneous Lorentz group $L$ defined by

$$L \equiv \mathrm{O}(1,3;\mathbb{R}) = \{\Lambda \in \mathrm{GL}(4;\mathbb{R}) | \Lambda^T \eta \Lambda = \eta\} \ . \tag{A.3}$$

Further, the subgroup of $L$ subject to the additional constraint,

$$\det \Lambda = 1 \qquad \Lambda^{0}{}_{0} \geq 1 \ , \tag{A.4}$$

is called the *proper orthochronous Lorentz group.* In the following discussion, we will deal only with this group. Consider an infinitesimal homogeneous Lorentz transformation

$$\Lambda^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu} \ , \tag{A.5}$$

with $\delta^{\mu}{}_{\nu}$ the Kronecker delta and $\omega^{\mu}{}_{\nu}$ a real infinitesimal parameter satisfying

$$\omega_{\mu\nu} = -\omega_{\nu\mu} \ . \tag{A.6}$$

Then, with these parameters, a representation for the transformation is given as

$$U(1+\omega) \simeq \mathbb{1} - \frac{i}{2}\omega_{\mu\nu}\mathcal{J}^{\mu\nu} \ , \tag{A.7}$$

with $\mathcal{J}_{\mu\nu} = -\mathcal{J}_{\nu\mu}$. In particular, if the target space is the spacetime itself, the generators are given as

$$(\mathcal{J}_{\rho\sigma})^{\mu}{}_{\nu} = i(\delta^{\mu}_{\rho}\eta_{\nu\sigma} - \delta^{\mu}_{\sigma}\eta_{\nu\rho}) \ . \tag{A.8}$$

The commutation relations of the generators are given as follows:

$$[\mathcal{J}_{\mu\nu}, \mathcal{J}_{\rho\sigma}] = -i(\eta_{\mu\rho}\mathcal{J}_{\nu\sigma} - \eta_{\nu\rho}\mathcal{J}_{\mu\sigma} - \eta_{\mu\sigma}\mathcal{J}_{\nu\rho} + \eta_{\nu\sigma}\mathcal{J}_{\mu\rho}) \ . \tag{A.9}$$

One can readily obtain the relations by subsequently carrying out infinitesimal Lorentz transformations, or by using the concrete representation of the generators (A.8). Also, we define

$$J_i \equiv \frac{1}{2}\epsilon_{ijk}\mathcal{J}^{jk} \ , \tag{A.10}$$

$$K_i \equiv \mathcal{J}^{0i} \ , \tag{A.11}$$

where $i, j, \ldots$ denote the space indices and run over $(1, 2, 3)$ and $\epsilon_{ijk}$ is the totally antisymmetric tensor. Then, Eq. (A.9) reads

$$[J_i, J_j] = i\epsilon_{ijk}J_k \ , \tag{A.12}$$

$$[J_i, K_j] = i\epsilon_{ijk}K_k \ , \tag{A.13}$$

$$[K_i, K_j] = -i\epsilon_{ijk}J_k \ . \tag{A.14}$$

Let us further define the three-vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ by

$$\boldsymbol{A} \equiv \frac{1}{2}(\boldsymbol{J} + i\boldsymbol{K}) \ , \qquad \boldsymbol{B} \equiv \frac{1}{2}(\boldsymbol{J} - i\boldsymbol{K}) \ . \tag{A.15}$$

Their commutation relations are easily obtained from the above relations as follows:

$$[A_i, A_j] = i\epsilon_{ijk}A_k \ , \tag{A.16}$$

$$[B_i, B_j] = i\epsilon_{ijk}B_k \ , \tag{A.17}$$

$$[A_i, B_j] = 0 \ . \tag{A.18}$$

Thus, $\boldsymbol{A}$ and $\boldsymbol{B}$ each satisfy the SU(2) algebra and their irreducible representations are labelled by a pair of integers and/or half-integers $(A, B)$.

Two-component spinors are fields that transform as $(A, B) = (\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$ fields. To distinguish the transformation properties, the former fields are expressed with undotted indices like $\xi_\alpha$, while the latter fields with dotted indices like $\bar{\eta}^{\dot{\alpha}}$. The spinor indices are raised and lowered by means of the antisymmetric tensor $\epsilon_{\alpha\beta}$, $\epsilon_{\dot{\alpha}\dot{\beta}}, \ldots$ with

$$\epsilon_{21} = \epsilon^{12} = 1, \quad \epsilon_{12} = \epsilon^{21} = -1, \quad \epsilon_{11} = \epsilon_{22} = 0, \tag{A.19}$$

and

$$\psi^{\alpha} = \epsilon^{\alpha\beta}\psi_{\beta}, \qquad \psi_{\alpha} = \epsilon_{\alpha\beta}\psi^{\beta}, \qquad \bar{\eta}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\eta}^{\dot{\beta}}, \qquad \bar{\eta}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\eta}_{\dot{\beta}} \ . \tag{A.20}$$

We contract repeated spinor indices put in the form like

$${}^{\alpha}{}_{\alpha} \qquad \text{or} \qquad {}_{\dot{\alpha}}{}^{\dot{\alpha}} \ . \tag{A.21}$$

Namely,

$$\psi\chi \equiv \psi^{\alpha}\chi_{\alpha} = -\psi_{\alpha}\chi^{\alpha} = \chi^{\alpha}\psi_{\alpha} = \chi\psi \ , \tag{A.22}$$

$$\bar{\psi}\bar{\chi} \equiv \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = -\bar{\psi}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} = \bar{\chi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = \bar{\chi}\bar{\psi} \ . \tag{A.23}$$

The $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ fields are related to each other via the Hermitian conjugation:

$$\bar{\chi} \equiv \chi^{\dagger} \ , \qquad \chi^{\dagger}_{\dot{\alpha}} \equiv (\chi_{\alpha})^{\dagger} = (\chi^{\dagger})_{\dot{\alpha}} \ . \tag{A.24}$$

So, it follows that

$$(\chi\psi)^{\dagger} = \bar{\psi}\bar{\chi} = \bar{\chi}\bar{\psi} \ . \tag{A.25}$$

The Pauli matrices

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.26}$$

also have the two-component spinor indices, $\sigma^{\mu}_{\alpha\dot{\alpha}}$. Further, we define

$$\bar{\sigma}^{\mu\dot{\alpha}\alpha} \equiv \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma^{\mu}_{\beta\dot{\beta}}, \tag{A.27}$$

and

$$\bar{\sigma}^0 = \sigma^0, \quad \bar{\sigma}^i = -\sigma^i \ \ (i = 1,2,3) \tag{A.28}$$

readily follows. These matrices are normalized such that

$$\mathrm{Tr}\ \sigma^{\mu}\bar{\sigma}^{\nu} = 2\eta^{\mu\nu} \ , \tag{A.29}$$

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\ \bar{\sigma}^{\dot{\beta}\beta}_{\mu} = 2\delta^{\beta}_{\alpha}\ \delta^{\dot{\beta}}_{\dot{\alpha}} \ , \tag{A.30}$$

$$(\sigma^{\mu}\bar{\sigma}^{\nu} + \sigma^{\nu}\bar{\sigma}^{\mu})^{\beta}_{\alpha} = 2\ \eta^{\mu\nu}\ \delta^{\beta}_{\alpha} \ . \tag{A.31}$$

In addition, some useful formulae are presented below:

$$\xi^{\dagger}\bar{\sigma}^{\mu}\chi = -\chi\sigma^{\mu}\xi^{\dagger} \ , \tag{A.32}$$

$$\xi\sigma^{\mu}\bar{\sigma}^{\nu}\chi = \chi\sigma^{\nu}\bar{\sigma}^{\mu}\xi \ , \tag{A.33}$$

$$\theta^{\alpha}\theta^{\beta} = -\frac{1}{2}\epsilon^{\alpha\beta}\theta\theta \ , \quad \theta_{\alpha}\theta_{\beta} = \frac{1}{2}\epsilon_{\alpha\beta}\theta\theta \ , \quad \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta} \ , \quad \bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta} \ , \tag{A.34}$$

$$\theta\sigma^{\mu}\bar{\theta}\theta\sigma^{\nu}\bar{\theta} = \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\eta^{\mu\nu} \ , \tag{A.35}$$

$$(\theta\phi)(\theta\psi) = -\frac{1}{2}(\phi\psi)(\theta\theta) \ , \qquad (\bar{\theta}\bar{\phi})(\bar{\theta}\bar{\psi}) = -\frac{1}{2}(\bar{\phi}\bar{\psi})(\bar{\theta}\bar{\theta}) \ , \tag{A.36}$$

$$\chi_{\alpha}(\xi\eta) = -\xi_{\alpha}(\eta\xi) - \eta_{\alpha}(\chi\xi) \ . \tag{A.37}$$

The last equation follows from the identity $\epsilon_{\alpha\beta}\epsilon_{\gamma\delta} - \epsilon_{\alpha\delta}\epsilon_{\gamma\beta} - \epsilon_{\gamma\alpha}\epsilon_{\delta\beta} = 0$. We also list the trace formulae for the Pauli matrices:

$$\mathrm{Tr}[\sigma^{\mu}\bar{\sigma}^{\nu}\sigma^{\lambda}\bar{\sigma}^{\tau}] = 2[\eta^{\mu\nu}\eta^{\lambda\tau} - \eta^{\mu\lambda}\eta^{\nu\tau} + \eta^{\mu\tau}\eta^{\nu\lambda} + i\epsilon^{\mu\nu\lambda\tau}] \ , \tag{A.38}$$

$$\mathrm{Tr}[\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\sigma}^{\lambda}\sigma^{\tau}] = 2[\eta^{\mu\nu}\eta^{\lambda\tau} - \eta^{\mu\lambda}\eta^{\nu\tau} + \eta^{\mu\tau}\eta^{\nu\lambda} - i\epsilon^{\mu\nu\lambda\tau}] \ . \tag{A.39}$$

By using the relations, one readily obtains

$$\sigma^\mu \bar{\sigma}^\nu \sigma^\lambda = \eta^{\mu\nu}\sigma^\lambda - \eta^{\mu\lambda}\sigma^\nu + \eta^{\nu\lambda}\sigma^\mu + i\epsilon^{\mu\nu\lambda\tau}\sigma_\tau \ , \tag{A.40}$$

$$\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^\lambda = \eta^{\mu\nu}\bar{\sigma}^\lambda - \eta^{\mu\lambda}\bar{\sigma}^\nu + \eta^{\nu\lambda}\bar{\sigma}^\mu - i\epsilon^{\mu\nu\lambda\tau}\bar{\sigma}_\tau \ . \tag{A.41}$$

It is convenient to construct antisymmetric matrices $\sigma^{\mu\nu}$ and $\overline{\sigma}^{\mu\nu}$ out of the Pauli matrices:

$$\left(\sigma^{\mu\nu}\right)_\alpha^{\ \beta} \equiv \frac{i}{2}\left(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu\right)_\alpha^{\ \beta} \ , \tag{A.42}$$

$$\left(\overline{\sigma}^{\mu\nu}\right)^{\dot{\alpha}}_{\ \dot{\beta}} \equiv \frac{i}{2}\left(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu\right)^{\dot{\alpha}}_{\ \dot{\beta}} \ . \tag{A.43}$$

For the matrices, we have

$$\begin{aligned} \epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma} &= 2i\sigma^{\mu\nu} \ , \\ \epsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\rho\sigma} &= -2i\bar{\sigma}^{\mu\nu} \ . \end{aligned} \tag{A.44}$$

By using the two-component spinors, we can compose the four-component Dirac spinors as

$$\psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix} \ , \tag{A.45}$$

or Majorana fermions as

$$\psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \ . \tag{A.46}$$

Also, the Dirac matrices are given by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \ . \tag{A.47}$$

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \ . \tag{A.48}$$

Spinors are essentially Grassmann objects. Thus, we need to define the Grassmann derivatives and integrals. In this thesis, all Grassmann derivatives should be understood as left-derivatives. Some formulae for the derivatives and integrals are summarized as follows:

$$\left(\frac{\partial}{\partial\theta^\alpha}\right)^\dagger = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \ . \tag{A.49}$$

This relation is justified by taking the Hermitian conjugate of $\frac{\partial}{\partial\theta^\alpha}\theta\theta = 2\theta_\alpha$.

