学位論文（要約）

A Study of Electron Velocity Distribution Function in TST-2 Spherical Tokamak Plasma by Double-Pass Thomson Scattering Diagnostics

（ダブルパストムソン散乱計測を用いたTST-2球状トカマクプラズマの電子速度分布関数に関する研究）

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Abstract

The electron distribution function of plasma tends to be an isotropic Maxwellian distribution function in an equilibrium. In high temperature fusion plasmas, however, the distribution functions can be distorted from various reasons, such as electric field, magnetic mirror effect, wave heating and neutral beam injection. Such anisotropy of velocity distribution function results in deformation of plasma pressure from flux function and destabilization of the ballooning modes. Therefore, it is important to measure the distribution function and to determine whether the distribution function is deformed or not. However, there was no diagnostics with which the anisotropy in the core of plasma can be measured.

Thomson scattering diagnostic is a method to measure electron temperature and density from Thomson scattering light of a laser beam injected to a plasma. The wavelength spectrum of the scattering light reflects Doppler shift of the electrons along the scattering vector. The width of the spectrum represents the electron temperature and the intensity of the scattering light is proportional to the electron density. While a laser beam passes through a plasma once in a standard Thomson scattering diagnostic, in a double-pass Thomson scattering diagnostic, the laser beam is reflected back to the plasma and forward- and backward-scattering lights can be measured by a single collection optics. Therefore, measurements with two different scattering vectors are possible, and we can obtain two dimensional velocity distribution function of thermal electrons.

A double-pass Thomson scattering diagnostic system was developed and installed on the TST-2 device. The system consists of a laser injection optics, a collection optics and a detection unit. The scheme of absolute and relative sensitivity calibrations were established successfully. The calibrations suppress the systematic error below 10%. In addition, the random error was evaluated in detail. Resultant measurement error was predicted with a Monte Carlo simulation by using the results of error analysis.

Deformation of velocity distribution function was simulated with several theoretical calculations under an Ohmic electric field. The calculations predicted that the velocity distribution function is determined by the collisions between thermal electrons and ions, and there should be a temperature anisotropy of around 10% without any shift.

Measurement of electron velocity distribution function of Ohmic heated plasmas were performed with the double-pass Thomson scattering system at the plasma center and the plasma

edge. The scattering spectrum was analyzed assuming several models. We found finite difference between the perpendicular and the parallel temperatures. Comparing the temperatures along the direction perpendicular and parallel to the magnetic field, there was temperature anisotropy of around 20% at the plasma center, while around 50% at the plasma edge. The estimated value of current density from the measurement is close to the spatially-averaged current density at both of the plasma center and edge, which suggests that the contribution of thermal electrons to plasma current is relatively large. These results are consistent with the theoretical calculations.

This is the first measurement of temperature anisotropy for thermal electrons in magnetically confined high temperature plasmas. Anisotropy of velocity distribution function appeared even in plasmas with a relatively high collision frequency (100 times higher than the fusion plasmas). The result of our investigation would contribute to the understanding of anisotropy of plasma velocity distribution function, and demonstrated the possibility of current density measurement with Thomson scattering.

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1 Introduction

1.1 Fundamental of Plasma Physics

1.1.1 Plasma

The state of matter consisting a group of atoms and molecules depends on its temperature. In general, the state changes from solid to liquid, and from liquid to gas with the increase of the temperature. When the temperature becomes higher, ions and electrons in atoms become free from the bond between them. Then, the state of matter changes from gas to this fourth state in which ions and electrons are free to move around. The matter in the fourth state is called as ”plasma”.

Plasma has an important ability to shield DC electric field, because ions and electrons move independently. When a DC voltage is applied to a plasma, ions and electrons move to cancel out the electric field. Therefore, DC electric field penetrates only a distance called as Debye length λ_{De}, which is written as

$$\lambda_{\mathrm{De}} = \sqrt{\frac{\epsilon_0 T_{\mathrm{e}}}{e^2 n_0}} = 2.35 \times 10^{-5} \sqrt{\frac{T_{\mathrm{k}}}{n_{20}}} \ \ [\mathrm{m}], \tag{1.1}$$

where the units for T_{k} and n_{20} are keV and 10^{20} m^3, respectively. From this characteristic, the local positive charge density due to ions are almost canceled by the local negative charge density due to electrons. This feature is referred to as quasi-neutrality of plasma, and it is expressed as

$$Z_{\mathrm{i}} e n_{\mathrm{i}} \simeq e n_{\mathrm{e}}. \tag{1.2}$$

1.1.2 Coulomb Collision

Charged particles in plasma collide each other many times. The collision is dominated by small-angle Coulomb scattering. Through this process, electrons tend to relax to a thermal equilibrium state described by a Maxwell distribution function. The collision frequency of electron with ion ν_{ei} is given as:

$$\nu_{\mathrm{ei}}(v_{\mathrm{e}}) \simeq \left(\frac{1}{4\pi} \frac{e^4 n_i}{\epsilon_0^2 m_{\mathrm{e}}^2} \ln \Lambda \right) \frac{1}{v_{\mathrm{e}}^3}. \tag{1.3}$$

A typical value of ν_{ei} is obtained by setting v_{e} to thermal velocity $\hat{v}_{\mathrm{e}} \equiv \sqrt{3T_{\mathrm{e}}/m_{\mathrm{e}}}$ as

$$\hat{\nu}_{\mathrm{ei}} = \nu_{\mathrm{ei}}|_{\hat{v}_{\mathrm{e}}} \simeq \frac{1}{\sqrt{3}} \frac{\omega_{\mathrm{pe}}}{\Lambda} \ln \Lambda = 1.33 \times 10^5 \frac{n_{20}}{T_k^{3/2}} \ [s^{-1}]. \tag{1.4}$$

Other collision frequencies of electron with electron, ion with ion and ion with electron are written as

$$\nu_{ee} \simeq \left(\frac{1}{2\pi}\frac{e^4 n_e}{\epsilon_0^2 m_e^2}\ln\Lambda\right)\frac{1}{v_e^3 + 1.3v_{T_e}^3} \simeq 1.2\hat{\nu}_{ei} \tag{1.5}$$

$$\nu_{ii} \simeq \left(\frac{1}{2\pi}\frac{e^4 n_i}{\epsilon_0^2 m_i^2}\ln\Lambda\right)\frac{1}{v_i^3 + 1.3v_{T_i}^3} \simeq 1.2\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2}\hat{\nu}_{ei} \tag{1.6}$$

$$\nu_{ie} \simeq \left(\frac{1}{2\pi}\frac{e^4 n_e}{\epsilon_0^2 m_e m_i}\ln\Lambda\right)\frac{1}{v_i^3 + 1.3v_{T_e}^3} \simeq 1.4\left(\frac{m_e}{m_i}\right)\hat{\nu}_{ei} \tag{1.7}$$

By using these values, we can calculate the amount of momentum and energy transfer between them. The collision frequency for momentum transfer is written as

$$\bar{\nu}_{ei} = \frac{\sqrt{2}}{12\pi^{3/2}}\frac{e^4 n_i}{\epsilon_0^2 m_e^{1/2} T^{3/2}}\ln\Lambda = \sqrt{\frac{2}{\pi}}\frac{\omega_{pe}}{\Lambda}\ln\Lambda = 1.8\times 10^5\frac{n_{20}}{T_k^{3/2}}\ [\mathrm{s}^{-1}]. \tag{1.8}$$

The collision frequency for energy transfer is written as

$$\bar{\nu}_{eq} = 2\frac{m_e}{m_i}\bar{\nu}_{ei} = 96\frac{n_{20}}{T_k^{3/2}}\ [s^{-1}]. \tag{1.9}$$

The rightmost formula represents the case of deuterium plasma.

1.1.3 Plasma Resistivity

The plasma current can be driven by an electric field, because plasma consists of charged particles. In analogy with metals, plasma resistivity arises from the collisions between electrons and ions. In equilibrium, the electric field $\boldsymbol{E}$ is proportional to the plasma current density $\boldsymbol{j}$. This is the Ohm's law in plasma, which is written as

$$\boldsymbol{E} = \eta\boldsymbol{j}, \tag{1.10}$$

where η is the plasma resistivity. By considering the equation of motion for electrons in a uniform plasma along a magnetic field $\boldsymbol{B}$,

$$mn_e\frac{\mathrm{d}\boldsymbol{u}_e}{\mathrm{d}t} = -en_\mathrm{e}\boldsymbol{E} + \boldsymbol{R}_{\mathrm{ei.}} \tag{1.11}$$

R_{ei} is the momentum gain or loss of the electrons caused by collisions with ions.

$$\boldsymbol{R}_{\mathrm{ei}} = -mn_{\mathrm{e}}\langle\nu_{\mathrm{ei}}\rangle(\boldsymbol{u}_{\mathrm{e}} - \boldsymbol{u}_{\mathrm{i}}). \tag{1.12}$$

Combining Eqs. (1.10) and (1.11), the plasma resistivity η can be written as

$$\eta = \frac{m\langle\nu_{\mathrm{ei}}\rangle}{n_{\mathrm{e}}e^2} = \frac{\sqrt{2}}{12\pi^{3/2}}\frac{e^2 m_e^{1/2}}{\epsilon_0^2 T_e^{3/2}}\ln\Lambda = 6.5\times10^{-8}\left(\frac{1}{T_k^{3/2}}\right)\ [\Omega\mathrm{m}]. \tag{1.13}$$

where we used Eq. (1.8).

More accurate calculations involving kinetic plasma models show that the actual resistive Ohm's law should be written as $\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} = \eta_{\parallel}\boldsymbol{J}_{\parallel} + \eta_{\perp}\boldsymbol{J}_{\perp}$. From kinetic theory, the more accurate value of the parallel resistivity is reduced by a factor of about 1/2, as $\eta_{\parallel} = 0.51\eta$.

1.1.4 Current Diffusion

As shown in Eq. (1.13), the resistivity decreases with temperature. Therefore, high temperature plasmas generated in fusion devices have very low resistivity and the current decaying time due to a finite resistivity is very long time. The current decaying process is a diffusion process as shown in the following. From Maxwell equations,

$$\nabla\times\boldsymbol{B} = \mu_0\boldsymbol{j}, \tag{1.14}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times\boldsymbol{E}. \tag{1.15}$$

From Eqs. (1.10, 1.14, 1.15), the diffusion equation of magnetic field can be written as

$$\frac{\partial\boldsymbol{B}}{\partial t} = \frac{\eta}{\mu_0}\nabla^2\boldsymbol{B}. \tag{1.16}$$

In analogy with diffusion equation of particle, the diffusion time of current τ_η can be written as

$$\tau_\eta = \frac{a^2\mu_0}{\eta}, \tag{1.17}$$

where a is the size of the plasma.

1.1.5 Magnetic Mirror

Total energy of a particle, that is, summation of kinetic energy and magnetic energy is conserved in a particle motion. As described in the following, a particle can be reflected back and trapped in a magnetic well like Fig. 1.1. Here, we introduce a conserved quantity, magnetic moment $\mu_{\mathrm{m}} = mv_{\perp}^2/2B$. From equation of motion,

$$m\boldsymbol{v}_{\perp} \cdot \frac{d\mathbf{v}_{\perp}}{dt} = \frac{d}{dt}\left(\frac{mv_{\perp}^2}{2}\right) = q(\mathbf{v}_{\perp} \cdot \mathbf{E}_{\perp}), \tag{1.18}$$

where m and q are the mass and the charge of the particle, and v_{perp} is the velocity perpendicular to the magnetic field. Defining $W_{\perp} = mv_{\perp}^2/2$, variation of W in a cycle of Larmor motion $\Delta W_{\perp}$ is

$$\Delta W_{\perp} = q\int(\boldsymbol{v}_{\perp} \cdot \boldsymbol{E}_{\perp})dt = q\oint \boldsymbol{E}_{\perp} \cdot d\boldsymbol{s} = q\int(\nabla \times \boldsymbol{E} \cdot \boldsymbol{n})dS = -q\int \frac{\partial \boldsymbol{B}}{\partial t} \cdot \boldsymbol{n}dS = |q|\pi\rho^2\frac{\partial B}{\partial t}, \tag{1.19}$$

where $\oint d\mathbf{s}$ represents integral over a period of Larmor motion along the Larmor orbit. Because variation of magnetic field after a period of Larmor motion is $\Delta B = (\partial B/\partial t)(2\pi/|\omega_{\mathrm{c}}|)$,

$$\Delta W_{\perp} = \frac{mv_{\perp}^2}{2}\frac{\Delta B}{B} = W_{\perp}\frac{\Delta B}{B}, \quad \Delta\mu_{\mathrm{m}} = \Delta\left(\frac{W_{\perp}}{B}\right) = \frac{\Delta B}{B}\left(\frac{\Delta W_{\perp}}{\Delta B} - \frac{W_{\perp}}{B}\right). \tag{1.20}$$

Therefore $\mu_{\mathrm{m}} = W_{\perp}/B = \mathrm{const.}$

When electric field is weak, kinetic energy $W = mv^2/2$ is conserved along a particle motion. Then

$$\frac{mv_{\parallel}^2}{2} = W - \mu_{\mathrm{m}}B. \tag{1.21}$$

When the magnetic field strength varies along the field line, there could be a position where $W = \mu_{\mathrm{m}}B_{\mathrm{max}}$ and $v_{\parallel} = 0$. In the case, the particle cannot enter the higher field region, and the particle is reflected. The ratio

$$R_{\mathrm{M}} = B_{\mathrm{max}}/B_{\mathrm{min}} \tag{1.22}$$

is called as mirror ratio. The condition of the reflection (i.e., trap in weak field region) is

$$\left(\frac{v_{\perp 0}}{v_0}\right)^2 > \frac{B_{\mathrm{min}}}{B_{\mathrm{max}}} = \frac{1}{R_{\mathrm{M}}}, \tag{1.23}$$

where $v_{\perp 0}$ and v_0 are velocity perpendicular to magnetic field and total velocity at the point where

$B = B_{\min}$.

Figure 1.1: A schematic of magnetic well and a trapped particle. B and s denote magnitude of magnetic field and trajectory coordinate of a particle along a field line, respectively.

1.1.6 Trapped and Passing Electron

The magnetic field strength is almost inversely proportional to major radius in a tokamak, since toroidal coil is located at the center of the torus, and the modification of the field strength due to plasma is small. Electrons in a tokamak move to toroidal and poloidal direction along magnetic field, and the electrons feel week magnetic field at the outboard side and strong magnetic field at the inboard side of the torus. Thus, the electrons can be trapped by the gradient of magnetic field. Such electrons are called as ”trapped electrons”, and electrons not trapped are called as ”passing electrons”. Note that ions can be trapped or untrapped in the same way. The fraction of trapped particles is calculated with some assumptions. The minimum (at the outboard side) and maximum (at the inboard side) magnetic field strengths in a tokamak are given by

$$B_{\min} = B_0 \frac{R_0}{R_0 + r}, \tag{1.24}$$

$$B_{\max} = B_0 \frac{R_0}{R_0 - r}, \tag{1.25}$$

where R_0 is the major radius of the torus, and r is the minor radius of the particle trajectory, and B_0 is the magnetic field strength at $R = R_0$ (Fig. 1.2). We consider a particle starting from the outboard midplane ($\theta = 0$) with velocity $\boldsymbol{v} = v_{\parallel}\boldsymbol{b} + \boldsymbol{v}_{\perp}$. Here θ is the poloidal angle. From Eq. (1.23), the

condition for particles to be trapped is written as

$$\left(\frac{v_\parallel}{v}\right)^2 < 1 - \frac{B_{\rm min}}{B_{\rm max}} \simeq 2\frac{r}{R_0}. \tag{1.26}$$

The critical angle $\theta_{\rm c}$ where the particle is reflected back is written as $\cos\theta_{\rm c} = v_\parallel/v \simeq (2r/R_0)^{1/2}$. The fraction f of trapped particles for a Maxwellian distribution function $F_{\rm M}(v)$ is calculated as

$$f = \frac{1}{n}\int_{\theta_{\rm c}}^{\pi-\theta_{\rm c}} \sin\theta {\rm d}\theta \int_0^{2\pi} {\rm d}\phi \int_0^\infty F_{\rm M}(v) v^2 {\rm d}v = \cos\theta_{\rm c} \simeq \sqrt{\frac{2r}{R_0}}. \tag{1.27}$$

The time for a particle to make a half-transit is given by $\tau_{1/2} = l/v_\parallel$, where l is the total parallel distance traveled by the particle as it covers a poloidal distance $l_{\rm p} = \pi r$. For a large aspect ratio tokamak

$$\frac{{\rm d}l}{B} \simeq \frac{{\rm d}l}{B_0} \simeq \frac{{\rm d}l_{\rm p}}{B_\theta}. \tag{1.28}$$

Therefore,

$$l(r) \simeq \frac{B_0}{B_\theta(r)} l_{\rm p} = \pi R_0 q(r). \tag{1.29}$$

Then, the full cycle bounce frequency of trapped particle $\nu_{\rm B}$ is

$$\nu_{\rm B} \simeq \frac{\overline{v}_\parallel}{l} \simeq \frac{v_\parallel}{4\pi R_0 q}, \tag{1.30}$$

where $\overline{v}_\parallel \simeq v_\parallel/2$ is the average parallel velocity of the particle during its bounce motion and $v_\parallel$ is the outboard parallel velocity at $\theta = 0$.

1.1.7 Fast Electron

An electron in a plasma is accelerated in an electric field. On the other hand, an electron slows down by collision with other electrons and ions. However, an accelerated electron faster than a certain velocity hardly slows down by collision, because collision frequency decreases with its velocity. On the tokamak operation, electrons are accelerated by electric field $-E_\parallel$ induced by time varying Ohmic coil current. On the other hand, electrons slow down by coulomb collision with other electrons and ions. Therefore, the equation of motion of a test electron is written as

$$\frac{{\rm d}v_{\rm e}}{{\rm d}t} = -\frac{e}{m_{\rm e}} E_\parallel - (\nu_{\rm ei} + \nu_{\rm ee}) v_{\rm e}. \tag{1.31}$$

Figure 1.2: Definitions of coordinates. R, r, ϕ and θ denote coordinate of major radius, minor radius, toroidal angle and poloidal angle, respectively. The dependence of strength of magnetic field on R is also described.

Therefore, electron velocity v_{e} has a threshold at which $\dot{v}_{\mathrm{e}}$ is always positive. Electrons, which are faster than this threshold, are accelerated indefinitely until relativistic effects become important or they are lost from the plasma. Such electrons are called as ”fast electrons” or ”runaway electrons”. In some cases, fast electrons can contribute to the plasma current significantly. From Eq. (1.31), the critical velocity for runaway electron can be written as

$$\frac{v_{\mathrm{e}}^2}{v_{T_{\mathrm{e}}}^2} \geq \frac{v_{\mathrm{c}}^2}{v_{T_{\mathrm{e}}}^2} = \frac{3}{8\pi}\frac{ne^3 \ln\Gamma}{\left|E_{\parallel}\right| \epsilon_0^2 T_{\mathrm{e}}} = 78\frac{n_{20}}{|E_{\parallel}|T_{\mathrm{k}}}. \tag{1.32}$$

v_{c} is the velocity at which electron transits from normal to runaway phase. We can estimate the fraction of runaway electrons from this expression when we assume a Maxwell distribution function. The number of runaway electrons n_{R} can be written as

$$\frac{n_{\mathrm{R}}}{n} = \frac{1}{\sqrt{\pi} v_{T_{\mathrm{e}}}} \int_{v_{\mathrm{c}}}^{\infty} e^{v_{\parallel}^2/v_{T_{\mathrm{e}}}^2} \mathrm{d}v_{\parallel} = \frac{1}{2}\left(1 - \Phi(\xi)\right) \simeq \frac{1}{2\sqrt{\pi}}\frac{e^{-\xi^2}}{\xi} (\xi >> 1), \tag{1.33}$$

where Φ is the error function, and

$$\xi = \frac{v_{\mathrm{c}}}{v_{T_{\mathrm{e}}}} = 8.8\sqrt{\frac{n_{20}}{|E_{\parallel}|\,T_{\mathrm{k}}}}. \tag{1.34}$$

Then, the fraction of runaway electrons can be calculated as

$$f_{\mathrm{n}} = \frac{n_{\mathrm{R}}}{n_{\mathrm{R}} + n}. \tag{1.35}$$

As a simple approximation, we can assume that all the runaway electrons accelerate to near relativistic speeds so that their energy density is given by $n_R m_e c^2/2$. Then, the relative energy density of runaway electron can be written as

$$\frac{U_{\mathrm{R}}}{3nT_{\mathrm{e}}/2} \simeq \frac{n_{\mathrm{R}} m_{\mathrm{e}} c^2/2}{3nT_{\mathrm{e}}/2} = \left(\frac{m_{\mathrm{e}}c^2}{3T_{\mathrm{e}}}\right)\left(\frac{n_{\mathrm{R}}}{n}\right). \tag{1.36}$$

In the same way, the relative current density of runaway electron can be written as

$$\frac{|J_{\mathrm{R}}|}{I/\pi a^2} = \left(\frac{\pi a^2 enc}{I}\right)\left(\frac{n_{\mathrm{R}}}{n}\right). \tag{1.37}$$

The energy and current fraction of runaway f_{U} and f_{J} are

$$f_{\mathrm{U}} = \frac{U_{\mathrm{R}}}{U_{\mathrm{R}} + 3nT_{\mathrm{e}}/2}, \tag{1.38}$$

$$f_{\mathrm{J}} = \frac{|J_{\mathrm{R}}|}{|J_{\mathrm{R}}| + I/\pi a^2}. \tag{1.39}$$

1.1.8 Penetration of Neutral Particles into Plasma

As shown in Sec. 1.4.3, the previous study with calculating kinetic equation shows collision of electron with neutral particles can play an important role of determining velocity distribution function. The main component of neutral particles in a plasma arises from the fuel gas, which is injected to the vacuum vessel. Deuterium is used in the TST-2 device. Neutral particle profile and collision frequency between electrons and neutrals can be estimated roughly by assuming a simplified one-dimentional model.