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha} \ . \tag{A.50}$$

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta\theta = 4 \ , \qquad \epsilon_{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\beta}}}\bar{\theta}\bar{\theta} = 4. \tag{A.51}$$

$$d^2\theta \equiv -\frac{1}{4}d\theta^\alpha d\theta^\beta\epsilon_{\alpha\beta} \ , \qquad d^2\bar{\theta} \equiv -\frac{1}{4}d\bar{\theta}_{\dot{\alpha}}d\bar{\theta}_{\dot{\beta}}\epsilon^{\dot{\alpha}\dot{\beta}} \ , \qquad d^4\theta \equiv d^2\theta d^2\bar{\theta} \ . \tag{A.52}$$

$$\int d^2\theta\ \theta\theta = 1, \qquad \int d^2\bar{\theta}\ \bar{\theta}\bar{\theta} = 1 \ . \tag{A.53}$$

## A.2 $D$-algebra

In this section, we summarize the properties of the *chiral covariant derivatives*, which are supersymmetric covariant derivatives with respect to the Grassmann coordinates $(\theta, \bar{\theta})$. They are defined by

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \qquad D^\alpha = -\frac{\partial}{\partial\theta_\alpha} + i(\bar{\theta}\bar{\sigma}^\mu)^\alpha\partial_\mu\ , \tag{A.54}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\ , \qquad \bar{D}^{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu\ , \tag{A.55}$$

with $\partial_\mu = \frac{\partial}{\partial x^\mu}$. These differential operators satisfy the following anticommutation relations,

$$\begin{aligned}\{D_\alpha, \bar{D}_{\dot{\alpha}}\} &= 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu, \\ \{D_\alpha, D_\beta\} &= \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0\ .\end{aligned} \tag{A.56}$$

By using the $y$-coordinate $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$, we can rewrite the operators as

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \qquad D^\alpha = -\frac{\partial}{\partial\theta_\alpha} + 2i(\bar{\theta}\bar{\sigma}^\mu)^\alpha\partial_\mu\ , \tag{A.57}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\ , \qquad \bar{D}^{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\ , \tag{A.58}$$

while with the $y^*$-coordinate $y^{*\mu} = x^\mu + i\theta\sigma^\mu\bar{\theta}$,

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha}\ , \qquad D^\alpha = -\frac{\partial}{\partial\theta_\alpha}\ , \tag{A.59}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + 2i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\ , \qquad \bar{D}^{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} - 2i(\bar{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu\ . \tag{A.60}$$

In the following discussion, $y^\mu$ and $y^{*\mu}$ are also referred to as $x^\mu_+$ and $x^\mu_-$, respectively.

We list useful formulae for the chiral covariant derivatives below. See Refs. [122, 123] for detail discussions.

1. $$[D_\alpha, \bar{D}^2] = 4i\sigma^\mu_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}\partial_\mu\ . \tag{A.61}$$

2. $$[\bar{D}_{\dot{\alpha}}, D^2] = -4iD^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\ . \tag{A.62}$$

3. $$\begin{aligned}\frac{1}{16}[D^2, \bar{D}^2] &= \frac{i}{2}D^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}\partial_\mu + \Box \\ &= -\frac{i}{2}\bar{D}^{\dot{\alpha}}\sigma^\mu_{\alpha\dot{\alpha}}D^\alpha\partial_\mu - \Box\ .\end{aligned} \tag{A.63}$$

4. $$\bar{D}^2D^2\bar{D}^2 = -16\bar{D}^2\Box\ . \tag{A.64}$$
$$D^2\bar{D}^2D^2 = -16D^2\Box\ . \tag{A.65}$$

5.

$$
\begin{aligned}
D^\alpha \bar{D}^2 D_\alpha &= \bar{D}_{\dot{\alpha}} D^2 \bar{D}^{\dot{\alpha}} \\
&= D^2 \bar{D}^2 - 4i D^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} \partial_\mu \ .
\end{aligned} \tag{A.66}
$$

6.

$$
D^2 \bar{D}^2 + \bar{D}^2 D^2 - 2 D^\alpha \bar{D}^2 D_\alpha = -16\Box \ . \tag{A.67}
$$

7.

$$
(D^\alpha \bar{D}^2 D_\alpha)^2 = 8 D^\alpha \bar{D}^2 D_\alpha \Box \ . \tag{A.68}
$$

8. For a chiral superfield $\Phi$,

$$
-\frac{1}{16}\frac{\bar{D}^2 D^2}{\Box}\Phi = \Phi \ , \qquad -\frac{1}{16}\frac{D^2 \bar{D}^2}{\Box}\Phi^\dagger = \Phi^\dagger \ . \tag{A.69}
$$

9. For arbitrary chiral superfields $\Phi$ and $\Psi$ ,

$$
\int d^4x d^2\theta \Phi \Psi = \int de4x d^4\theta \Phi \left(\frac{1}{4}\frac{D^2}{\Box}\right)\Psi \ . \tag{A.70}
$$

# Appendix B

# Renormalization group equations

In this chapter, we summarize the renormalization group equations (RGEs) for couplings and mass parameters which we use in this thesis.

## B.1 QCD corrections

Below the electroweak scale, the QCD corrections become much significant due to the asymptotic free nature of the strong interaction. For this reason, we take into account the QCD corrections at two-loop level. On the other hand, the QED contribution is quite small, so we neglect it in the following calculation.

To begin with, we write down the two-loop RGEs for the strong coupling constant below the electroweak scale:

$$\mu\frac{d\alpha_s}{d\mu} = (2b_1)\frac{\alpha_s^2}{4\pi} + (2b_2)\frac{\alpha_s^3}{(4\pi)^2} \ , \tag{B.1}$$

with $\alpha_s \equiv g_s^2/(4\pi)$ the strong coupling constant and

$$b_1 = -\frac{11N_c - 2N_f}{3} \ , \qquad b_2 = -\frac{34}{3}N_c^2 + \frac{10}{3}N_cN_f + 2C_FN_f \ , \tag{B.2}$$

where $N_c = 3$ is the number of colors, $N_f$ denotes the number of quark flavors in an effective theory and $C_F$ is the quadratic Casimir invariant defined by

$$C_F \equiv \frac{N_c^2 - 1}{2N_c} \ . \tag{B.3}$$

The two-loop RGEs for the $\overline{\text{MS}}$ quark masses are given as

$$\mu\frac{dm(\mu)}{d\mu} = \left[\gamma_m^{(1)}\frac{\alpha_s}{4\pi} + \gamma_m^{(2)}\frac{\alpha_s^2}{(4\pi)^2}\right]m(\mu) \ , \tag{B.4}$$

with

$$\gamma_m^{(1)} = -6C_F \ , \qquad \gamma_m^{(2)} = -C_F\left(3C_F + \frac{97}{3}N_c - \frac{10}{3}N_f\right) \ . \tag{B.5}$$

## B.2 RGEs for gauge and Yukawa couplings

Here, we present the RGEs of the gauge and Yukawa coupling constants. We use the $\overline{\mathrm{DR}}$ scheme [190] in this work. The two-loop beta functions [219] are used for the gauge couplings, while the one-loop RGEs are used for the Yukawa couplings.

### B.2.1 Generic results: Non-SUSY

First, we consider a generic field theory described by the following Lagrangian,

$$\mathcal{L} = -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + \frac{1}{2}(D_\mu\phi_i)^2 + \psi_i^\dagger\bar{\sigma}^\mu iD_\mu\psi_i - (y_k^{ij}\psi_i\psi_j\phi_k + \text{h.c.}) \ . \tag{B.6}$$

Here, fermions are expressed in terms of the two-component spinors.

The beta-function for the gauge coupling constant is given in Ref. [219] for a generic field theory up to two-loop order in the $\overline{\mathrm{MS}}$ scheme:

$$\begin{aligned}\beta(g) \equiv& \mu\frac{dg}{d\mu} \\ =& -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}C(G) - \frac{4}{3}\kappa S(F) - \frac{1}{6}S(S)\right] - \frac{g^3}{(4\pi)^4}2\kappa Y_4(F) \\ &- \frac{g^5}{(4\pi)^4}\left[\frac{34}{3}\{C(G)\}^2 - \kappa\{4C(F) + \frac{20}{3}C(G)\}S(F) - \{2C(S) + \frac{1}{3}C(G)\}S(S)\right] ,\end{aligned} \tag{B.7}$$

where $C(G)$, $C(F)$, and $C(S)$ are the Casimir invariants for gauge, fermion and scalar fields, respectively. Also, $S(F)$ and $S(S)$ are the Dynkin indices for fermion and scalar representations, respectively. The factor $\kappa$ is $\kappa = 1/2$ ($\kappa = 1$) for two-component (four-component) fermions. Scalar fields are written in terms of real fields so that they form a real representation of a given gauge group. The invariant $Y_4(F)$ is defined by

$$Y_4(F) \equiv \frac{1}{d(G)}\mathrm{Tr}\big[C(F)y_k y_k^\dagger\big] \ , \tag{B.8}$$

with $d(G)$ is the dimension of the Lie algebra of a group $G$. Since the gauge couplings at two-loop level are actually independent of the renormalization schemes, the above formula is also valid in the $\overline{\mathrm{DR}}$ scheme.