When we consider the diffusion of neutral as random walk of charge exchange, diffusion coeffi-

cient of neutral D_n is written as

$$D_\mathrm{n} \simeq \nu_\mathrm{cx}\lambda_\mathrm{cx}^2 \simeq \frac{v_\mathrm{t,i}^2}{n_\mathrm{i}\langle\sigma_\mathrm{cx}v_\mathrm{i}\rangle}. \tag{1.40}$$

From 1D diffusion equations,

$$\frac{\partial n_\mathrm{i}}{\partial t} = D\frac{\partial^2 n_\mathrm{i}}{\partial x^2} + n_\mathrm{i}n_\mathrm{n}\langle\sigma_\mathrm{ion}v_\mathrm{e}\rangle, \tag{1.41}$$

$$\frac{\partial n_\mathrm{n}}{\partial t} = \frac{\partial}{\partial x}\left(D_\mathrm{n}\frac{\partial n_\mathrm{n}}{\partial x}\right) - n_\mathrm{i}n_\mathrm{n}\langle\sigma_\mathrm{ion}v_\mathrm{e}\rangle. \tag{1.42}$$

Boundary conditions are

$$n_\mathrm{i} \to n_\mathrm{i\infty} \qquad n_\mathrm{n} \to 0 \qquad D_\mathrm{n} \to D_\mathrm{n\infty}, \tag{1.43}$$

where ∞ denotes plasma center. Then, the ion and neutral density profiles are

$$n_\mathrm{i}(x) = n_\mathrm{i\infty}\tanh(x/x_0) \tag{1.44}$$

$$n_\mathrm{n}(x) = n_\mathrm{n0}\,\mathrm{sech}^2(x/x_0) \tag{1.45}$$

$$x_0 = \sqrt{\frac{4D_\mathrm{n\infty}}{\langle\sigma_\mathrm{ion}v_\mathrm{e}\rangle n_\mathrm{i\infty}}} \tag{1.46}$$

$$= \frac{2v_\mathrm{t,i}}{n_\mathrm{i\infty}\sqrt{\langle\sigma_\mathrm{ion}v_\mathrm{e}\rangle\langle\sigma_\mathrm{cx}v_\mathrm{i}\rangle}} \tag{1.47}$$

$$n_\mathrm{n0} = \frac{Dn_\mathrm{i\infty}}{2D_\mathrm{n\infty}}. \tag{1.48}$$

Here,

$$\mathrm{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}. \tag{1.49}$$

$$N_\mathrm{all} = \frac{p_\mathrm{fill}}{k_\mathrm{B}T_\mathrm{n}} \times \pi a^2 \times 2\pi R = N_\mathrm{n,all} + N_\mathrm{i,all} + N_\mathrm{e,all}. \tag{1.50}$$

From the condition of quasi-neutrality, $N_\mathrm{i,all} = N_\mathrm{e,all}$.

$$N_\mathrm{n,all} = 2\pi n_\mathrm{n0}\int_0^a x\,\mathrm{sech}(\mathrm{x}/\mathrm{x}_0)^2\mathrm{d}x, \tag{1.51}$$

$$N_\mathrm{i,all} = 2\pi n_\mathrm{i\infty}\int_0^a x\tanh(\mathrm{x}/\mathrm{x}_0)\mathrm{d}x, \tag{1.52}$$

$$n_\mathrm{n0} = \frac{N_\mathrm{n,all}}{2\pi\int_0^a x\,\mathrm{sech}(\mathrm{x}/\mathrm{x}_0)^2\mathrm{d}x}. \tag{1.53}$$

We assume that the temperature profile is similar to the density profile

$$T_{\mathrm{e}}(x) = T_{\mathrm{e}\infty} \tanh(x/x_0). \tag{1.54}$$

The collision frequency of an electron with neutrals is written as

$$\nu_{\mathrm{en}} = n_{\mathrm{n}} \langle \sigma_{\mathrm{ion}} v_{\mathrm{e}} \rangle, \tag{1.55}$$

by using impact ionization cross section σ_{ion}. We use a room temperature as the velocity of neutral v_{n}. A typical value of σ_{ion} is

$$\langle \sigma_{\mathrm{ion}} v_{\mathrm{e}} \rangle \simeq 10^{-14}\ \mathrm{m}^3/\mathrm{s}. \tag{1.56}$$

These calculations are used in Sec. 4.2.

1.1.9 Boltzmann Equation and Fokker-Planck Equation

A standard method to calculate evolution of velocity distribution function is to solve Fokker-Planck equation. Fokker-Planck equation is a consequence of kinetic theory. Kinetic theory describes the behavior of plasmas in terms of the particle motions. Generally, the distribution is a function of the position $\boldsymbol{q}$ and the canonical momentum $\boldsymbol{p}$, that is $f = f(\boldsymbol{q}, \boldsymbol{p})$. The rate of change of f along a trajectory in phase space $\mathrm{d}f/\mathrm{d}t$ is given by the divergence of the phase space flow

$$\frac{\mathrm{d}f}{\mathrm{d}t} = -f\left(\frac{\partial}{\partial \boldsymbol{q}} \cdot \dot{\boldsymbol{q}} + \frac{\partial}{\partial \boldsymbol{p}} \cdot \dot{\boldsymbol{p}}\right), \tag{1.57}$$

where

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \dot{\boldsymbol{q}} \cdot \frac{\partial}{\partial \boldsymbol{q}} + \dot{\boldsymbol{p}} \cdot \frac{\partial}{\partial \boldsymbol{p}}. \tag{1.58}$$

By substituting Hamilton's equations

$$\dot{\boldsymbol{q}} = \frac{\partial H}{\partial \boldsymbol{p}}, \qquad \dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{q}}. \tag{1.59}$$

to Eq. (1.57), the resulting equation is

$$\frac{\partial f}{\partial t} + \dot{\boldsymbol{q}} \cdot \frac{\partial f}{\partial \boldsymbol{q}} + \dot{\boldsymbol{p}} \cdot \frac{\partial f}{\partial \boldsymbol{q}} = 0 \tag{1.60}$$

Finally, the kinetic equation for electrons is written as

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} + \frac{e\boldsymbol{E}}{m} \cdot \frac{\partial f}{\partial \boldsymbol{v}} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{c}}. \tag{1.61}$$

This kind of kinetic equation is called Boltzmann equation. The right-hand side represents collision effect. The Boltzmann equation in which the Fokker-Planck term is used as the collision term is called as Fokker-Planck equation. The rate of change of the distribution function due to collisions during a short time Δt is given by

$$\left(\frac{\partial f}{\partial t}\right)_{\mathrm{c}} = \frac{f(\boldsymbol{x}, \boldsymbol{v}, t + \Delta t) - f(\boldsymbol{x}, \boldsymbol{v}, t)}{\Delta t}. \tag{1.62}$$

Since this change in f is a result of collision, which causes a change in velocity from $v - \Delta v$ to v, various collision (i.e., various Δv) should be considered. Thus, f after the short time can be written as

$$f(\boldsymbol{x}, \boldsymbol{v}, t + \Delta t) = \int f(\boldsymbol{x}, \boldsymbol{v} - \Delta \boldsymbol{v}, t)\, \psi(\boldsymbol{v} - \Delta \boldsymbol{v}, \Delta \boldsymbol{v})\, \mathrm{d}(\Delta \boldsymbol{v}), \tag{1.63}$$

where $\psi(\boldsymbol{v}, \Delta \boldsymbol{v})$ is the transition probability that a particle with velocity $\boldsymbol{v}$ will be scattered by $\Delta \boldsymbol{v}$ in Δt. Taylor expansion of the integrand is

$$f(\boldsymbol{x}, \boldsymbol{v} - \Delta \boldsymbol{v}, t)\, \psi(\boldsymbol{v} - \Delta \boldsymbol{v}, \Delta \boldsymbol{v}) = f(\boldsymbol{x}, \boldsymbol{v}, t)\, \psi(\boldsymbol{v}, \Delta \boldsymbol{v}) - \sum_{\alpha} \frac{\partial}{\partial v_\alpha}(f\psi)\Delta v_\alpha + \frac{1}{2}\sum_{\alpha,\beta} \frac{\partial^2}{\partial v_\alpha \partial v_\beta}(f\psi)\Delta v_\alpha \Delta v_\beta. \tag{1.64}$$

Here we retain up to second order terms. The Fokker-Planck coefficients defined by

$$\langle \Delta v_\alpha \rangle = \int \psi \Delta v_\alpha\, \mathrm{d}(\Delta \boldsymbol{v})/\Delta t \tag{1.65}$$

$$\langle \Delta v_\alpha \Delta v_\beta \rangle = \int \psi \Delta v_\alpha \Delta v_\beta\, \mathrm{d}(\Delta \boldsymbol{v})/\Delta t \tag{1.66}$$

represent the average time rate of change of Δv_α and $\Delta v_\alpha \Delta v_\beta$ due to collisions. Then, the Fokker-Planck collision term is written as

$$\left(\frac{\partial f}{\partial t}\right)_{\mathrm{c}} = -\sum_{\alpha} \frac{\partial}{\partial v_\alpha}(\langle \Delta v_\alpha \rangle f) + \frac{1}{2}\sum_{\alpha,\beta} \frac{\partial^2}{\partial v_\alpha \partial v_\beta}(\langle \Delta v_\alpha \Delta v_\beta f). \tag{1.67}$$

The quantity $\langle \Delta v_\alpha \rangle$ is referred to as a coefficient of dynamic friction and $\langle \Delta v_\alpha \Delta v_\beta \rangle$ is referred to as a diffusion tensor. For example, taking a Maxwellian velocity distribution function for the background specie, i.e.,

$$f(v) = n\sqrt{m}2\pi kT \exp\left[-\frac{m}{2kT}v^2\right], \tag{1.68}$$

the Fokker-Planck term is written as

$$\left(\frac{\partial f}{\partial t}\right)_{\mathrm{c}} = -\nabla_{\mathrm{v}} \cdot [(\langle \delta v_\parallel \rangle f - \nabla_\parallel (D_\parallel f) - D_\perp \nabla_\perp f]. \tag{1.69}$$

$D_\parallel$ and $D_\perp$ are the diffusion coefficients defined as

$$D_\parallel = \frac{1}{2}\langle (\Delta v_\parallel)^2 \rangle, \tag{1.70}$$

$$D_\perp = \frac{1}{2}\langle (\Delta v_{\perp\alpha})^2 \rangle = \frac{1}{2}\langle (\Delta v_{\perp\beta})^2 \rangle. \tag{1.71}$$

$\nabla_{\mathrm{v}}\cdot$ indicates the divergence operator in velocity space. $\Delta v_\parallel$ is along v, and $\nabla_\parallel$ and $\nabla_\perp$ are the component of ∇ parallel and perpendicular to v.

1.2 Tokamak and Spherical Tokamak

1.2.1 Magnetic Confinement Fusion

Nuclear fusion is an attractive energy source because huge electric power can be generated in a clean and safety way. Particularly, D-T reaction (Fig. 1.3) is the most suitable reaction for the fusion power, because deuterium (D) can be found in seawater and tritium (T) is generated by another fusion reaction in fusion reactors. D-T reaction is expressed as [1]

$$\mathrm{D} + \mathrm{T} \rightarrow \alpha + \mathrm{n} + 17.6\ \mathrm{MeV}, \tag{1.72}$$

and tritium is produced by the reaction

$$_3\mathrm{Li}^7 + \mathrm{n(slow)} \rightarrow \alpha + \mathrm{T} + 4.8\mathrm{MeV}. \tag{1.73}$$

Cross section of the D-T reaction depends on temperature and has a maximum at a quite high temperature $\simeq$ 100 keV [1]. In such a high temperature environment, the state of components like deuterium, tritium and lithium is plasma.

The concept of ignition is important in nuclear fusion power plant. The alpha particles generated by the reaction of Eq. (1.72) is charged particles that are confined by the magnetic field and thus remain within the plasma, providing a source of heat. On the other hand, plasma energy is lost by Bremsstrahlung and thermal conduction. The assumption of ignition is that in steady state the alpha power heating is sufficiently large to balance the combined Bremsstrahlung and thermal conduction losses, without the need for any external power, i.e.,

$$S_\alpha = S_\mathrm{B} + S_\kappa, \tag{1.74}$$

S_α, S_B and S_κ are powers of alpha particle heating, Bremsstrahlung loss and thermal conduction loss, respectively. The critical values of $p\tau_\mathrm{E}$ for ignition is a function of temperature, where p and τ_E represent the plasma pressure and confinement time, respectively [1]. The minimizing value of T and the corresponding value of $p\tau_\mathrm{E}$ are given by

$$T_\mathrm{min} = 15\ \mathrm{keV}, \tag{1.75}$$

$$(p\tau_\mathrm{E})_\mathrm{min} = 8.3\ \mathrm{atm\ s}. \tag{1.76}$$

For $p\tau_E < (p\tau_E)_{min}$ ignition is not possible.