On the other hand, the one-loop beta functions of the Yukawa couplings are given as [220]

$$\begin{aligned}\beta_{y_k} \equiv& \mu\frac{d}{d\mu}y_k \\ =& \frac{1}{(4\pi)^2}\left[\frac{1}{2}\{Y_2^\dagger(F)y_k + y_k Y_2(F)\} + 2y_l y_k^\dagger y_l + 2\kappa y_l \mathrm{Tr}(y_l^\dagger y_k) - 3g^2\{C(F), y_k\}\right] .\end{aligned} \tag{B.9}$$

Here,

$$Y_2(F) \equiv y_k^\dagger y_k \ . \tag{B.10}$$

In addition, let us show the one-loop relations between the couplings in the $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ schemes for convenience. Such a relation for the renormalized gauge coupling constants are given as [221]

$$\alpha(\mu)_{\overline{\mathrm{DR}}} = \alpha(\mu)_{\overline{\mathrm{MS}}}\left(1 + \frac{C(G)}{12\pi}\alpha(\mu)_{\overline{\mathrm{MS}}}\right) , \tag{B.11}$$

where $C(G)$ is the quadratic Casimir invariant for the adjoint representation of a single group $G$; *e.g.*, for SU($N$), $C(\mathrm{SU}(N)) = N$, while for U(1), $C(\mathrm{U}(1)) = 0$. By using the relation, one can easily check a well-known result that the beta function for the gauge coupling is scheme-independent up to two-loop level.

The one-loop relation between the Yukawa couplings is, on the other hand, presented in Ref. [222] as

$$y_k^{ij}|_{\overline{\mathrm{DR}}} = y_k^{ij}|_{\overline{\mathrm{MS}}}\left[1 - \frac{\alpha}{8\pi}\{C(r_i) + C(r_j) - 2C(r_k)\}\right] . \tag{B.12}$$

Here, $C(r_i)$ denotes the quadratic Casimir invariant for the field with subscript $i$. Notice that in SUSY theories the Yukawa couplings are totally symmetric in the $\overline{\mathrm{DR}}$ scheme, which respects supersymmetry, while not in the $\overline{\mathrm{MS}}$ scheme because of the quantum corrections.

### B.2.2 Generic results: SUSY

Next, we consider a generic SUSY theory with the superpotential for Yukawa terms

$$W = \frac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k . \tag{B.13}$$

In this case, we can rewrite the generic result for the gauge coupling beta functions as [223]

$$\mu \frac{d}{d\mu} g = \frac{1}{16\pi^2} b_g^{(1)} + \frac{1}{(16\pi^2)^2} b_g^{(2)} , \tag{B.14}$$

with

$$b_g^{(1)} = g^3 [S(R) - 3C(G)] , \tag{B.15}$$

$$b_g^{(2)} = g^5 \{-6[C(G)]^2 + 2C(G)S(R) + 4S(R)C(R)\} - g^3 \frac{C(r_k)}{d(G)} y^{ijk} (y^{ijk})^* . \tag{B.16}$$

Here $S(R)$ is the Dynkin index summed over all chiral matter fields and $S(R)C(R)$ is the sum of the Dynkin indices multiplied by the Casimir invariant.

The one-loop beta-functions for the Yukawa matrices are readily obtained from the wave-function renormalization of each chiral superfield thanks to the non-renormalization theorem [116]. The result is

$$\mu \frac{d}{d\mu} y^{ijk} = \frac{1}{16\pi^2} \left[ y^{ijl} \gamma_l^{(1)k} + y^{ilk} \gamma_l^{(1)j} + y^{ljk} \gamma_l^{(1)i} \right] , \tag{B.17}$$

with

$$\gamma_i^{(1)j} = \frac{1}{2} (y^*)_{ikl}\, y^{jkl} - 2\delta_i^j g^2 C(r_i) , \tag{B.18}$$

where $C(r_i)$ is the Casimir invariant for the chiral superfield indicated by the subscript $i$.

### B.2.3 Above SUSY scale

By using the results presented in the previous section, we can readily obtain the two-loop RGEs for the gauge coupling constants in the MSSM. The resultant expression is given as

$$\mu\frac{dg_a}{d\mu} = \frac{1}{16\pi^2}b_a^{(1)}g_a^3 + \frac{g_a^3}{(16\pi^2)^2}\left[\sum_{b=1}^{3} b_{ab}^{(2)} g_b^2 - \sum_{k=u,d,e} c_{ak}\ \mathrm{Tr}(f_k^\dagger f_k)\right] , \tag{B.19}$$

where

$$b_a^{(1)} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} , \quad b_{ab}^{(2)} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix} , \tag{B.20}$$

and

$$c_{ak} = \begin{pmatrix} 26/5 & 14/5 & 18/5 \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix} . \tag{B.21}$$

Similarly, the one-loop RGEs for the Yukawa matrices are given as

$$\begin{aligned}
\mu\frac{d}{d\mu}f_u &= \frac{1}{16\pi^2}\left[3\mathrm{Tr}(f_u^\dagger f_u) + 3f_u f_u^\dagger + f_d f_d^\dagger - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right] f_u , \\
\mu\frac{d}{d\mu}f_d &= \frac{1}{16\pi^2}\left[\mathrm{Tr}(3f_d^\dagger f_d + f_e^\dagger f_e) + 3f_d f_d^\dagger + f_u f_u^\dagger - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right] f_d , \\
\mu\frac{d}{d\mu}f_e &= \frac{1}{16\pi^2}\left[\mathrm{Tr}(3f_d^\dagger f_d + f_e^\dagger f_e) + 3f_e f_e^\dagger - \frac{9}{5}g_1^2 - 3g_2^2\right] f_e .
\end{aligned} \tag{B.22}$$

### B.2.4 Below SUSY scale

Below the SUSY breaking scale ($M_S$), squarks, sleptons, higgsinos, and the heavy Higgs bosons are decoupled so that the theory is regarded as the SM with gauginos. In such a case, we need to exploit the results presented in Sec. B.2.1. Then, we find that the contribution of gauginos and the SM particles to the coefficients of the beta functions is given as

$$b_a^{(1)} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix}_{\mathrm{SM}} + \begin{pmatrix} 0 \\ 4/3 \\ 2 \end{pmatrix}_{\mathrm{gaugino}} , \tag{B.23}$$

$$b_{ab}^{(2)} = \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix}_{\mathrm{SM}} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 64/3 & 0 \\ 0 & 0 & 48 \end{pmatrix}_{\mathrm{gaugino}} , \tag{B.24}$$

and

$$c_{ak} = \begin{pmatrix} 17/10 & 1/2 & 3/2 \\ 3/2 & 3/2 & 1/2 \\ 2 & 2 & 0 \end{pmatrix}_{\mathrm{SM}} , \tag{B.25}$$

where the subscripts "SM" and "gaugino" indicate that the contributions are of the SM particles and gauginos, respectively. The running of the Yukawa couplings in this case is given as follows:

$$
\begin{aligned}
\mu\frac{d}{d\mu}y_u &= \frac{1}{16\pi^2}\left[\frac{3}{2}(y_u y_u^\dagger - y_d y_d^\dagger) + Y_2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right]y_u, \\
\mu\frac{d}{d\mu}y_d &= \frac{1}{16\pi^2}\left[\frac{3}{2}(y_d y_d^\dagger - y_u y_u^\dagger) + Y_2 - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right]y_d, \\
\mu\frac{d}{d\mu}y_e &= \frac{1}{16\pi^2}\left[\frac{3}{2}y_e y_e^\dagger + Y_2 - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right]y_e,
\end{aligned}
\tag{B.26}
$$

where

$$
Y_2 \equiv \mathrm{Tr}(3y_u y_u^\dagger + 3y_d y_d^\dagger + y_e y_e^\dagger) \ . \tag{B.27}
$$

# Appendix C

# Renormalization factors of effective operators

Here, we obtain the RGEs for the effective operators which appear in this thesis.

## C.1 Long-distance corrections

First, we demonstrate the evaluation of the long-distance QCD corrections to the baryon-number violating dimension-six operators below the electroweak scale down to the hadronic scale $\simeq$ 1 GeV. There are two kinds of such operators; one is the operators in which all of the fermions have the same chirality, like $\mathcal{O}^{(3)}_{ijkl}$ defined in the next section, and the other is those who have mixed quark chiralities; *e.g.*, $\mathcal{O}^{(1)}_{ijkl}$ in Eq. (4.7). Here, we present the two-loop calculation for both types [205]. We neglect the small QED contribution in the following computation.

The long-distance factor, $A_L$, is determined by the ratio of the Wilson coefficients for the effective operators at the scale of $m_Z$ and the hadron scale $\mu_H$:

$$A_L \equiv \frac{C(\mu_H)}{C(m_Z)} \ . \tag{C.1}$$

In this thesis, we take $\mu_H = 2$ GeV. The coefficient $C(\mu)$ satisfies the following RGE at two-loop level [205]:

$$\mu\frac{d}{d\mu}C(\mu) = -\left[4\frac{\alpha_s}{4\pi} + \left(\frac{14}{3} + \frac{4}{9}N_f + \Delta\right)\frac{\alpha_s^2}{(4\pi)^2}\right]C(\mu) \ , \tag{C.2}$$

where $\Delta = 0$ for the operators including fermions with the same chirality, while $\Delta = -10/3$ for the operators with mixed chiralities. The solution of the equation is

$$\frac{C(\mu)}{C(\mu_0)} = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{-\frac{2}{b_1}} \left[\frac{4\pi b_1 + b_2\alpha_s(\mu)}{4\pi b_1 + b_2\alpha_s(\mu_0)}\right]^{\left(\frac{2}{b_1} - \frac{42+4N_f+9\Delta}{18b_2}\right)} \ , \tag{C.3}$$

with $b_1$ and $b_2$ given in Eq. (B.2). Thus, $A_L$ is given as

$$A_L = \left[\frac{\alpha_s(\mu_H)}{\alpha_s(m_b)}\right]^{\frac{6}{25}} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_Z)}\right]^{\frac{6}{23}} \left[\frac{\alpha_s(\mu_H) + \frac{50\pi}{77}}{\alpha_s(m_b) + \frac{50\pi}{77}}\right]^{-\frac{2047}{11550}} \left[\frac{\alpha_s(m_b) + \frac{23\pi}{29}}{\alpha_s(m_Z) + \frac{23\pi}{29}}\right]^{-\frac{1375}{8004}} \ , \tag{C.4}$$

for $\Delta = 0$, and

$$A_L = \left[\frac{\alpha_s(\mu_H)}{\alpha_s(m_b)}\right]^{\frac{6}{25}} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_Z)}\right]^{\frac{6}{23}} \left[\frac{\alpha_s(\mu_H) + \frac{50\pi}{77}}{\alpha_s(m_b) + \frac{50\pi}{77}}\right]^{-\frac{173}{825}} \left[\frac{\alpha_s(m_b) + \frac{23\pi}{29}}{\alpha_s(m_Z) + \frac{23\pi}{29}}\right]^{-\frac{430}{2001}} , \tag{C.5}$$

for $\Delta = -10/3$. Numerically,

$$A_L = \begin{cases} 1.250 & (\text{for} \quad \Delta = 0) \\ 1.247 & (\text{for} \quad \Delta = -10/3) \end{cases} . \tag{C.6}$$

## C.2 Four-Fermi operators

The baryon-number violating four-Fermi operators invariant under the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ are summarized as follows [202, 203, 204][1]:

$$\begin{aligned} \mathcal{O}^{(1)}_{ijkl} &= \epsilon_{abc}(u^a_{Ri} d^b_{Rj})(Q^c_{Lk} \cdot L_{Ll}) , \\ \mathcal{O}^{(2)}_{ijkl} &= \epsilon_{abc}(Q^a_{Li} \cdot Q^b_{Lj})(u^c_{Rk} e_{Rl}) , \\ \mathcal{O}^{(3)}_{ijkl} &= \epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}(Q^a_{Li\alpha} Q^b_{Lj\gamma})(Q^c_{Lk\delta} L_{Ll\beta}) , \\ \mathcal{O}^{(4)}_{ijkl} &= \epsilon_{abc}(u^a_{Ri} d^b_{Rj})(u^c_{Rk} e_{Rl}) . \end{aligned} \tag{C.7}$$

Here we explicitly write the way of contracting the $\mathrm{SU}(2)_L$ indices for $\mathcal{O}^{(3)}_{ijkl}$. The effective Lagrangian below the SUSY breaking scale is then written as

$$\mathcal{L}_{\text{eff}} = \sum_{I=1}^{4} C^{ijkl}_{(I)} \mathcal{O}^{(I)}_{ijkl} , \tag{C.8}$$

where $C^{ijkl}_{(I)}$ denote the Wilson coefficients of the effective operators.

We evaluate the RGEs for the Wilson coefficients at one-loop level. We have

$$\begin{aligned} \mu\frac{d}{d\mu}C^{ijkl}_{(1)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{11}{10}\right) + \frac{\alpha_2}{4\pi}\left(-\frac{9}{2}\right) + \frac{\alpha_3}{4\pi}(-4)\right]C^{ijkl}_{(1)} , \\ \mu\frac{d}{d\mu}C^{ijkl}_{(2)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{23}{10}\right) + \frac{\alpha_2}{4\pi}\left(-\frac{9}{2}\right) + \frac{\alpha_3}{4\pi}(-4)\right]C^{ijkl}_{(2)} , \\ \mu\frac{d}{d\mu}C^{ijkl}_{(3)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{1}{5}\right) + \frac{\alpha_2}{4\pi}(-3) + \frac{\alpha_3}{4\pi}(-4)\right]C^{ijkl}_{(3)} + \frac{\alpha_2}{4\pi}(-4)\big(C^{jikl}_{(3)} + C^{kjil}_{(3)} + C^{ikjl}_{(3)}\big) , \\ \mu\frac{d}{d\mu}C^{ijkl}_{(4)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{6}{5}\right) + \frac{\alpha_3}{4\pi}(-4)\right]C^{ijkl}_{(4)} + \frac{\alpha_1}{4\pi}(-4)C^{kjil}_{(4)} . \end{aligned} \tag{C.9}$$

The contribution to the RGEs of the gauge interactions is consistent with that presented in Ref. [204].

[1] We have slightly changed the labels of the operators as well as the order of fermions from those presented in Ref. [204].

## C.3 $F$-term operators

From this section, we consider the renormalization of the effective operators in the SUSY theories. In the SUSY theories, SUSY-invariant operators can be written in terms of the superspace integral. There are two types of such operators; the $F$-term and $D$-term type operators. In this section, we discuss the former, and consider the latter in the next section.

For the effective operators written in the form of superpotential, the renormalization effects are readily obtained from the wave-function renormalization of each chiral superfield in the operators. This is a direct consequence of the non-renormalization theorem [116]. A typical example of such operators is the dimension five operators $\mathcal{O}^{5L}_{ijkl}$ and $\mathcal{O}^{5R}_{ijkl}$.

Let us consider a generic $F$-term type operator $\mathcal{O}_{i_1\dots i_n}$ defined by

$$\mathcal{O}_{i_1\dots i_n} \equiv \int d^2\theta\ \Phi_{i_1}\dots\Phi_{i_n}\ , \tag{C.10}$$

with its Wilson coefficient $C^{i_1\cdots i_n}$. The RGE for the Wilson coefficient is then given as follows:

$$\mu\frac{d}{d\mu}C^{i_1\dots i_n}(\mu) = \sum_k C^{i_1\dots i'_k\dots i_n}\ \gamma_{i'_k}{}^{i_k}\ , \tag{C.11}$$

where $\gamma_i{}^j$ are the anomalous dimension matrices associated with the chiral superfields $\Phi_i$. In Ref. [223], they are given upto two-loop level in a generic SUSY theory. Here, we only consider the one-loop contribution:

$$\gamma_i{}^j = \frac{1}{16\pi^2}\gamma_i^{(1)j}\ , \tag{C.12}$$

with $\gamma_i^{(1)j}$ displayed in (B.18).

By using the above formula, the RGEs for the Wilson coefficients $C^{ijkl}_{5L}$ and $C^{ijkl}_{5R}$ of the dimension-five proton decay operators at one-loop level are given as

$$\begin{aligned}
\mu\frac{d}{d\mu}C^{ijkl}_{5L}(\mu) &= \frac{1}{16\pi^2}\biggl[\bigl(-\frac{2}{5}g_1^2 - 6g_2^2 - 8g_3^2\bigr)C^{ijkl}_{5L} + (f_uf_u^\dagger + f_df_d^\dagger)^i{}_{i'}C^{i'jkl}_{5L}\\
&\qquad + (f_uf_u^\dagger + f_df_d^\dagger)^j{}_{j'}C^{ij'kl}_{5L} + (f_uf_u^\dagger + f_df_d^\dagger)^k{}_{k'}C^{ijk'l}_{5L} + (f_ef_e^\dagger)^l{}_{l'}C^{ijkl'}_{5L}\biggr]\ ,\\
\mu\frac{d}{d\mu}C^{ijkl}_{5R}(\mu) &= \frac{1}{16\pi^2}\biggl[\bigl(-\frac{12}{5}g_1^2 - 8g_3^2\bigr)C^{ijkl}_{5R} + C^{i'jkl}_{5R}(2f_u^\dagger f_u)_{i'}{}^i\\
&\qquad + C^{ij'kl}_{5R}(2f_e^\dagger f_e)_{j'}{}^j + C^{ijk'l}_{5R}(2f_u^\dagger f_u)_{k'}{}^k + C^{ijkl'}_{5R}(2f_d^\dagger f_d)_{l'}{}^l\biggr]\ .
\end{aligned} \tag{C.13}$$

Improvement of the results to the two-loop level is straightforward.

## C.4 $D$-term operators

Next, we consider the renormalization of the Kähler type effective operators. On the contrary to the superpotential type operators, these Kähler type operators receive the renormalization effects other than the wave-function renormalization. First, we will derive a simple formula for the one-loop level renormalization factors of any higher-dimensional operators in the Kähler potential in Sec. C.4.1. Then, we evaluate the renormalization factors of the dimension-six proton decay operators at two-loop level in Sec. C.4.2. In the following calculation, we only take the gauge interactions into account, *i.e.*, we neglect the superpotential. A justification for this is that the experimental constraints on the effective operators are particularly severe when the external lines of the operators are of the first and/or second generations, and in such a case the effects of the Yukawa couplings are negligible.

Before moving to the calculation, we first introduce our convention used below. The relationship between bare and renormalized operators is written in the following fashion:

$$\mathcal{O}_B = Z_{\mathcal{O}}\mathcal{O} \ , \tag{C.14}$$

where the subscript $B$ indicates the operator is bare. Then, the Wilson coefficients $C$ for the operators $\mathcal{O}$ obey the differential equations,

$$\mu\frac{d}{d\mu}C(\mu) = \gamma_{\mathcal{O}}C(\mu) \ , \tag{C.15}$$

with $\gamma_{\mathcal{O}}$ the anomalous dimensions for the operators defined as

$$\gamma_{\mathcal{O}} \equiv \mu\frac{d}{d\mu}\ln Z_{\mathcal{O}} \ . \tag{C.16}$$

The anomalous dimensions are obtained by analyzing the vertex functions (or the effective action) in which the operators are inserted. Their RGEs are given as

$$\left[\mu\frac{\partial}{\partial\mu} + \beta_\alpha\frac{\partial}{\partial g_\alpha} - \sum_i \gamma_i + \gamma_{\mathcal{O}}\right]\Gamma_{\mathcal{O}} = 0 \ . \tag{C.17}$$

Here, $\Gamma_{\mathcal{O}}$ are the four-point vertex functions with an insertion of the operators $\mathcal{O}$. The gauge coupling constants and their beta functions are denoted by $g_\alpha$ and $\beta_\alpha$, respectively, and the sum over each gauge group is implicit. Further, $\gamma_i$ shows the anomalous dimension of each superfield contained in the operators.

### C.4.1 One-loop results

In this section, we present a generic one-loop formula [224] for the renormalization factors using the effective Kähler potential for generic four dimensional $N = 1$ SUSY theories computed in Refs. [225, 226]. In the calculation, the dimensional reduction scheme ($\overline{\rm DR}$) [190] is employed for the regularization. According to the results, the one-loop correction to the Kähler potential is given as

$$\Delta K_1 = -\sum_\alpha \frac{1}{16\pi^2}\mathtt{Tr}M^2_{C(\alpha)}\big(2 - \ln\frac{M^2_{C(\alpha)}}{\bar{\mu}^2}\big) \ , \tag{C.18}$$

where $\bar{\mu}^2 \equiv 4\pi e^{-\gamma}\mu^2$ defines the $\overline{\rm MS}$ renormalization scale, and the mass matrix $M^2_{C(\alpha)}$ is defined by

$$(M^2_{C(\alpha)})_{AB} \equiv 2g_\alpha^2\bar{\phi}_a(T_A^{(\alpha)})^a{}_bG^b{}_c(T_B^{(\alpha)})^c{}_d\phi^d \ , \tag{C.19}$$

with $\phi$ the background for the chiral superfield $\Phi$ and $G^a{}_b$ the Kähler metric

$$G^a{}_b \equiv \frac{\partial^2}{\partial\bar{\phi}_a\partial\phi^b}K(\bar{\phi},\phi) \ . \tag{C.20}$$

In Eq. (C.18), `Tr` denotes the trace over the adjoint representation of a gauge group whose coupling constant is $g_\alpha$ and generators are given by $T_A^{(\alpha)}$.