In order to generate electric power by confining such a high pressure plasma, it is necessary to support the force of pressure (gradient) by somewhat huge force. Solid material is not appropriate, because the solid material melts by the high temperature plasma, and plasma get cold. The most efficient way to support (and confine) a high pressure plasma is confinement by magnetic field. In particular, tokamak confinement scheme is the best way at present. In this method, current is driven in the plasma and the gradient of plasma pressure is supported by the current and magnetic field. The principle is explained in section 1.2.2 in detail. At an equilibrium, the force balance equation is written as

$$\nabla p = \boldsymbol{J} \times \boldsymbol{B}, \tag{1.77}$$

where ∇p is the gradient of plasma pressure, $\boldsymbol{J}$ is the plasma current and $\boldsymbol{B}$ is the external magnetic field.

Therefore, the balance between plasma current and external magnetic field is important for plasma confinement. After several decades of tokamak research, several empirical rules have been established, which are used to predict the performance and features of fusion reactor. In particular, density and confinement time are closely related to plasma current. By using data from a number of tokamaks, Greenwald has generalized an empirical result on the upper boundary of the density, which is written as

$$\frac{\bar{n}}{10^{20}} = \frac{I_{MA}}{\pi a^2}, \tag{1.78}$$

where $\bar{n}$ is averaged density in [10^{20} m^3], I is total plasma current in [MA] and a is plasma minor radius in [m] [2]. On the other hand, the empirical formula on confinement time τ_E is given by Goldston [2], written as

$$\tau_E \propto \frac{I^2}{nT} g, \tag{1.79}$$

where g is a geometrical factor. In summary, density n, temperature T and plasma current I are the key parameters for high performance fusion plasma, and these quantities should be measured and controlled in fusion devices..

In order to realize power generation by nuclear fusion, international cooperation project international thermonuclear experimental reactor (ITER), which is based on tokamak configuration, is now under construction [3].

Figure 1.3: D-T reaction.

1.2.2 Tokamak

For nuclear fusion, it is necessary to confine the plasma in a certain finite volume for long time and it is also necessary to keep an appropriate temperature and a density. For these purposes, it's possible to use the feature of charged particles. Charged particles rotate and travel along magnetic field, this gyro motion is called Larmor motion (Fig. 1.4). Therefore, charged particles can be confined when the magnetic field forms a torus shape [2]. Plasma is confined in a vacuum vessel of torus shape.

Firstly, we define the coordinate system of axisymmetric torus. Figure 1.7 shows the coordinate system consisting of three directions: radial direction, toroidal direction and poloidal direction. Particles of plasma are confined in toroidal direction when a toroidal magnetic field is generated by toroidal field coil (TF coil, Fig. 1.5), because the particles travel along the magnetic field. However, in this case, particles can escape along the radial direction due to $\boldsymbol{E} \times \boldsymbol{B}$ drift, because charge separation occurs due to $\nabla \boldsymbol{B}$ drift as shown in Figure 1.5. When current flows in the toroidal direction, poloidal field is generated and the magnetic field is twisted (Fig. 1.6). This twist connects upper plasma and lower plasma, and suppress the charge separation as a result. The plasma current is generated by Ohmic coil (Fig. 1.8). Increasing the coil current of Ohmic coil, electric field is induced by the Faraday's law along the toroidal direction. Pre-ionized gas is accelerated by the electric field and resultant current along the toroidal direction (i.e., plasma current) generates the poloidal magnetic field. This type of configuration is called tokamak.

Figure 1.4: Larmor motion of electron.

Figure 1.5: Toroidal field coil, charge separation and $\boldsymbol{E} \times \boldsymbol{B}$ drift.

Figure 1.6: Toroidal magnetic field, plasma current and twisted magnetic field.

Figure 1.7: Coordinate system of axisymmetric torus.

Figure 1.8: Schematic diagram of the Ohmic coil.

1.2.3 Spherical Tokamak

Electric power generated by fusion increases with the plasma pressure. Tokamak confines a higher pressure plasma when the magnetic field strength increases. However, the magnitude of magnetic field is restricted to a certain value from technical reasons. Therefore, the efficiency of high pressure confinement for a given field strength is important. The index of normalized pressure β is defined as

$$\beta \equiv \frac{2\mu_0 \langle p \rangle}{\langle B^2 \rangle}, \tag{1.80}$$

where $\langle\rangle$ denotes the volume averaged value. The device is more efficient when β is higher. The value of β is mainly restricted by MHD instabilities. Many numerical studies have been carried out to determine the β limit against ideal MHD instabilities. One of the most widely used results is usually referred as the Troyon limit, which is written as

$$\beta \leq 0.072 \left(\frac{1 + \kappa^2}{2} \right) \epsilon, \tag{1.81}$$

where κ is the elongation and $\epsilon \equiv a/R_0$ is the inverse aspect ratio. In the numerical studies the following major ideal MHD modes are considered; those are external kink modes and ballooning-kink modes, internal modes, localized modes, etc. In usual, the external ballooning-kink mode is the most dangerous mode. We see $\beta \propto \epsilon$ from Eq. (1.81). Therefore, β can be increased when the aspect ratio $A = R_0/a$ is small. This is the original motivation of the spherical tokamak (ST). ST is a kind of tokamak. The tokamak is called ST when the aspect ratio is small enough (A $<$ 1.5). The shape tends to become sphere due to hoop force [2].

In tokamak configuration, the curvature of magnetic field is directed to higher pressure region at the inboard side of the torus, where the directions are favorable from the view point of stability. Such curvature is called as ”good curvature”. In contrast, the curvature is directed to lower pressure region at the outboard side of the torus, where the directions are unfavorable. Such curvature is called as ”bad curvature”. In ST configuration, plasma particles spend longer time at the good curvature region when we compare the time for a standard tokamak. This is the reason why a high β can be sustained in ST.

In the ST configuration, the ratio of magnetic field strength at the inboard side to that at the outboard side is higher than in a standard tokamak. Therefore, the mirror ratio is higher and the

fraction of trapped particles is high in ST. This fact results in a significant spontaneous driven plasma current, called as ”Bootstrap current”. This is another advantage of ST fusion reactors.

1.2.4 Tokyo Spherical Tokamak-2 (TST-2)

The Tokyo Spherical Tokamak -2 (TST-2) device [4] was constructed at the University of Tokyo. The TST-2 is located at the Kashiwa Campus of the University of Tokyo. The photograph of the present TST-2 is shown in Fig. 1.9. The TST-2 device has the following typical parameters for Ohmic discharge: major radius $R \leq 0.38$ m, minor radius $a \leq 0.25$ m and aspect ratio $A = R/a \leq 1.5$. The definition of R and a are shown in Fig. 1.10. Because the aspect ratio is small, the TST-2 device is classified as a spherical tokamak. 24 toroidal field (TF) coils, a 239 turn Ohmic heating (OH) center solenoid (CS) and 5 pairs of poloidal field (PF) coils are installed in TST-2 (Fig. 1.11). The TF coils generate the toroidal magnetic field $B_t \leq 0.3$ T. The PF coils are used to control position and shape of the plasma. In order to pre-ionize neutral gas, electron cyclotron wave (ECW) from a magnetron (2.45 GHz / 5 kW) is used. There are two types of operation in the TST-2 device: the Ohmic heated discharge and non-inductive start-up discharge driven by radio frequency (RF) wave. In the Ohmic heated discharge, plasma is generated by joule heating using induction by the Ohmic coil. Plasma current grows up to 120 kA and the discharge duration is about 40 ms in a typical. In the RF start-up discharge, plasma is (non-inductively) generated and sustained by RF wave alone, such as ECW and lower hybrid (LH). Plasma current grows up to 15 kA and the typical discharge duration is about 120 ms.

Figure 1.9: Photograph of the TST-2.

As shown in Fig. 1.11, the 239 turn Ohmic coil is installed in the center stack (CS) of TST-2. The current flowing in this coil generates magnetic field along the center stack. The inductive current

flows along the circumference direction of the torus. The joule heating by this inductive current heats the plasma.

Various diagnostics are available or under development on TST-2 (including the TS). Electron density and temperature can be measured by the TS system. Line-integrated electron density is measured by a microwave interferometer. Plasma current, coil currents and magnetic fields are measured by various magnetics. Electron density, electron temperature and floating potential at the plasma edge can be measured by a Langmuir probe. Impurity ion temperature is measured by a visible spectrometer. A CCD camera captures visible images of the plasma. Line emissions from atomic hydrogen such as H_α and H_β are monitored by photodiodes. SBD array, PIN diode array and Absolute Extreme Ultraviolet (AXUV) detector are used to measure soft X-ray emission.

1.2.5 Plasma Heating and Current Drive

As described in Sec. 1.2.1, the cross section of the D-T reaction is a function of temperature. In order to operate fusion power plant with a high efficiency, it is required to heat the plasma to an optimum temperature. In addition to the temperature, plasma current is another key parameter. The performance of the plasma confinement becomes better for higher plasma current There are several methods for heating and current drive, such as injecting wave, high energy neutral beam and etc. In principle, such heating and current drive may induce temperature anisotropy. For example, the electron cyclotron resonance heating increases the temperature perpendicular to the magnetic field. The lower hybrid current drive increases momentum parallel to the magnetic field. Different heating or current drive method may induce difference between $T_{e,\perp}$ and $T_{e,\perp}$. While these method enhance the temperature anisotropy, collisions have the role of reducing the anisotropy.

Figure 1.10: Definition of major and minor radii of the plasma.

Figure 1.11: Schematic diagram of the coils on TST-2.

1.3 Plasma Pressure Anisotropy

1.3.1 Deformation of Plasma Pressure from Flux Function

In tokamak plasma, plasma pressure gradient $\nabla \cdot \mathbf{P}$ is balanced by the plasma current $\mathbf{j}$ and external magnetic field $\mathbf{B}$, i.e., $\nabla \cdot \mathbf{P} = \mathbf{j} \times \mathbf{B}$. When the pressure is isotropic, pressure is scalar quantity and is flux function. When the pressure is anisotropic, $\mathbf{P}$ is tensor quantity. Here, we define the tensor as

$$\mathbf{P} = \mathbf{I} p_\perp + \mathbf{bb} p_\Delta, \tag{1.82}$$

where $p_\Delta \equiv p_\parallel - p_\perp$ and $\mathbf{b}$ is an unit vector along the magnetic field. Then, the force balance expressed written as

$$\nabla \cdot \mathbf{P} = \nabla p_\perp + \mathbf{b}\left[\nabla_\parallel p_\Delta + p_\Delta \nabla \cdot \mathbf{b}\right] + p_\Delta \kappa = \mathbf{j} \times \mathbf{B}, \tag{1.83}$$

where $\nabla_\parallel \equiv \mathbf{b} \cdot \nabla$, $\mathbf{B} = B\mathbf{b}$ and $\kappa \equiv \mathbf{b} \cdot \nabla \mathbf{b}$. κ represents the curvature of the magnetic field. Here, we used the Ampere's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$. From a simple calculation, the equation reduces to

$$\frac{1}{\mu_0} \mathbf{K} \times \mathbf{B} = \nabla p_\parallel - \frac{p_\parallel - p_\perp}{B} \nabla B, \tag{1.84}$$

where $\mathbf{K}$ is defined as

$$\mathbf{K} \equiv \nabla \times \left[\left(1 - \mu_0 \frac{p_\parallel - p_\perp}{B^2}\right) \mathbf{B}\right]. \tag{1.85}$$

The parallel component of Eq. (1.84) is written as

$$\nabla_\parallel p_\parallel - \frac{p_\parallel - p_\perp}{B} \nabla_\parallel B = 0. \tag{1.86}$$

Therefore, the plasma pressure is not a flux function when $\nabla_\parallel B \neq 0$ [5].