In order to obtain the renormalization factors for the higher-dimensional effective operators, we consider the Kähler potential

$$K = \bar{\phi}_a \phi^a + C\mathcal{O} + C\mathcal{O}^\dagger \ , \tag{C.21}$$

with $C$ the Wilson coefficient of the operator $\mathcal{O}$. In this case, the Kähler metric reads

$$G^a{}_b = \delta^a_b + C\mathcal{O}^a{}_b + C\mathcal{O}^{\dagger a}{}_b \ , \tag{C.22}$$

with $\mathcal{O}^a{}_b \equiv \partial^2 \mathcal{O}/\partial\bar{\phi}_a \partial\phi^b$. By substituting the above equations to Eq. (C.18), we have

$$\Delta K_1 = -\sum_\alpha \frac{g_\alpha^2}{16\pi^2} 2(1+\ln\bar{\mu}^2)[C_\alpha(a)\bar{\phi}_a\phi^a + \{C(\bar{\phi}T_A^{(\alpha)})_a \mathcal{O}^a{}_b (T_A^{(\alpha)}\phi)^b + \text{h.c.}\}] \ , \tag{C.23}$$

where $C_\alpha(i)$ are the quadratic Casimir group theory invariants for the superfield $\Phi_i$, defined in terms of the Lie algebra generators $T_A$ by $(T_A^{(\alpha)}T_A^{(\alpha)})^a_b = C_\alpha(i)\delta^a_b$. Further, we keep only the terms up to the first order with respect to the Wilson coefficient, $C$, and do not show the terms including the logarithmic dependence on the background fields, which are not relevant to the present calculation. At the first order in the perturbation theory, the RGE (C.17) then leads to

$$\gamma_{\mathcal{O}}^{(1)}\mathcal{O} = \sum_i \gamma_i^{(1)}\mathcal{O} + \sum_\alpha \frac{g_\alpha^2}{16\pi^2} 4(\bar{\phi}T_A^{(\alpha)})_a \mathcal{O}^a{}_b (T_A^{(\alpha)}\phi)^b \ . \tag{C.24}$$

Here, the superscript (1) of the anomalous dimensions denotes that they are evaluated at one-loop level. In SUSY theories, $\gamma_i^{(1)}$ is given by Eq. (B.18).

Now we evaluate the second term in Eq. (C.24). To that end, we analyze the structure of the term on a general basis in order to derive the formula for the one-loop renormalization factor of any operator. Consider the following operator which contains an arbitrary number of both chiral and anti-chiral superfields and is singlet under a given global symmetry $G$ as a whole:

$$\mathcal{O} = \bar{\lambda}_a^{i_1\dots i_m} \lambda^a_{j_1\dots j_n} \overline{\Phi}_{i_1}\dots\overline{\Phi}_{i_m} \Phi^{j_1}\dots\Phi^{j_n} \ . \tag{C.25}$$

Here, the coefficients $\lambda^a_{j_1\dots j_n}$ and $\bar{\lambda}_a^{i_1\dots i_m}$ make the set of superfields $G$ singlet. When $G$ is localized (gauged), the operator invariant under both supersymmetry and the gauge symmetry is

$$\int d^2\theta d^2\bar{\theta}\ (\bar{\lambda}_a^{i_1\dots i_m}\overline{\Phi}_{i_1}\dots\overline{\Phi}_{i_m})\big[e^{2gV_G^A T^A}\big]^a{}_b (\lambda^b_{j_1\dots j_n}\Phi^{j_1}\dots\Phi^{j_n}) \ , \tag{C.26}$$

where $g$ and $V_G^A$ are the coupling constant and the gauge vector superfields of the gauge group $G$, respectively. Moreover, $T^A$ are assumed to be the generators for an irreducible representation, which are relevant to the transformation properties of the composite chiral superfield $\Phi_{j_1}\dots\Phi_{j_n}$; under the gauge transformation, $\Phi_{j_1}\dots\Phi_{j_n}$ is transformed as

$$(\lambda^a_{j_1\dots j_n}\Phi^{j_1}\dots\Phi^{j_n}) \to (e^{ig\Lambda^A T^A})^a{}_b (\lambda^b_{j_1\dots j_n}\Phi^{j_1}\dots\Phi^{j_n}) \ , \tag{C.27}$$

with $\Lambda^A$ any chiral superfields. Further, we write the generators for each chiral superfield $\Phi$ as $t^A$, *i.e.*, $\Phi^j \to (e^{ig\Lambda^A t^A})^j{}_{j'}\Phi^{j'}$. Then, since the coefficients $\lambda^a_{j_1\cdots j_n}$ and $\bar{\lambda}^{i_1\cdots i_m}_a$ assemble the transformation properties of each chiral superfield into that of the composite operator $\lambda^a_{j_1\cdots j_n}\Phi^{j_1}\ldots\Phi^{j_n}$, it follows that

$$(T^A)^a{}_b\lambda^b_{j_1\cdots j_n} = \lambda^a_{j'_1 j_2\cdots j_n}(t^A)^{j'_1}_{\ \ j_1} + \cdots + \lambda^a_{j_1\cdots j_{n-1}j'_n}(t^A)^{j'_n}_{\ \ j_n} \ , \tag{C.28}$$

and similarly for the anti-chiral superfields,

$$\bar{\lambda}^{i_1\cdots i_m}_b(T^A)^b{}_a = (t^A)^{i_1}_{\ \ i'_1}\ \bar{\lambda}^{i'_1 i_2\cdots i_m}_b + \cdots + (t^A)^{i_m}_{\ \ i'_m}\ \bar{\lambda}^{i_1\cdots i_{m-1}i'_m}_b \ . \tag{C.29}$$

These expressions imply that $\lambda^a_{j_1\cdots j_n}$ and $\bar{\lambda}^{i_1\cdots i_m}_a$ are invariant tensors under $G$. By using the relations, we now evaluate the second term in Eq. (C.24). It goes as follows:

$$\begin{aligned}
(\bar{\phi}t^A)_a\mathcal{O}^a{}_b(t^A\phi)^b &= [(t^A)^{i_1}_{\ \ i'_1}\ \bar{\lambda}^{i'_1 i_2\cdots i_m}_b + \cdots + (t^A)^{i_m}_{\ \ i'_m}\ \bar{\lambda}^{i_1\cdots i_{m-1}i'_m}_b] \\
&\times [\lambda^a_{j'_1 j_2\cdots j_n}(t^A)^{j'_1}_{\ \ j_1} + \cdots + \lambda^a_{j_1\cdots j_{n-1}j'_n}(t^A)^{j'_n}_{\ \ j_n}]\bar{\phi}_{i_1}\ldots\bar{\phi}_{i_m}\phi^{j_1}\ldots\phi^{j_n} \\
&= \bar{\lambda}^{i_1\cdots i_m}_b(T^A)^b{}_a(T^A)^a{}_c\lambda^c_{j_1\cdots j_n}\bar{\phi}_{i_1}\ldots\bar{\phi}_{i_m}\phi^{j_1}\ldots\phi^{j_n} \\
&= C_G^{\rm comp}\ \mathcal{O} \ ,
\end{aligned} \tag{C.30}$$

where $C_G^{\rm comp}$ is defined by $T^AT^A = C_G^{\rm comp}\mathbb{1}$; it corresponds to the Casimir invariant for the composite chiral superfield $\lambda^a_{j_1\cdots j_n}\Phi^{j_1}\ldots\Phi^{j_n}$. Substituting the expression into Eq. (C.24), we finally obtain a generic formula for the one-loop renormalization factors of arbitrary operators [224]:

$$\gamma^{(1)}_{\mathcal{O}} = \sum_\alpha \frac{g_\alpha^2}{16\pi^2}\left[4C_\alpha^{\rm comp} - 2\sum_i C_\alpha(i)\right] \ , \tag{C.31}$$

with $C_\alpha^{\rm comp}$ the Casimir invariants of the gauge group $\alpha$ for the chiral part of the operators.

Now we apply the formula to the dimension-six proton decay operators

$$\mathcal{L}_6^{\rm eff} = C^{ijkl}_{6(1)}\mathcal{O}^{6(1)}_{ijkl} + C^{ijkl}_{6(2)}\mathcal{O}^{6(2)}_{ijkl} \ , \tag{C.32}$$

where

$$\mathcal{O}^{6(1)}_{ijkl} = \int d^2\theta d^2\bar{\theta}\ \epsilon_{abc}\epsilon_{\alpha\beta}\big(\overline{U}^*_i\big)^a\big(\overline{D}^*_j\big)^b e^{-\frac{2}{3}g'B}\big(e^{2g_3G}Q^\alpha_k\big)^c L^\beta_l \ , \tag{C.33}$$

$$\mathcal{O}^{6(2)}_{ijkl} = \int d^2\theta d^2\bar{\theta}\epsilon_{abc}\epsilon_{\alpha\beta}\ Q^{a\alpha}_i Q^{b\beta}_j e^{\frac{2}{3}g'B}\big(e^{-2g_3G}\overline{U}^*_k\big)^c\overline{E}^*_l \ . \tag{C.34}$$

For the operators, we find $C_3^{\rm comp} = C_3(\Box) = 4/3$ in the case of $SU(3)_C$ and $C_2^{\rm comp} = 0$ in the case of $SU(2)_L$ for both $\mathcal{O}^{6(1)}$ and $\mathcal{O}^{6(2)}$. Here, $\Box$ denotes the fundamental representation of the corresponding group, and we have used $C_3(\Box) = C_3(\overline{\Box})$. Note that the latter equation for $SU(2)_L$ follows from the fact that the $SU(2)_L$ non-singlet superfields in the effective operators have the same chirality and form an $SU(2)_L$ singlet. For $U(1)_Y$ contributions, on the other hand, we obtain different results for the operators $\mathcal{O}^{6(1)}$ and $\mathcal{O}^{6(2)}$: $C_Y^{\rm comp} = (Y_Q + Y_L)^2$ for $\mathcal{O}^{6(1)}$ and $C_Y^{\rm comp} = (2Y_Q)^2$ for $\mathcal{O}^{6(2)}$. As a result, we obtain the RGEs for the Wilson coefficients as

$$\begin{aligned}
\mu\frac{d}{d\mu}C^{ijkl}_{6(1)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{11}{15}\right) + \frac{\alpha_2}{4\pi}(-3) + \frac{\alpha_3}{4\pi}\left(-\frac{8}{3}\right)\right]C^{ijkl}_{6(1)} \ , \\
\mu\frac{d}{d\mu}C^{ijkl}_{6(2)} &= \left[\frac{\alpha_1}{4\pi}\left(-\frac{23}{15}\right) + \frac{\alpha_2}{4\pi}(-3) + \frac{\alpha_3}{4\pi}\left(-\frac{8}{3}\right)\right]C^{ijkl}_{6(2)} \ .
\end{aligned} \tag{C.35}$$

These results are totally consistent with those in Ref. [227].