1.3.2 Destabilization of Ballooning Modes

The stability of ideal-MHD ballooning modes are affected by pressure anisotropy [6, 7]. Here, we define the anisotropic pressure as

$$p_\perp = p_0(1 + \eta \cos\theta), \tag{1.87}$$

where θ is poloidal angle ($\theta = 0$) and η is a parameter corresponding to pressure anisotropy. The ballooning stability of anisotropic tokamak equilibria is determined by the anisotropic ballooning

equation:

$$\mathbf{B}\cdot\nabla\left[\frac{1}{R^2B_{\rm p}^2}(1+G^2)\mathbf{B}\cdot\nabla F\right]+\frac{F}{RB_{\rm p}}\left(K_\psi-GK_S\right)\left[\frac{\partial}{\partial\psi}\left(p_\parallel+p_\perp\right)-\frac{B_\phi}{RB_{\rm p}^2B}\frac{K_\perp}{K_S}\mathbf{B}\cdot\nabla(p_\parallel+p_\perp)\right]=0, \tag{1.88}$$

where $B_{\rm p}$ and B_ϕ denote the poloidal and toroidal components of magnetic field, respectively. The poloidal magnetic flux is ψ and principal and geodesic curvatures are:

$$K_\psi = \frac{\nabla\psi\cdot\nabla(p_\perp+B^2/2)}{B^2(1-\sigma)\,|\nabla\psi|}, \tag{1.89}$$

$$K_S = \frac{\mathbf{B}\times\nabla\psi\cdot\nabla(p_\perp+B^2/2)}{B^3(1-\sigma)\,|\nabla\psi|}, \tag{1.90}$$

with $\sigma=(p_\parallel-p_\perp)/B^2$. The quantity G is defined by

$$G=\frac{R^2B_{\rm p}^2}{B}\int_{\chi_0}^{\chi}\frac{\partial}{\partial\psi}\left(\frac{IJ}{R^2}\right)\mathrm{d}\chi, \tag{1.91}$$

where χ is the poloidal coordinate with $\nabla\chi$ orthogonal to $\nabla\phi$ and $\nabla\psi$ and J is the Jacobian. We use the approximation of large aspect ratio and circular cross-section. Then, Eq. (1.88) reduces to

$$\frac{\partial}{\partial\theta}\left[g(\theta)(1+G^2)\frac{\partial F}{\partial\theta}\right]+g^2F(\cos\theta+G\sin\theta)\left(\alpha+\frac{\alpha\eta}{2}\cos\theta+\frac{\eta\beta q^2}{2\epsilon}g\cos\theta\right)=0. \tag{1.92}$$

s and α are the magnetic shear and pressure gradient, defined as

$$s \equiv \frac{d(\ln q)}{d(\ln r)} \tag{1.93}$$

$$\alpha \equiv -\left(\frac{2Rq^2}{B^2}\right)\frac{\mathrm{d}p}{\mathrm{d}r}, \tag{1.94}$$

where q is safety factor. If the poloidal magnetic field is constant round a magnetic surface, then

$$g \rightarrow 1, \tag{1.95}$$

$$G \rightarrow s(\theta-\theta_0)-\alpha\left(1+\frac{\eta\beta}{4\epsilon}\right)(\sin\theta-\sin\theta_0)-\frac{\eta}{16}\left[\alpha\left(1+\frac{\eta\beta}{\epsilon}\right)-\frac{\beta q^2}{\epsilon}\right](\sin 2\theta-\sin 2\theta_0). \tag{1.96}$$

ϵ denotes aspect ratio. Stability to interchange modes is determined by the Mercier criterion [8]. The resultant criterion is written as

$$s^2 > \eta \left[\alpha + \frac{\beta q^2}{\epsilon} - \frac{1}{2}\frac{\alpha^2 \beta}{\epsilon} + \frac{\eta}{16}\left(\alpha + \frac{\beta q^2}{\epsilon}\right)\frac{\beta q^2}{\epsilon}\left(1 - \frac{\eta \alpha}{2q^2}\right)\right]. \tag{1.97}$$

The stability region becomes narrower in the anisotropic case compared to in the isotropic case.

1.4 Review of Previous Studies

In this section, we will overview the previous studies on plasma velocity distribution function including advanced methods. After describing them, objectives of this thesis are presented. Here, we focus only on noncollective Thomson scattering, while collective Thomson scattering can be used for the measurement of high energy ions.

The method of noncollective Thomson scattering diagnostics is an established and matured method for electron temperature and electron density measurement in the area of fusion plasma studies. Because the principle of measurement is simple, the method is regarded as the most reliable diagnostic. Recently, additional applications of Thomson scattering measurement are developed to investigate new physical properties of fusion plasma. Since the Thomson scattering spectrum reflects the electron distribution function directly, it contains various information on the plasma such as pressure anisotropy [9], current density [10, 11, 12, 13, 14] and fraction of high energy electrons [15, 16, 17]. Some advanced schemes to measure these quantities are shown in the next.

1.4.1 Temperature Anisotropy Measurement

Thomson scattering spectrum reflects the electron velocity distribution function along the direction of scattering vector. The measured direction is determined from the directions of laser and viewing sight. By using this feature of Thomson scattering measurement, bidirectional electron temperature can be measured. An experiment to obtain the temperatures $T_{\mathrm{e},\perp}$ and $T_{\mathrm{e},\parallel}$ with Thomson scattering measurement for an electron cyclotron resonance plasma was performed [9]. Here, $T_{\mathrm{e},\perp}$ denotes the temperature along magnetic field and $T_{\mathrm{e},\parallel}$ denotes that perpendicular to the field. Plasma is produced by a Radio Frequency (RF) wave power injection under an unidirectional constant magnetic field. $T_{\mathrm{e},\perp}$ and $T_{\mathrm{e},\parallel}$ were measured with Thomson scattering diagnostics. Since the plasma production is based on electron cyclotron resonance heating, $T_{\mathrm{e},\perp}$ is expected to be higher than $T_{\mathrm{e},\parallel}$. In fact, such relationship was observed at low density. On the other hand, the degree of temperature anisotropy decreases at high density due to high collisionality.

1.4.2 Current Density Measurement on the RTP Tokamak Device

Current density measurement with a Ruby laser Thomson scattering diagnostic was performed on the RTP tokamak device [11]. The laser was injected vertically, and the scattered light was measured from two horizontal directions. The measurements in the directions parallel and perpendicular to the toroidal magnetic field reflect the distribution functions parallel and perpendicular to the magnetic field, respectively. They estimated current density from these signals. Spectrum is detected by a CCD camera. The current density profile was estimated from the shift of spectrum. Since the error of wavelength shift measurement is about 100 % from one measurement alone, data was averaged for 36 discharges in order to increase the accuracy. They estimated the plasma current density by assuming that the electron distribution function is a simple shifted Maxwellian and the shift contributes to the plasma current. In this model, the wavelength shift of Thomson scattering spectrum is given by

$$\Delta\lambda = -\frac{2}{c}\lambda_0 v_0 \sin\left(\phi - \frac{\theta}{2}\right)\cos\gamma, \tag{1.98}$$

where c is the speed of light and λ_0 is the wavelength of the incident laser. Several geometrical factors: the scattering angle θ, the angle γ between the scattering plane and v_0 and angle ϕ between k_0 and the projection of v_0 onto the scattering plane, are included in the equation. Then, the electron current density j_e is written as

$$j_\mathrm{e} = e n_\mathrm{e} v_0. \tag{1.99}$$

They also estimated current density from the first moment of spectrum. Since the spectrum corresponds to the velocity distribution function, its first moment corresponds to the plasma current. The estimated value agrees with the values estimated by the spectrum shift.

Although this experiment was performed successfully, they used Ruby laser of which energy is quite intensive (25J) but repetition rate is very low. This laser is inappropriate for future devices because of the quite low repetition rate. In addition, there is no confidence that a shifted Maxwellian for the velocity distribution function is always valid. For example, under a certain condition, distribution function can be distorted without a shift [18]. Therefore, we must investigate the applicability domain of the shifted Maxwellian model.

1.4.3 Calculation of Fokker-Planck Equation Including a DC Electric Field

In conventional plasma start-up phase, the plasma current is driven by an electric field generated by electromagnetic induction from the Ohmic coil. The current arises from the distortion (including the shift) of the velocity distribution function. There are theoretical studies in which Fokker-Planck equation was solved to investigate the shape of the velocity distribution function of Ohmic heated plasma [18]. In Ref. [18], velocity distribution function is assumed to be Maxwellian and $T_\mathrm{e} = T_\mathrm{i} = T_\mathrm{n}$ before the electric field was applied. Fokker-Planck equation with a constant electric field is written as [18, 19]

$$\frac{\partial f_\mathrm{e}}{\partial t} = \frac{e\boldsymbol{E}}{m_\mathrm{e}}\frac{\partial f_\mathrm{e}}{\partial \boldsymbol{v}} + \left(\frac{\partial f_\mathrm{e}}{\partial t}\right)_\mathrm{FP} + \left(\frac{\partial f_\mathrm{e}}{\partial t}\right)_\mathrm{en}, \tag{1.100}$$

where

$$\frac{1}{\Gamma}\left(\frac{\partial f_\mathrm{e}}{\partial t}\right)_\mathrm{FP} = A_1\frac{\partial^2 f_\mathrm{e}}{\partial V^2} + A_2\frac{\partial f_\mathrm{e}}{\partial V} + A_3\frac{\partial f_\mathrm{e}}{\partial \theta} + A_4\frac{\partial^2 f_\mathrm{e}}{\partial \theta^2} + A_5 f_\mathrm{e}, \tag{1.101}$$

$$A_1 = \frac{1}{2}\frac{\partial^2 G}{\partial V^2}, \tag{1.102}$$

$$A_2 = \frac{1}{V^2}\frac{\partial G}{\partial V} + \frac{\partial C}{\partial V}, \tag{1.103}$$

$$A_3 = \frac{\cot\theta}{2V^3}\frac{\partial G}{\partial V}, \tag{1.104}$$

$$A_4 = \frac{1}{2V^3}\frac{\partial G}{\partial V}, \tag{1.105}$$

$$A_5 = 4\pi\left(f_\mathrm{e} + \sum_\mathrm{i} Z_\mathrm{i}^2 f_\mathrm{e}\right), \tag{1.106}$$

and

$$\frac{\partial G}{\partial V} = \sum_\mathrm{j} n_\mathrm{j} Z_\mathrm{j}^2\left[\frac{V_\mathrm{tj}}{V}\sqrt{2\pi}\exp\left(-\frac{V^2}{2V_\mathrm{tj}^2}\right) + \left(1 - \frac{V_\mathrm{tj}^2}{V^2}\right)\mathrm{erf}\left(\frac{V}{\sqrt{2}V_\mathrm{tj}}\right)\right]. \tag{1.107}$$

$$\frac{\partial C}{\partial V} = \sum_\mathrm{i} n_\mathrm{i} Z_\mathrm{i}^2\left[\frac{1}{V_\mathrm{ti}V}\sqrt{2\pi}\exp\left(-\frac{V^2}{2V_\mathrm{ti}^2}\right) - \left(\frac{1}{V^2}\right)\mathrm{erf}\left(\frac{V}{\sqrt{2}V_\mathrm{ti}}\right)\right]. \tag{1.108}$$

As explained in Sec. 1.1.9, the second and third terms in Eq. (1.100) represent collision effects, including dynamic friction and diffusion. The factor Γ is

$$\Gamma = 4\pi\left(\frac{e^2}{4\pi\epsilon_0 kT_\mathrm{e}}\right)^2 \ln\Lambda. \tag{1.109}$$

where

$$\Lambda = \frac{3}{2e^3}\frac{(4\pi\epsilon_0 kT_\mathrm{e})^{3/2}}{\sqrt{\pi n_\mathrm{e}}}. \tag{1.110}$$

In Ref. [18], they found two types of conditions depending on the collision frequency of an electron with neutral particles. The velocity distribution functions for the cases that collision frequency of an electron with neutral particles ν_en is larger and smaller were calculated. The collision frequency and the electric field are normalized as

$$\hat{E} = E/2E_\mathrm{D} \qquad \hat{\nu}_\mathrm{n} = \nu_\mathrm{en}/\nu_1 \qquad \nu_1 = n_\mathrm{e}\Gamma/V_\mathrm{te}^3, \tag{1.111}$$

where $E_\mathrm{D} = m_\mathrm{e}\nu_1 V_0/2e$ is the Dreicer electric field. In large $\hat{\nu}_\mathrm{n}$ case, the distribution is similar to the shifted-Maxwellian. On the other hand, the distribution is distorted from the Maxwellian without any shift in small $\hat{\nu}_\mathrm{n}$ case. The condition of Ohmic heated plasma in the TST-2 device is similar to the case of low $\hat{\nu}_\mathrm{n}(< 0.1)$ as shown in Sec. 4.2, and $\hat{E} \simeq 0.1$.

1.4.4 In-situ Calibration Method by Using Double-Pass Scheme

A notable advantage of the double-pass scheme is its high accuracy. As an example of application, in-situ calibration method by using double-pass scheme is described in this section. This method is developed in TST-2 as a collaboration study between the author and a collaborator.

In a fusion reactor or in large tokamaks such as ITER, enormous quantities of neutron are generated by fusion reactions. It is known that optical transmission decreases when the optical component exposed to neutron irradiation. In addition, plasma facing materials may deposit to optical windows, and the optical transmission is degraded wavelength dependence [20, 21]. In such case, diagnostics depending on wavelength spectrum like Thomson scattering can not be used. However, the problem is solved if the transmission dependence on wavelength is calibrated at the same time as measurement. Such in-situ calibration is possible with double-pass Thomson scattering when there is no plasma temperature anisotropy.

We assume that the measurement point is same in the first- and second-pass, and there is no plasma anisotropy, that is, the true value of electron density and temperature measured by first- and second-pass are completely the same in the double-pass measurement. In that case, the ratio of signal intensity between first- and second-pass Thomson scattering $R_\mathrm{th,j}$ on the channel j is written as

[22, 23]

$$R_{\mathrm{th,j}} = \frac{C_{\mathrm{j}} n_{\mathrm{e}}}{C_{\mathrm{j}} n_{\mathrm{e}}} \frac{L_{\mathrm{s},2} E_2}{L_{\mathrm{s},1} E_1} \frac{G_{\mathrm{j}}(\theta_2, T_{\mathrm{e}})}{G_{\mathrm{j}}(\theta_1, T_{\mathrm{e}})}. \tag{1.112}$$

Since the decrease of transmission C_{j} and electron density n_{e} are common value in first- and second-pass, they are canceled out. The remains are constants, the scattering length L_{s} and the laser energy E depending on the optics. In addition, the term G depending on scattering angle θ and electron temperature T_{e} also remains. Therefore, the electron temperature T_{e} is determined by the signals independently of the decrease of transmission C_{j}. Because the term G is determined by T_{e}, we can determine the $n_{\mathrm{e}}\eta_{\mathrm{c,j}}$ by minimizing the following chi square value:

$$\chi^2_{\mathrm{R}} = \sum_{\mathrm{j}} \sum_{k=1}^{2} \frac{(V_{m,j,k} - n_{\mathrm{e}}\eta_{c,j} E_k L_{s,k} G_{j,k}(T_{\mathrm{e}}, \theta_k \lambda_i))^2}{\sigma^2_{j,k}}. \tag{1.113}$$

In Ref. [23], the in-situ calibration method was examined to be valid by experiment of plasma measurement. The Ohmic heated plasma was measured with double-pass Thomson scattering diagnostic system in TST-2 device. The temperature measured by the in-situ calibration method was found to agree with the temperature measured by conventional Thomson scattering diagnostic method in broad range of temperature [23]. In addition, experiments with detection system of which wavelength dependence was changed by filters in order to simulate the decrease of transmission by neutron and attachment of hydrocarbon compound. The decrease of transmission by filters was successfully estimated by the in-situ calibration method with density and temperature measurement.

1.5 Thesis Objectives and Outline

The double-pass Thomson scattering diagnostic system on the TST-2 device is the first effective tool to measure two directional temperature simultaneously and directly. The author has totally developed the system and examined the quality of the system. Furthermore, the author has researched the important area, measurement of velocity distribution function in order to supply a method to measure temperature anisotropy and current density profile by using Thomson scattering diagnostic.

The information supplied in this thesis is useful for, in particular, the research and development of fusion plasma proceeded in the world. Measurement of velocity distribution function inside plasma has not been achieved in previous studies. From the result of this thesis, we can choose more appropriate model of distribution function, which is better expression than simple Maxwell-Boltzmann distribution. The modification of model can help to solve the problems such as modification of plasma confinement and suppression of plasma instability. After expanded the understanding of velocity distribution function, it becomes possible to measure plasma current by Thomson scattering diagnostic. Then, the parameters of fusion plasma which is needed to monitor, density, temperature and current can be measured by only Thomson scattering diagnostic. Moreover, by improving the understanding about generation of plasma anisotropy, Thomson scattering diagnostic can be used even in a severe environment such as neutron irradiation by the in-situ calibration method shown as in section 1.4.4. Therefore, it is significant to summarize the specification and performance of the double-pass Thomson scattering system, and the results of researches about velocity distribution function inside plasma.

In summary, the thesis objectives are:

1. To summarize detail of the double-pass Thomson scattering diagnostic system.
2. To examine performance of the system.
3. To propose several models to express Ohmic heated plasma appropriately.
4. To summarize the result of plasma temperature anisotropy measurement by comparing with the models.
5. To confirm the availability of the system to measure current density.

The outline of this thesis is as follows.

Chapter 2 introduces the principle of the double-pass Thomson scattering measurement. A conventional expression of Thomson scattering spectrum is introduced. The cross sections of Rayleigh and Raman scattering and related calculations are also summarized. These scatterings are used for calibration of absolute sensitivity in the system to determine electron density. Then, theory of electromagnetic wave propagation is also described, which is important to design a laser injection optics for Thomson scattering system.

Chapter 3 describes detail of the double-pass Thomson scattering measurement system. Information of almost all components of the system is summarized. The method and result of calibrations are also summarized. Since the cross section of Thomson scattering is small, random error can dominates the total error. Monte Carlo simulation method to estimate the random error is introduced in this chapter.

Chapter 4 supposes the models of velocity distribution function for Ohmic heated plasma in TST-2 device.

Chapter 5 presents the experiment results about velocity distribution function by using the double-pass Thomson scattering diagnostic system..

Chapter 6 gives discussion and conclusions of this thesis.

2 Principle of Double-Pass Thomson Scattering Diagnostics

2.1 Thomson Scattering Spectrum

In this section, calculations of Thomson scattering spectrum are summarized.

The electric field scattered by an accelerated electron at a distance R from the electron, in the scattering direction $\hat{s}$ is generally given by

$$E_{\mathrm{s}}(R,t) = \frac{e^2}{mc^2}\sqrt{1-\beta^2}\frac{\left\{\hat{s}\times[(\hat{s}-\boldsymbol{\beta})\times(\boldsymbol{E}_{\mathrm{i}}+\boldsymbol{\beta}\times(\hat{i}\times\boldsymbol{E}_{\mathrm{i}})-\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}_{\mathrm{i}}))]\right\}}{R(1-\hat{s}\cdot\boldsymbol{\beta})^3}\cos(k_{\mathrm{i}}\cdot\boldsymbol{r}(t^1)-\omega_{\mathrm{i}}t^1), \quad (2.1)$$

where $\boldsymbol{\beta} \equiv \boldsymbol{v}/c$, $\boldsymbol{v}(t^1)$ is the velocity of the electron, ω_{i}, $\boldsymbol{E}_{\mathrm{i}}$, $\boldsymbol{k}_{rmi}$ and $\hat{i}$ are respectively the angular frequency, the electric field, and the wave vector of the incident wave and the unit vector in the incident direction $\boldsymbol{k}_{\mathrm{i}}$, e is the elementary charge, m is the static electron mass, and c is the speed of light. These quantities are evaluated at the retarded time

$$t^1 = t - \frac{|\boldsymbol{R}-\boldsymbol{r}(t^1)|}{c} = t - \frac{|\boldsymbol{R}^1|}{c}. \quad (2.2)$$

Here, we treat only the standard application of incoherent scattering where the incident plane wave is linearly polarized, and the scattering plane is perpendicular to $\boldsymbol{E}_{\mathrm{i}}$, i.e, $\hat{s}\cdot\boldsymbol{E}_{\mathrm{i}} = 0$. The time-averaged power of the scattered wave is

$$P_{\mathrm{sc}}(R)\mathrm{d}\Omega = \frac{cR^2\mathrm{d}\Omega}{4\pi}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}|\boldsymbol{E}_{\mathrm{s}}(R,t)|^2\,\mathrm{d}t, \quad (2.3)$$

where $\mathrm{d}\Omega$ is the element of solid angle over which the scattered light accepted. The factor $(1-\hat{s}\cdot\boldsymbol{\beta})$ is introduced into Eq. (2.3) because of the finite transit time effect [24]. Finally, the scattered power from a single electron is expressed as

$$P_{\mathrm{sc}}(R)\mathrm{d}\Omega = \frac{P_{\mathrm{i}}}{A_0}\cdot r_0^2\mathrm{d}\Omega\frac{(1-\beta)^2}{(1-\beta_{\mathrm{s}})^5}[(1-\beta_{\mathrm{i}})(1-\beta_{\mathrm{s}})-(1-\cos\theta)\beta_{\mathrm{E}}^2]^2, \quad (2.4)$$

with the incident power

$$P_{\mathrm{i}} = \frac{CE_{\mathrm{i}}^2A_0}{8\pi}, \quad (2.5)$$

and the classical electron radius

$$r_0 = \frac{e^2}{mc^2}. \tag{2.6}$$

A_0 is the cross sectional area of the incident beam of radiation, θ is the scattering angle and $\beta_\mathrm{s} \equiv \hat{s} \cdot \boldsymbol{\beta}$, $\beta_\mathrm{i} \equiv \hat{i} \cdot \boldsymbol{\beta}$, $\beta_\mathrm{E} = \hat{E}_\mathrm{i} \cdot \boldsymbol{\beta}$. We consider a plasma with electron temperature T_e and density $n_\mathrm{e} = N/V$. An incident wave is scattered from a volume V, which contains N electrons. The frequencies and wave vectors of the scattered and incident lights are connected by the following conservation laws,