### C.4.2 Two-loop results

Next, we discuss the two-loop level contribution. Again, we use the results in Ref. [226]. The radiative corrections to the Kähler potential at two-loop level are described by

$$\Delta K_2 = \frac{1}{2} R^{b\ d}_{\ a\ c} J^{a\ c}_{\ b\ d}(M^2) - \sum_\alpha f^{(\alpha)}_{ABC} f^{(\alpha)}_{DEF} I^{BDEAFC}(M^2_{V(\alpha)})$$
$$- \sum_\alpha (G T^{(\alpha)}_A \phi)^b{}_{;c} (\bar{\phi} T^{(\alpha)}_B G)_a{}^{;d} H^{a\ c\ AB}_{\ b\ d}(M^2, M^2_{V(\alpha)}) \ , \tag{C.36}$$

with $f^{(\alpha)}_{ABC}$ the structure constants of the gauge group $\alpha$. The mass functions and the geometric factors appear in Eq. (C.36) are defined as follows:

$$J^{a\ c}_{\ b\ d}(M^2) = \frac{2}{(16\pi^2)^2} (\ln \bar{\mu}^2) \sum_{\alpha,\beta} (M^2_\alpha G^{-1})^a{}_b (M^2_\beta G^{-1})^c{}_d \ , \tag{C.37}$$

$$I^{ABCDEF}(M^2_{V(\alpha)}) = -\frac{1}{2} \frac{g^2_\alpha}{(16\pi^2)^2} (\ln \bar{\mu}^2) \big[ 4 (M^2_{V(\alpha)})_{AB} \delta_{CD} \delta_{EF}$$
$$- \delta_{AB} (M^2_{V(\alpha)} \ln M^2_{V(\alpha)})_{CD} \delta_{EF} - \delta_{AB} \delta_{CD} (M^2_{V(\alpha)} \ln M^2_{V(\alpha)})_{EF} \big] + \text{cycl.} \ , \tag{C.38}$$

where the +cycl. denotes the cyclic permutation of the labels $AB, CD, EF$, and

$$H^{a\ c\ AB}_{\ b\ d}(M^2, M^2_{V(\alpha)}) = -\frac{g^2_\alpha}{(16\pi^2)^2} (\ln \bar{\mu}^2) \Bigg[ \sum_\beta \delta_{AB} \{ 2 (M^2_\beta G^{-1})^a{}_b (G^{-1})^c{}_d + 2 (G^{-1})^a{}_b (M^2_\beta G^{-1})^c{}_d$$
$$- (G^{-1})^a{}_b (M^2_\beta \ln\{M^2_\beta\} G^{-1})^c{}_d - (M^2_\beta \ln\{M^2_\beta\} G^{-1})^a{}_b (G^{-1})^c{}_d \}$$
$$+ 2 (G^{-1})^a{}_b (G^{-1})^c{}_d (M^2_{V(\alpha)})_{AB} + (G^{-1})^a{}_b (G^{-1})^c{}_d (M^2_{V(\alpha)} \ln M^2_{V(\alpha)})_{AB} \Bigg] \ , \tag{C.39}$$

Here, we drop the terms independent of the scale $\mu$ or containing two logarithms. The latter terms give rise to the logarithmic terms after differentiation, which cancel other logarithmic terms in the RGEs. The mass parameters are defined as

$$(M^2_\alpha)^a{}_b \equiv 2 g^2_\alpha (T^{(\alpha)}_A \phi)^a (\bar{\phi} T^{(\alpha)}_A G)_b \ , \tag{C.40}$$

and

$$(M^2_{V(\alpha)})_{AB} \equiv \frac{1}{2} \big[ (M^2_{C(\alpha)})_{AB} + (M^2_{C(\alpha)})_{BA} \big] \ . \tag{C.41}$$

Further, $G^{-1}$ is inverse of the Kähler metric $G^a{}_b$ defined in Eq. (C.20), and the curvature $R^{a\ c}_{\ b\ d}$ is given by

$$R^{a\ c}_{\ b\ d} \equiv \frac{\partial^2}{\partial \bar{\phi}_a \partial \phi^b} G^c{}_d - \left( \frac{\partial}{\partial \bar{\phi}_a} G^c_e \right) (G^{-1})^e{}_f \left( \frac{\partial}{\partial \phi^b} G^f{}_d \right) \ . \tag{C.42}$$

The third term in Eq. (C.36) includes the shorthand notations, $(G T_A \phi)^b{}_{;c}$ and $(\bar{\phi} T_B G)_a{}^{;d}$, which are defined as

$$(G T_A \phi)^a{}_{;b} \equiv G^a{}_c (T_A)^c{}_b + \left( \frac{\partial}{\partial \phi^c} G^a{}_b \right) (T_A \phi)^c$$
$$= (T_A)^a{}_c G^c{}_b + (\bar{\phi} T_A)_c \left( \frac{\partial}{\partial \bar{\phi}_c} G^a{}_b \right) \equiv (\bar{\phi} T_A G)_b{}^{;a} \ . \tag{C.43}$$

Here, the second line follows from the gauge invariance of the Kähler potential.

By using them, we readily obtain the two-loop corrections to the vertex functions. The RGE in Eq. (C.17) at two-loop level is given as

$$\mu\frac{\partial\Gamma^{(2)}_{\mathcal{O}}}{\partial\mu}+\sum_{\alpha}\frac{1}{16\pi^2}b^{(1)}_{\alpha}g^3_{\alpha}\frac{\partial}{\partial g_{\alpha}}\Gamma^{(1)}_{\mathcal{O}}-\sum_{i}\gamma^{(1)}_i\Gamma^{(1)}_{\mathcal{O}}-\sum_{i}\gamma^{(2)}_i\Gamma^{(0)}_{\mathcal{O}}+\gamma^{(1)}_{\mathcal{O}}\Gamma^{(1)}_{\mathcal{O}}+\gamma^{(2)}_{\mathcal{O}}\Gamma^{(0)}_{\mathcal{O}}=0\ . \tag{C.44}$$

Here, the subscripts (0–2) indicate the quantities are evaluated at tree, one-loop, and two-loop level, respectively. One-loop anomalous dimensions $\gamma^{(1)}_i$ are given in Eq. (B.18), while the two-loop ones are given as [223]

$$\gamma^{(2)}_i=\frac{1}{(16\pi^2)^2}\sum_{\alpha,\beta}2g^2_{\alpha}C_{\alpha}(i)\left[g^2_{\alpha}b^{(1)}_{\alpha}\delta_{\alpha\beta}+2g^2_{\beta}C_{\beta}(i)\right]\ . \tag{C.45}$$

Here, $b^{(1)}_{\alpha}$ are the one-loop beta function coefficients for gauge coupling constants, given in Eq. (B.20). From the RGE in Eq. (C.44), we now obtain the two-loop anomalous dimensions for the effective operators. Again, we parametrize them as follows [224]:

$$\begin{aligned}\gamma^{(2)}_{\mathcal{O}^{(I)}}=&\frac{g^4_3}{(16\pi^2)^2}\left[\gamma^{(2)}_{\mathcal{O}^{(I)}}\right]_{33}+\frac{g^4_2}{(16\pi^2)^2}\left[\gamma^{(2)}_{\mathcal{O}^{(I)}}\right]_{22}+\frac{g^4_1}{(16\pi^2)^2}\left[\gamma^{(2)}_{\mathcal{O}^{(I)}}\right]_{11}\\&+\frac{g^2_2g^2_3}{(16\pi^2)^2}\left[\gamma^{(2)}_{\mathcal{O}^{(I)}}\right]_{23}+\frac{g^2_1g^2_2}{(16\pi^2)^2}\left[\gamma^{(2)}_{\mathcal{O}^{(I)}}\right]_{12}+\frac{g^2_1g^2_3}{(16\pi^2)^2}\left[\gamma^{(2)}_{\mathcal{O}^{(I)}}\right]_{13}\ .\end{aligned} \tag{C.46}$$

Then, we have

$$\left[\gamma^{(2)}_{\mathcal{O}^{6(1)}}\right]_{33}=\left[\gamma^{(2)}_{\mathcal{O}^{6(2)}}\right]_{33}=\frac{64}{3}+8b^{(1)}_3\ , \tag{C.47}$$

$$\left[\gamma^{(2)}_{\mathcal{O}^{6(1)}}\right]_{22}=\left[\gamma^{(2)}_{\mathcal{O}^{6(2)}}\right]_{22}=\frac{9}{2}+3b^{(1)}_2\ , \tag{C.48}$$

$$\begin{aligned}\left[\gamma^{(2)}_{\mathcal{O}^{6(1)}}\right]_{11}=&\frac{113}{150}+b^{(1)}_1\ ,\\\left[\gamma^{(2)}_{\mathcal{O}^{6(2)}}\right]_{11}=&\frac{91}{50}+\frac{9}{5}b^{(1)}_1\ ,\end{aligned} \tag{C.49}$$

$$\begin{aligned}\left[\gamma^{(2)}_{\mathcal{O}^{(1)}}\right]_{23}=12\ ,\\\left[\gamma^{(2)}_{\mathcal{O}^{(2)}}\right]_{23}=20\ ,\end{aligned} \tag{C.50}$$

$$\begin{aligned}\left[\gamma^{(2)}_{\mathcal{O}^{(1)}}\right]_{12}=\frac{6}{5}\ ,\\\left[\gamma^{(2)}_{\mathcal{O}^{(2)}}\right]_{12}=\frac{2}{5}\ ,\end{aligned} \tag{C.51}$$

$$\begin{aligned}\left[\gamma^{(2)}_{\mathcal{O}^{(1)}}\right]_{13}=\frac{68}{15}\ ,\\\left[\gamma^{(2)}_{\mathcal{O}^{(2)}}\right]_{13}=\frac{76}{15}\ .\end{aligned} \tag{C.52}$$

# Appendix D

# Input parameters

In this chapter, we list the set of input parameters which we use in our calculation. Basically, we follow the conventions presented in the Supersymmetry Les Houches Accord (SLHA) [228, 229]. The values of the parameters are taken from Ref. [8].

## D.1 Standard Model parameters

### D.1.1 Electroweak sector

The electroweak sector is fixed by the following four parameters:

- $m_{H^0}$: Mass of Higgs boson, $H^0$
$$m_{H^0} = 125.9 \pm 0.4 \text{ GeV} . \tag{D.1}$$
The value is obtained by the ATLAS and CMS Collaborations at the LHC.

- $m_Z$: Mass of $Z$-boson
$$m_Z = 91.1876 \pm 0.0021 \text{ GeV} . \tag{D.2}$$
It has been determined at LEP 1 [230].

- $G_F$: Fermi constant
$$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2} , \tag{D.3}$$
which is derived from the muon lifetime measurement.

- $\alpha_{\text{em}}(m_Z)^{\overline{\text{MS}}}$: Electromagnetic coupling at $m_Z$ in the $\overline{\text{MS}}$ scheme with $N_f = 5$:
$$[\alpha_{\text{em}}(m_Z)^{\overline{\text{MS}}}]^{-1} = 127.944 \pm 0.014 . \tag{D.4}$$
It is evaluated from the fine structure constant, $\alpha = 1/137.035999074(44)$, which is determined from the electron anomalous magnetic dipole moment. The uncertainty mainly comes from the low-energy contribution to vacuum polarization.