$$\omega_\mathrm{s} = \omega_\mathrm{i} + \boldsymbol{k} \cdot \boldsymbol{\beta} c, \tag{2.7}$$

$$\boldsymbol{k}_\mathrm{s} = \boldsymbol{k}_\mathrm{i} + \boldsymbol{k}, \tag{2.8}$$

where ω_s and $\boldsymbol{k}_\mathrm{s}$ are respectively the angular frequency and the wave vector of the scattered wave, and $\boldsymbol{k}$ is the scattering vector. The spectral profile of the scattered wave is derived from the velocity distribution of electrons along the scattering vector. Here, we assume the velocity distribution function is the relativistic Maxwellian distribution function,

$$f(\beta) = \frac{\alpha(1-\beta^2)^{-5/2} \exp[-2\alpha(1-\beta^2)^{-1/2}]}{2\pi K_2(2\alpha)}, \tag{2.9}$$

where

$$\alpha = \frac{mc^2}{2T_\mathrm{e}}. \tag{2.10}$$

K_2 is the modified Bessel function of second kind and second order. The frequency spectrum of the scattered wave is given by

$$P_\mathrm{sc}(R, \omega_\mathrm{s})\mathrm{d}\omega_\mathrm{s}\mathrm{d}\Omega = N \int\int\int_{-c}^{+c} \mathrm{d}v_\mathrm{k}\mathrm{d}v_{\mathrm{k}\perp}\mathrm{d}v_\mathrm{kT} P_\mathrm{sc}(R)\mathrm{d}\Omega \frac{f(\beta)}{c^3} \delta\left(v_\mathrm{k} - \frac{\omega_\mathrm{s} - \omega_\mathrm{i}}{k}\right) \mathrm{d}v_\mathrm{k}. \tag{2.11}$$

where δ is the delta function and v_k, $v_{\mathrm{k}\perp}$ and v_kT are the velocities along the scattering vector, perpendicular to the scattering vector within the plane of the $\hat{s}$ and $\hat{i}$ vectors, and perpendicular to the plane of $\hat{s}$ and $\hat{i}$, respectively. Thus,

$$\beta_\mathrm{i} = -\beta_\mathrm{k} \cos\gamma + \beta_{\mathrm{k}\perp} \sin\gamma, \tag{2.12}$$

$$\beta_\mathrm{s} = -\beta_\mathrm{k} \cos(\gamma + \theta) + \beta_{\mathrm{k}\perp} \sin(\gamma + \theta), \tag{2.13}$$

$$\beta_\mathrm{E} = \beta_\mathrm{kT} \tag{2.14}$$

and

$$\mathrm{d}v_{\mathrm{k}} = \frac{(\omega_{\mathrm{i}}^2 + \omega_{\mathrm{i}}\omega_{\mathrm{s}})(1-\cos\theta)}{k^2c^2} \cdot \frac{\mathrm{d}\omega_{\mathrm{s}}}{k}. \tag{2.15}$$

γ is the angle between $\boldsymbol{k}_{\mathrm{i}}$ and $\boldsymbol{k}$. Therefore, the scattered spectrum is written as

$$\begin{aligned} P_{\mathrm{sc}}(R,\omega_{\mathrm{s}})\mathrm{d}\omega_{\mathrm{s}}\mathrm{d}\Omega &= N\int\int\int_{-1}^{+1}\mathrm{d}\beta_{\mathrm{k}}\mathrm{d}\beta_{\mathrm{k}\perp}\mathrm{d}\beta_{\mathrm{kT}}\frac{P_{\mathrm{i}}}{A_0}r_0^2\mathrm{d}\Omega(1-\beta^2)Q \\ &\times\ f(\beta)\delta\left(\beta_{\mathrm{k}} - \frac{\omega_{\mathrm{s}}-\omega_{\mathrm{i}}}{kc}\right)\frac{(\omega_{\mathrm{i}}^2+\omega_{\mathrm{i}}\omega_{\mathrm{s}})(1-\cos\theta)}{k^2c^2}\cdot\frac{\mathrm{d}\omega_{\mathrm{s}}}{kc}, \end{aligned} \tag{2.16}$$

where

$$Q = \frac{[(1+\beta_{\mathrm{k}}\cos\gamma - \beta_{\mathrm{k}\perp}\sin\gamma)\{1+\beta_{\mathrm{k}}\cos(\gamma+\theta)-\beta_{\mathrm{k}\perp}\sin(\gamma+\theta)\} - (1-\cos\theta)\beta_{\mathrm{kT}}^2]^2}{\{1+\beta_{\mathrm{k}}\cos(\gamma+\theta)-\beta_{\mathrm{k}\perp}\sin(\gamma+\theta)\}^5} \tag{2.17}$$

The relation between the element of the angular frequency and the wavelength for the scattered wave is

$$\mathrm{d}\omega_{\mathrm{s}} = -\frac{2\pi c\mathrm{d}\lambda_{\mathrm{s}}}{\lambda_{\mathrm{s}}^2} = -\frac{\omega_{\mathrm{i}}}{(1+\epsilon)^2}\mathrm{d}\epsilon, \tag{2.18}$$

where

$$\epsilon = \frac{\Delta\lambda}{\lambda_{\mathrm{i}}} \qquad \Delta\lambda = \lambda_{\mathrm{s}} - \lambda_{\mathrm{i}}. \tag{2.19}$$

We consider the reduction a formula to a second order approximation in β below. The factor $1/\alpha$ is the second order in β for tokamak plasmas. Then

$$K_2(2\alpha) \simeq \sqrt{\frac{\pi}{4\alpha}}\exp(-2\alpha)\left(1+\frac{15}{16\alpha}\right), \tag{2.20}$$

$$\exp[-2\alpha(1-\beta^2)^{-1/2}] \simeq \exp(-2\alpha)\exp(-\alpha\beta^2)\left(1-\frac{3}{4}\alpha\beta^4\right), \tag{2.21}$$

$$(1-\beta^2)^{-5/2} \simeq 1+\frac{5}{2}\beta^2. \tag{2.22}$$

Therefore, $f(\beta)$ is reduced to

$$f(\beta) \simeq \left(\frac{\alpha}{\pi}\right)^{3/2}\exp(-\alpha\beta^2)\left(1+\frac{5}{2}\beta^2 - \frac{15}{16\alpha} - \frac{3}{4}\alpha\beta^4\right), \tag{2.23}$$

where the term $(\alpha/\pi)^{3/2}\exp(-\alpha\beta^2)$ is the nonrelativistic Maxwellian distribution function. Then we

find that

$$
\begin{aligned}
P_{\rm sc}^{(2)}(R,\omega_{\rm s})\mathrm{d}\omega_{\rm s}\mathrm{d}\Omega &= N\frac{P_{\rm i}}{A_0}r_0^2\mathrm{d}\Omega\sqrt{\frac{\alpha}{\pi}}\exp\left[-\frac{\alpha(\omega_{\rm s}-\omega_{\rm i})^2}{k^2c^2}\right]\frac{(\omega_{\rm i}^2+\omega_{\rm i}\omega_{\rm s})(1-\cos\theta)}{k^2c^2}\frac{\mathrm{d}\omega_{\rm s}}{kc} \\
&\times \left[1-\frac{1}{4\alpha}\left\{\frac{43}{8}-2\cos\theta-\sin^2\gamma+6\sin\gamma\sin(\gamma+\theta)-6\sin^2(\gamma+\theta)\right\}\right. \\
&+ \{2\cos\gamma-3\cos(\gamma+\theta)\}\frac{\omega_{\rm s}-\omega_{\rm i}}{kc}+\left\{\frac{3}{4}+\cos^2\gamma+4\cos\gamma\cos(\gamma+\theta)\right. \\
&+ \left.\left.16\cos^2(\gamma+\theta)\right\}\frac{(\omega_{\rm s}-\omega_{\rm i})^2}{k^2c^2}-\frac{3}{4}\alpha\frac{(\omega_{\rm s}-\omega_{\rm i})^4}{k^4c^4}\right]. \qquad (2.24)
\end{aligned}
$$

Now we keep terms to second order in ϵ only. Then

$$
\begin{aligned}
\mathrm{d}\omega_{\rm s} &= -\omega_{\rm i}(1-2\epsilon+3\epsilon^2)\mathrm{d}\epsilon, && (2.25)\\
\frac{1}{kc} &= \frac{1}{\omega_{\rm i}\sqrt{2(1-\cos\theta)}}\left\{1+\frac{\epsilon}{2}-\frac{1}{8}\left(5+\frac{2}{1-\cos\theta}\right)\epsilon^2\right\}, && (2.26)\\
\frac{\omega_{\rm s}-\omega_{\rm i}}{kc} &= -\frac{\epsilon}{\sqrt{2(1-\cos\theta)}}\left\{1-\frac{\epsilon}{2}-\frac{1}{8}\left(1+\frac{2}{1-\cos\theta}\right)\epsilon^2\right\}, && (2.27)\\
\frac{(\omega_{\rm i}^2+\omega_{\rm i}\omega_{\rm s})}{k^2c^2} &= \frac{1}{1-\cos\theta}\left[1+\frac{\epsilon}{2}-\left\{1+\frac{1}{2(1-\cos\theta)}\right\}\epsilon^2\right], && (2.28)\\
\sin\gamma &= \sqrt{\frac{1+\cos\theta}{2}}\left(1-\frac{\epsilon}{2}\right), && (2.29)\\
\cos\gamma &= \sqrt{\frac{1-\cos\theta}{2}}\left\{1+\frac{3+\cos\theta}{2(1-\cos\theta)}\epsilon\right\}. && (2.30)
\end{aligned}
$$

Therefore, the scattered spectrum is rewritten as

$$
\begin{aligned}
P_{\rm sc}^{(2)}(R,\epsilon)\mathrm{d}\epsilon\mathrm{d}\Omega &= N\frac{P_{\rm i}}{A_0}r_0^2\mathrm{d}\Omega\frac{\mathrm{d}\epsilon}{\sqrt{2(1-\cos\theta)}}\sqrt{\frac{\alpha}{\pi}}\exp\left[-\frac{\alpha\epsilon^2}{2(1-\cos\theta)}\right] \\
&\times \left[1-\frac{7}{2}\epsilon+\frac{\alpha\epsilon^3}{2(1-\cos\theta)}-\frac{1}{8\alpha}\left(\frac{39}{4}-5\cos\theta\right)+\frac{1}{8}\left(29+\frac{5}{1-\cos\theta}\right)\epsilon^2\right. \\
&- \left.\frac{1}{16(1-\cos\theta)}\left(28-\frac{1}{1-\cos\theta}\right)\alpha\epsilon^4+\frac{\alpha^2\epsilon^6}{8(1-\cos\theta)^2}\right]. \qquad (2.31)
\end{aligned}
$$

The second order approximate relativistic spectral density function $S_R^{(2)}(\epsilon)$ is

$$S_R^{(2)}(\epsilon) = S_N(\epsilon)\left[1 - \frac{7}{2}\epsilon + \frac{\alpha\epsilon^3}{2(1-\cos\theta)} - \frac{1}{8\alpha}\left(\frac{39}{4} - 5\cos\theta\right) + \frac{1}{8}\left(29 + \frac{5}{1-\cos\theta}\right)\epsilon^2 \right.$$
$$\left. - \frac{1}{16(1-\cos\theta)}\left(28 - \frac{1}{1-\cos\theta}\right)\alpha\epsilon^4 + \frac{\alpha^2\epsilon^6}{8(1-\cos\theta)^2}\right]. \tag{2.32}$$