Table D.1: Input masses for the SM fermions [8].

| | Masses (GeV) | Comments |
|---|---|---|
| $m_u(2\text{ GeV})^{\overline{\text{MS}}}$ | $(2.3^{+0.7}_{-0.5}) \times 10^{-3}$ | $u$ quark $\overline{\text{MS}}$ mass at $\mu = 2$ GeV. |
| $m_d(2\text{ GeV})^{\overline{\text{MS}}}$ | $(4.8^{+0.5}_{-0.3}) \times 10^{-3}$ | $d$ quark $\overline{\text{MS}}$ mass at $\mu = 2$ GeV. |
| $m_s(2\text{ GeV})^{\overline{\text{MS}}}$ | $(95 \pm 5) \times 10^{-3}$ | $s$ quark $\overline{\text{MS}}$ mass at $\mu = 2$ GeV. |
| $m_c(m_c)^{\overline{\text{MS}}}$ | $1.275 \pm 0.025$ | $c$ quark $\overline{\text{MS}}$ mass at $\mu = m_c$ . |
| $m_b(m_b)^{\overline{\text{MS}}}$ | $4.18 \pm 0.03$ | $b$ quark $\overline{\text{MS}}$ mass at $\mu = m_b$ . |
| $m_t$ | $173.07 \pm 0.52 \pm 0.72$ | $t$ quark pole mass. |
| $m_e$ | $0.510998928(11) \times 10^{-3}$ | electron pole mass. |
| $m_\mu$ | $105.6583715(35) \times 10^{-3}$ | muon pole mass. |
| $m_\tau$ | $1.77682(16)$ | $\tau$ pole mass. |

All of the other electroweak parameters are in principle determined from the above parameters. However, in the analysis presented in this thesis, we sometimes also use the following parameters as input parameters:

$$m_W = 80.385(15)\text{ GeV} , \tag{D.5}$$

$$\sin^2\theta(m_Z)^{\overline{\text{MS}}} = 0.23116(12) . \tag{D.6}$$

The difference between the values and those evaluated from the above four parameters are only relevant to beyond the leading-order calculations.

### D.1.2 Strong coupling constant

We use the strong coupling constant at the $Z$-boson mass in the $\overline{\text{MS}}$ scheme with five flavors as an input parameter:

$$\alpha_s(m_Z) = 0.1184(7) . \tag{D.7}$$

### D.1.3 Fermion masses

The input values for the SM fermion masses are summarized in Table. D.1. We use the current quark masses at 2 GeV in the $\overline{\text{MS}}$ scheme for the light quarks, $u$, $d$, and $s$. The running masses in the $\overline{\text{MS}}$ scheme are used for $c$ and $b$ quarks. For top quark and the charged leptons, we use the on-shell masses. Neutrino masses are ignored in this thesis.

For the quark masses, the one-loop relation between the pole mass $m_q$ and the $\overline{\text{MS}}$ mass $m_q^{\overline{\text{MS}}}$ is given as

$$m_q = m_q^{\overline{\text{MS}}}(m_q^{\overline{\text{MS}}})\left(1 + \frac{4}{3\pi}\alpha_s(m_q^{\overline{\text{MS}}})\right) . \tag{D.8}$$

### D.1.4 CKM matrix

The CKM matrix elements given in Eq. (1.11) are expressed in terms of the Wolfenstein parametrization as

$$\begin{aligned} s_{12} &= \lambda \ , \\ s_{23} &= A\lambda^2 \ , \\ s_{13}e^{i\delta} &= \frac{A\lambda^3(\overline{\rho}+i\overline{\eta})\sqrt{1-A^2\lambda^4}}{\sqrt{1-\lambda^2}[1-A^2\lambda^4(\overline{\rho}+i\overline{\eta})]} \ . \end{aligned} \tag{D.9}$$

These four parameters, $\lambda$, $A$, $\overline{\rho}$ and $\overline{\eta}$, are obtained as follows:

$$\begin{aligned} \lambda &= 0.22535 \pm 0.00065 \ , \\ A &= 0.811^{+0.022}_{-0.012} \ , \\ \overline{\rho} &= 0.131^{+0.026}_{-0.013} \ , \\ \overline{\eta} &= 0.345^{+0.013}_{-0.014} \ . \end{aligned} \tag{D.10}$$

## D.2 Hadron matrix elements

In this section, we give the hadron matrix elements of baryon-number violating operators. For the values we use those estimated with lattice QCD simulation. There are two ways for the calculation; one is the *direct method* and the other is the *indirect method*. In the former method, the matrix elements of the operators between nucleon and meson states are measured directly from the three-point functions including an effective operator, a nucleon current, and a meson current. On the other hand, in the latter method, the calculation reduces to the determination of two low-energy constants by means of the chiral perturbation theory. In this thesis, we use the values obtained with the direct method unless otherwise noted. The results are given in Sec. D.2.1. We also discuss the indirect method in the subsequent section and compare both results in Sec. D.2.3.

### D.2.1 Direct method

In Ref. [206], the proton decay matrix elements are evaluated using the direct method with $N_f = 2+1$ flavor lattice QCD, where $u$ and $d$ quarks are degenerate in mass respecting the isospin symmetry. In the simulation, *domain-wall fermions (DWFs)* [231, 232, 233] are used; the fermions are defined on a five-dimensional spacetime so that chiral fermions are realized on a four-dimensional domain wall in the fifth-dimension without introducing extra *doubler* fermions. To begin with, let us define the hadron matrix elements of baryon-number violating operators. Here, we only discuss the dimension-six operators. Such operators include three quarks and a lepton, having a form like $\overline{l^c}\mathcal{O}^{\Gamma\Gamma'}_{qqq}$ where $c$ denotes the charge conjugation and the three-quark operator $\mathcal{O}^{\Gamma\Gamma'}_{qqq}$ is defined as

$$\mathcal{O}^{\Gamma\Gamma'}_{qqq} = (qq)_\Gamma q_{\Gamma'} \equiv \epsilon_{abc}(q^{aT}CP_\Gamma q^b)P_{\Gamma'}q^c \ . \tag{D.11}$$

Here, $C$ denotes the charge conjugation matrix and $\Gamma,\ \Gamma' = R$ or $L$ represents the chirality. $P_\Gamma$ is the projection operator defined by $P_\Gamma \equiv (1\mp\gamma_5)/2$ for $(\Gamma = L, R)$, respectively. The operators

Table D.2: Matrix elements obtained by direct calculation in Ref. [206].

| Matrix element | Value ($\mathrm{GeV}^2$) | Matrix element | Value ($\mathrm{GeV}^2$) |
|---|---|---|---|
| $\langle\pi^0\|(ud)_R u_L\|p\rangle$ | $-0.103(23)(34)$ | $\langle K^0\|(us)_R u_L\|p\rangle$ | $0.098(15)(12)$ |
| $\langle\pi^0\|(ud)_L u_L\|p\rangle$ | $0.133(29)(28)$ | $\langle K^0\|(us)_L u_L\|p\rangle$ | $0.042(13)(8)$ |
| $\langle\pi^+\|(ud)_R d_L\|p\rangle$ | $-0.146(33)(48)$ | $\langle K^+\|(us)_R d_L\|p\rangle$ | $-0.054(11)(9)$ |
| $\langle\pi^+\|(ud)_L d_L\|p\rangle$ | $0.188(41)(40)$ | $\langle K^+\|(us)_L d_L\|p\rangle$ | $0.036(12)(7)$ |
| $\langle\eta^0\|(ud)_R u_L\|p\rangle$ | $0.015(14)(17)$ | $\langle K^+\|(ud)_R s_L\|p\rangle$ | $-0.093(24)(18)$ |
| $\langle\eta^0\|(ud)_L u_L\|p\rangle$ | $0.088(21)(16)$ | $\langle K^+\|(ud)_L s_L\|p\rangle$ | $0.111(22)(16)$ |
| | | $\langle K^+\|(ds)_R u_L\|p\rangle$ | $-0.044(12)(5)$ |
| | | $\langle K^+\|(ds)_L u_L\|p\rangle$ | $-0.076(14)(9)$ |

induce nucleon ($N = p, n$) decay into a pseudo-scalar meson ($M = \pi, K, \eta$) and an anti-lepton ($l^c$). The transition amplitude is

$$\langle M(\boldsymbol{p}), l^c(\boldsymbol{q}, s')|[\overline{l^c}\mathcal{O}^{\Gamma\Gamma'}_{qqq}]|N(\boldsymbol{k}, s)\rangle = \overline{v^c}(\boldsymbol{q}, s')\langle M(\boldsymbol{p})|[\mathcal{O}^{\Gamma\Gamma'}_{qqq}]|N(\boldsymbol{k}, s)\rangle \ , \tag{D.12}$$

with $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{p}$. $|N(\boldsymbol{k}, s)\rangle$ and $|M(\boldsymbol{p})\rangle$ denote the one-particle states[1] of the initial nucleon and the final meson with three-momenta $\boldsymbol{k}$ and $\boldsymbol{p}$, respectively, and $\overline{v^c}(\boldsymbol{q}, s')$ is the wave function coefficient of the final anti-lepton with spin $s'$. The amplitude is a function of $\not{q}$ and $q^2$. By using the on-shell condition, we obtain $\not{q}v_l = -m_l v_l$ and $q^2 = m_l^2$ with $m_l$ the mass of the final lepton. Since $m_l^2 \ll m_N^2$ ($m_N$ is the nucleon mass), we can safely ignore the $\not{q}$ and $q^2$ dependence, so we set $\not{q} = q^2 = 0$ in the following calculation. Then, the right-hand side of the above expression reduces to

$$\overline{v^c}(\boldsymbol{q}, s')\langle M(\boldsymbol{p})|[\mathcal{O}^{\Gamma\Gamma'}_{qqq}]|N(\boldsymbol{k}, s)\rangle = \langle M|\mathcal{O}^{\Gamma\Gamma'}_{qqq}|N\rangle\ \overline{v^c}(\boldsymbol{q}, s')P_{\Gamma'}u_N(\boldsymbol{k}, s) \ . \tag{D.13}$$

Now, all we have to do is to evaluate the hadron matrix elements $\langle M|\mathcal{O}^{\Gamma\Gamma'}_{qqq}|N\rangle$.[2]

The results are summarized in Table. D.2. In the table, we use an abbreviated notation like

$$\langle\pi^0|(ud)_R u_L|p\rangle = \langle\pi^0|\epsilon_{abc}(u^{aT}CP_R d^b)P_L u^c|p\rangle \ . \tag{D.14}$$

The first and second parentheses represent statistical and systematic errors, respectively. The matrix elements are evaluated at the scale of $\mu = 2$ GeV. Here, we only show the matrix elements for proton. As for neutron, we can readily obtain the matrix elements through the following

[1] We use the invariant normalization for the one-particle states: *e.g.*, $\langle\boldsymbol{p}|\boldsymbol{q}\rangle = 2E_{\boldsymbol{p}}(2\pi)^3\delta^{(3)}(\boldsymbol{p} - \boldsymbol{q})$.

[2] Notice that we define $\langle M|\mathcal{O}^{\Gamma\Gamma'}_{qqq}|N\rangle$ to be a scalar; a spinor factor is removed in the definition.

relations:

$$
\begin{aligned}
\langle \pi^0 | (ud)_\Gamma u_L | p \rangle &= \langle \pi^0 | (du)_\Gamma d_L | n \rangle \ , \\
\langle \pi^+ | (ud)_\Gamma d_L | p \rangle &= -\langle \pi^- | (du)_\Gamma u_L | n \rangle \ , \\
\langle K^0 | (us)_\Gamma u_L | p \rangle &= -\langle K^+ | (ds)_\Gamma d_L | n \rangle \ , \\
\langle K^+ | (us)_\Gamma d_L | p \rangle &= -\langle K^0 | (ds)_\Gamma u_L | n \rangle \ , \\
\langle K^+ | (ud)_\Gamma s_L | p \rangle &= -\langle K^0 | (du)_\Gamma s_L | n \rangle \ , \\
\langle K^+ | (ds)_\Gamma u_L | p \rangle &= -\langle K^0 | (us)_\Gamma d_L | n \rangle \ , \\
\langle \eta^0 | (ud)_\Gamma u_L | p \rangle &= -\langle \eta^0 | (du)_\Gamma d_L | n \rangle \ .
\end{aligned}
\tag{D.15}
$$

Further, in the case of the other two combinations of chirality $(\Gamma, \Gamma') = (L, R)$ and $(R, R)$, the matrix elements are derived from the above results through the parity transformation.

### D.2.2 Indirect method

Next, we discuss the indirect method. In the method, we divide the evaluation of the hadron matrix elements into two steps. First, we express them in terms of the low-energy constants $\alpha_p$ and $\beta_p$ by using the chiral perturbation techniques [234, 235, 236]. Then, the low-energy constants are obtained from the results of lattice simulations [237].

To begin with, let us give a brief introduction to the chiral perturbation theory. For detail, see Refs. [238, 239]. In the formalism, the octet meson and baryon fields are written with the traceless $3 \times 3$ matrix as

$$
\Phi = \sqrt{2}\Phi^A T^A = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -2\frac{\eta^0}{\sqrt{6}} \end{pmatrix} \ , \tag{D.16}
$$

$$
B = \sqrt{2}B^A T^A = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix} \ . \tag{D.17}
$$

The transformation properties of the meson fields under the $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ chiral symmetry are expressed in terms of

$$
U \equiv \exp\left(\frac{i\sqrt{2}\Phi}{f_\pi}\right) \tag{D.18}
$$

as follows:

$$
U \mapsto U' = g_R U g_L^\dagger \ , \qquad g_L \in \mathrm{SU}(3)_L \ , \quad g_R \in \mathrm{SU}(3)_R \ . \tag{D.19}
$$

Here, $f_\pi \simeq 92.2$ MeV [8] is the pion decay constant. Further, we define a matrix $\xi$ by

$$
\xi \equiv \exp\left(\frac{i\Phi}{\sqrt{2}f_\pi}\right) \ , \tag{D.20}
$$

which of course satisfies $\xi^2 = U$. The transformation properties of $\xi$ is given by

$$
\xi \mapsto \xi' = g_R \xi h(g, \pi)^\dagger = h(g, \pi) \xi g_L^\dagger \ , \tag{D.21}
$$

where $h(g,\pi)$ is a SU(3)-valued function. With the function, the transformation properties of the baryon field are give as

$$B \to B' = h(g,\pi) B h(g,\pi)^\dagger \ . \tag{D.22}$$

By using the quantities, we can construct the effective Lagrangian of the meson and baryon fields:

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U(\partial^\mu U)^\dagger] + \mathrm{Tr}[\overline{B}(i\not{D} - M_B)B] \\ &- \frac{D}{2}\mathrm{Tr}\big(\overline{B}\gamma^\mu\gamma_5\{\xi_\mu, B\}\big) - \frac{F}{2}\mathrm{Tr}\big(\overline{B}\gamma^\mu\gamma_5[\xi_\mu, B]\big) \ ,\end{aligned} \tag{D.23}$$

with appropriate mass terms for the meson/baryon mass spectrum. The covariant derivative of the baryon field is given by

$$D_\mu B \equiv \partial_\mu B + [\Gamma_\mu, B], \tag{D.24}$$

where

$$\Gamma_\mu \equiv \frac{1}{2}[\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger], \tag{D.25}$$

is called the chiral connection. Further, the definition of $\xi_\mu$ is

$$\xi_\mu \equiv i[\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger], \tag{D.26}$$

which is referred to as the chiral vielbein. The low-energy constants $D$ and $F$ are determined by fitting the semi-leptonic decays $B \to B' + e^- + \bar{\nu}_e$ at tree level [240]: $D = 0.80(1),\ F = 0.47(1)$. $M_B$ represents the baryon mass parameter, which we choose as $M_B \simeq (m_{\Sigma^0} + m_{\Lambda^0})/2$ with $m_{\Sigma^0}$ and $m_{\Lambda^0}$ the masses of $\Sigma^0$ and $\Lambda^0$, respectively.

In addition, the baryon number violating operators, which are written in terms of the quark fields, are expressed with the meson and baryon fields according to their transformation properties under the $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$. In this step, all of the long-distance QCD effects are included into just two parameters, $\alpha_p$ and $\beta_p$, and all other things are completely determined from symmetry arguments. Detailed discussion is given in Refs. [234, 235]. The low-energy constants $\alpha_p$ and $\beta_p$ are defined as

$$\begin{aligned}\langle 0|(ud)_R u_L|p\rangle &= \alpha_p P_L u_p \ , \\ \langle 0|(ud)_L u_L|p\rangle &= \beta_p P_L u_p \ ,\end{aligned} \tag{D.27}$$

where $|0\rangle$ is the vacuum state and $u_p$ is the four-component spinor wave function of proton. We have used the abbreviation introduced in the previous section. The values of the constants are extracted from the results of lattice simulations [237]:

$$\begin{aligned}\alpha_p &= -0.0112 \pm 0.0012_{(\rm stat)} \pm 0.0022_{(\rm syst)}\ \mathrm{GeV}^3 \ , \\ \beta_p &= 0.0120 \pm 0.0013_{(\rm stat)} \pm 0.0023_{(\rm syst)}\ \mathrm{GeV}^3 \ ,\end{aligned} \tag{D.28}$$

where they are evaluated at $\mu = 2$ GeV.

Now, by using the interactions, we can calculate the amplitudes of proton decay into a meson and an antilepton. The corresponding hadron matrix elements are given as follows:

$$\langle\pi^0|(ud)_R u_L|p\rangle = \frac{\alpha_p}{2f_\pi}(1+D+F) \ , \tag{D.29}$$

$$\langle\pi^0|(ud)_L u_L|p\rangle = \frac{\beta_p}{2f_\pi}(1+D+F) \ , \tag{D.30}$$

$$\langle\pi^+|(ud)_R d_L|p\rangle = \frac{\alpha_p}{\sqrt{2}f_\pi}(1+D+F) \ , \tag{D.31}$$

$$\langle\pi^+|(ud)_L d_L|p\rangle = \frac{\beta_p}{\sqrt{2}f_\pi}(1+D+F) \ , \tag{D.32}$$

$$\langle K^0|(us)_R u_L|p\rangle = -\frac{\alpha_p}{\sqrt{2}f_\pi}\left(1+(D-F)\frac{m_p}{M_B}\right) \ , \tag{D.33}$$

$$\langle K^0|(us)_L u_L|p\rangle = +\frac{\beta_p}{\sqrt{2}f_\pi}\left(1-(D-F)\frac{m_p}{M_B}\right) \ , \tag{D.34}$$

$$\langle K^+|(us)_R d_L|p\rangle = \frac{\alpha_p}{\sqrt{2}f_\pi}\left(\frac{2D}{3}\frac{m_p}{M_B}\right) \ , \tag{D.35}$$

$$\langle K^+|(us)_L d_L|p\rangle = \frac{\beta_p}{\sqrt{2}f_\pi}\left(\frac{2D}{3}\frac{m_p}{M_B}\right) \ , \tag{D.36}$$

$$\langle K^+|(ud)_R s_L|p\rangle = \frac{\alpha_p}{\sqrt{2}f_\pi}\left(1+\frac{D+3F}{3}\frac{m_p}{M_B}\right) \ , \tag{D.37}$$

$$\langle K^+|(ud)_L s_L|p\rangle = \frac{\beta_p}{\sqrt{2}f_\pi}\left(1+\frac{D+3F}{3}\frac{m_p}{M_B}\right) \ , \tag{D.38}$$

$$\langle K^+|(ds)_R u_L|p\rangle = +\frac{\alpha_p}{\sqrt{2}f_\pi}\left(1+\frac{D-3F}{3}\frac{m_p}{M_B}\right) \ , \tag{D.39}$$

$$\langle K^+|(ds)_L u_L|p\rangle = -\frac{\beta_p}{\sqrt{2}f_\pi}\left(1-\frac{D-3F}{3}\frac{m_p}{M_B}\right) \ , \tag{D.40}$$

$$\langle\eta^0|(ud)_R u_L|p\rangle = -\frac{\alpha_p}{2\sqrt{3}f_\pi}\Big(1+(D-3F)\Big) \ , \tag{D.41}$$

$$\langle\eta^0|(ud)_L u_L|p\rangle = +\frac{\beta_p}{2\sqrt{3}f_\pi}\Big(3-(D-3F)\Big) \ . \tag{D.42}$$

### D.2.3 Comparison

Finally, we compare the values obtained by the direct and indirect methods for convenience. Such comparison is given in Ref. [206] and we just quote the results. They are summarized in Fig. D.1. As one can see from the figure, these results are consistent with each other within error of calculation.

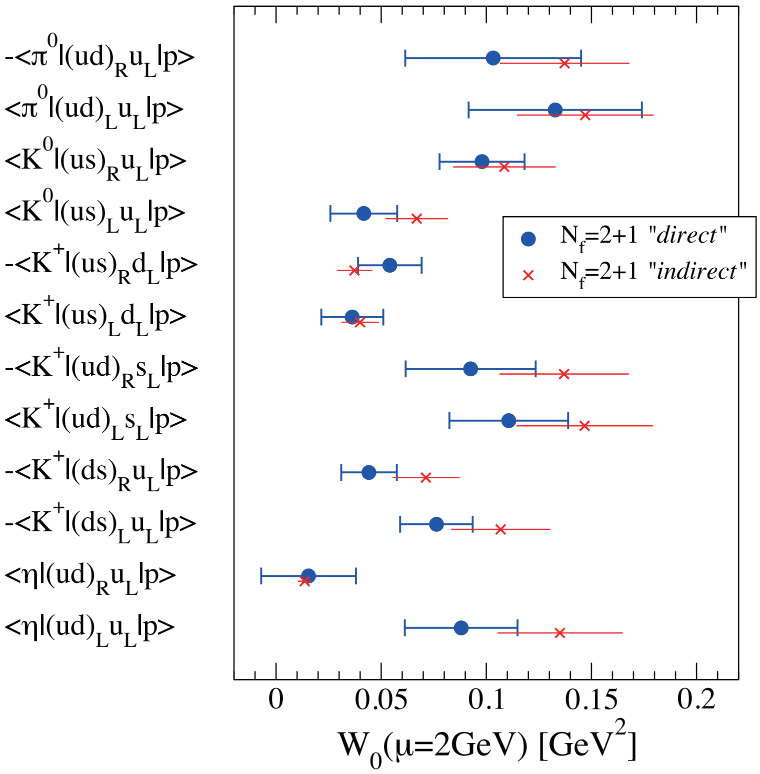


Figure D.1: Summary of hadron matrix elements evaluated in direct (blue dot) and indirect (red cross) methods. This figure is taken from Ref. [206].

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