$S_N(\epsilon)$ is the nonrelativistic spectral density function

$$S_N(\epsilon) = \left(\frac{\alpha}{\pi}\right)^{1/2} \frac{1}{\sqrt{2(1-\cos\theta)}} \exp\left\{\frac{-\alpha\epsilon^2}{2(1-\cos\theta)}\right\} \tag{2.33}$$

$$\epsilon = \frac{\Delta\lambda}{\lambda_i} \qquad \Delta\lambda = \lambda_s - \lambda_i \qquad \alpha = \frac{m_0 c^2}{2T_\mathrm{e}} \tag{2.34}$$

2.2 Effect of Magnetic Field

In the tokamak system, there always is strong toroidal magnetic field. In this situation, the orbits of the electrons take on a helical form, as a Larmor motion. Then, Thomson scattering spectrum also depends on the cyclotron frequency [25, 26, 27, 28, 29]. The cyclotron resonance condition is written as

$$k_\parallel v_\parallel = \omega_s - \omega_i - l\omega_e (l = -\infty \to \infty), \tag{2.35}$$

where ω_s and ω_i are frequency of scattered and incident light. $\omega_e = qB/m_e$ is the cyclotron frequency. In this situation, Thomson spectrum is modified as

$$s(\omega) = \frac{\exp(-a)}{\sqrt{\pi b}} \sum_{n=-\infty}^{+\infty} I_n(a) \exp\left\{-\frac{(\omega - \omega_1 - n\omega_g)^2}{b\omega_g^2}\right\}, \tag{2.36}$$

where a and b correspond to $k_\perp$ and $k_\parallel$, respectively, written as

$$a = \frac{k_s^2 \cos^2\beta kT}{m\omega_g^2}, \tag{2.37}$$

$$b = \frac{k_s^2 \sin^2\beta 2kT}{m\omega_g^2}. \tag{2.38}$$

β is an angle like $\beta = \angle(\boldsymbol{k}_s, \boldsymbol{B}) - \pi/2$. In the case of large a, the function I reduces to

$$I_l(a) = \frac{1}{\sqrt{2\pi a}} \exp\left(a - \frac{l^2}{2a}\right). \tag{2.39}$$

Then total spectrum s reduces to

$$s(\omega) = \frac{1}{\pi\sqrt{2ab}} \exp\left(-\frac{l^2}{2a}\right) \exp\left[-\left(\frac{\omega_s - \omega_i - l\omega_e}{\sqrt{b}\omega_e}\right)^2\right]. \tag{2.40}$$

Now, we estimate the effect of magnetic field on Thomson scattering spectrum in the configuration of TST-2. We assume a plasma of $T_e = 10$ [eV]. The wavelength of incident laser $\lambda_i = 1064.14$ [nm] and toroidal magnetic field $B_t = 0.3$ T. In the TST-2 configuration, the solid angle of measurement of Thomson scattering is about 0.3 [sr]. Therefore, the scattering angle spread about 0.3 [rad] from the averaged value. This means the angle β cannot be lower than 0.3 [rad].

Fig. 2.1 shows a result of calculation for $s(\omega)$ in Eq. (2.40) in three conditions. In fact, b and β must be greater than 4.6 and $0.03^\circ \simeq 5 \times 10^{-4}$ rad in the actual condition of experiment in the TST-2 operation. Therefore, it is concluded that there is no effect caused by magnetic field into Thomson scattering spectrum.

Figure 2.1: Thomson scattering spectrum in three cases of parameters. Abscissa and vertical represent wavelength and intensity, respectively.

2.3 Scatterings for Absolute Sensitivity Calibration

In order to measure the plasma density and current density, it is necessary to calibrate absolute sensitivity of the measurement system. The calculations for absolute calibration are summarized in this section.

The power density of Thomson scattering Eq. (2.31) is written as

$$P_{\mathrm{TS}}^{(2)}(R,\epsilon)\,\mathrm{d}\epsilon\mathrm{d}\Omega = n_{\mathrm{e}}LP_{\mathrm{i}}r_{\mathrm{e}}^{2}S_{R}^{(2)}(\epsilon)\,\mathrm{d}\epsilon\mathrm{d}\Omega. \tag{2.41}$$

This is the value measured from a direction perpendicular to the polarization of linear polarized incident laser. When we take into account the angle between scattered direction and polarization of linear polarized incident laser β, signals measured on i-th channel y_{i} is written as

$$y_{\mathrm{i}} = \int\int C(\epsilon)\,P_{\mathrm{TS}}^{(2)}(R,\epsilon)\;f_{\mathrm{i}}(\epsilon)\left(\frac{\mathrm{d}I}{\mathrm{d}\Omega}\right)_{\mathrm{TS}}\mathrm{d}\epsilon\mathrm{d}\Omega \tag{2.42}$$

$$\simeq n_{\mathrm{e}}LP_{\mathrm{i}}r_{\mathrm{e}}^{2}\Delta\Omega\left(\frac{\mathrm{d}I}{\mathrm{d}\Omega}\right)_{\mathrm{TS}}\Bigg|_{\beta=\pi/2}\int C(\epsilon)f_{\mathrm{i}}(\epsilon)S_{R}^{(2)}(\epsilon)\,\mathrm{d}\epsilon \tag{2.43}$$

where the dependence on the angle β is

$$\left(\frac{\mathrm{d}I}{\mathrm{d}\Omega}\right)_{\mathrm{TS}} = \frac{3}{8\pi}\sin^{2}\beta \tag{2.44}$$

$C(\lambda)$ is total transmissivity of optical components such as mirror, fiber and etc. except for the sensitivity of interference filter facing to APD. f_{i} denotes sensitivity of the interference filter. Therefore, The intensity of Thomson scattering signal depends not only on electron density n_{e} but also on instrumental condition such as scattering length, incident laser power and transmissivity of optical components. In general, it is difficult to measure or estimate each value accurately. The most useful way to calibrate the absolute sensitivity including these values is to measure other scattering phenomenon of which cross section is well-known. We use two kinds of scattering, Rayleigh scattering and Raman scattering. In this section, method and result of the absolute calibration are described.

2.3.1 Rayleigh Scattering

One of an useful scattering to calibrate absolute sensitivity is Rayleigh scattering. Rayleigh scattering is an elastic scattering (i.e., the wavelength of scattered light is the same as the wavelength of incident

wave). The wavelength of scattered light is 1064 nm when the incident beam is Nd:YAG laser. For the measurement of Rayleigh scattering, we must replace the interference filter in the polychromator to a bandpass filter which passes only around the wavelength of 1064 nm.

When we use nitrogen gas, Rayleigh scattering signal y^{Rayleigh} is

$$y^{\mathrm{Rayleigh}} \simeq n(\mathrm{N}_2)LP_{\mathrm{i}}\sigma_{\mathrm{Rayleigh}}\Delta\Omega\left(\frac{dI}{d\Omega}\right)_{Rayleigh} C(\lambda_0)f(\lambda_0), \tag{2.45}$$

where

$$\left(\frac{dI}{d\Omega}\right)_{\mathrm{Rayleigh}} = \left(\frac{dI}{d\Omega}\right)_{\mathrm{TS}} = \frac{3}{8\pi}\sin^2\beta. \tag{2.46}$$

2.3.2 Rotational Raman Scattering (RRS)

In the double-pass Thomson scattering scheme, stray light can be a serious noise source as background light because laser beam travels a long distance. Then calibration with Rayleigh scattering is inaccurate, since the wavelength of Rayleigh scattering and that of stray light are same. In such case, calibration with Rotational Raman Scattering (RRS, Raman scattering) is useful. Although the cross section of Raman scattering is smaller than that of Rayleigh scattering, there are many advantages to use Raman scattering. One of those is that Raman scattering is an inelastic scattering. Therefore, we can use the interference filter which is also used for Thomson scattering measurement. Then much amount of stray light is shut off by the interference filter. In fact, the method of calibration with Raman scattering is widely prevalent in many large devices [30, 31].

Signal of i-th channel by Raman scattering with nitrogen gas is given as

$$y_{\mathrm{i}}^{\mathrm{Raman}} \simeq n(\mathrm{N}_2)LP_{\mathrm{i}}\Delta\Omega\left(\frac{\mathrm{d}I}{\mathrm{d}\Omega}\right)_{\mathrm{Ram}}\Bigg|_{\beta=\pi/2}\sum_J C(\epsilon_J)w_J\sigma_{\mathrm{a,s}J}(\epsilon_J)f_{\mathrm{i}}(\epsilon_J), \tag{2.47}$$

where

$$\left(\frac{dI}{d\Omega}\right)_{\mathrm{Ram}} = \frac{3}{8\pi}\frac{6+\sin^2\beta}{10} \qquad \text{and} \qquad \epsilon_J = \frac{\lambda_J}{\lambda_0} \tag{2.48}$$

Here, J is the initial rotational-angular-momentum quantum number, w_J is the population of the initial rotational state, σ_J is the cross section of Raman scattering (suffix ”a” and ”s” specify anti-

Stokes and Stokes, respectively) [30]. Raman shifted wavelength can be calculated as

$$\frac{1}{\lambda_{\mathrm{a},J}} = \frac{1}{\lambda_0} + (2B_0 - 3D_0)(2J-1) - D_0(2J-1)^3 \qquad (J \to J-2,\ \text{anti Stokes}), \tag{2.49}$$

$$\frac{1}{\lambda_{\mathrm{s},J}} = \frac{1}{\lambda_0} + (-2B_0 + 3D_0)(2J+3) + D_0(2J+3)^3 \qquad (J \to J+2,\ \text{Stokes}). \tag{2.50}$$

Here, $\lambda_0 = 1064.14$ nm is the wavelength of the incident laser, $J \to J-2$ (called as O-branch) and $J \to J+2$ (called as S-branch) represent transitions of anti-Stokes and Stokes RRS, respectively, B_0 is the rotational constant and D_0 is the centrifugal distortion constant [32] [33]. w_J is calculated as

$$w_J = \frac{1}{W} g_J (2J+1) \exp\left(-F(J)\frac{hc}{kT}\right). \tag{2.51}$$

Here, g_J is the statistical weighting of the rotational levels due to nuclear spin (Table 1), T is the temperature of nitrogen in thermal equilibrium. The normalization factor W is determined from

$$\sum_{J=0}^{\infty} w_j = 1. \tag{2.52}$$

$F(J)$ is the rotational energy term value,

$$F(J) = B_0 J(J+1) - D_0[J(J+1)]^2 + H_0[J(J+1)]^3. \tag{2.53}$$

RRS cross sections σ_{aJ} and σ_{sJ} are written as

$$\sigma_{\mathrm{a},J} = \frac{112\pi^4}{45} \frac{3J(J-1)}{2(2J+1)(2J-1)} \frac{\gamma^2}{\lambda_J^4} \int [(1-\rho)\cos^2\psi + \rho]\, \mathrm{d}\psi \tag{2.54}$$

$$= \frac{112\pi^5}{90} \frac{3J(J-1)}{2(2J+1)(2J-1)} \frac{\gamma^2}{\lambda_J^4} (1+\rho), \tag{2.55}$$

and

$$\sigma_{\mathrm{s},J} = \frac{112\pi^4}{45} \frac{3(J+1)(J+2)}{2(2J+1)(2J+3)} \frac{\gamma^2}{\lambda_J^4} \int [(1-\rho)\cos^2\psi + \rho]\, \mathrm{d}\psi \tag{2.56}$$

$$= \frac{112\pi^5}{90} \frac{3(J+1)(J+2)}{2(2J+1)(2J+3)} \frac{\gamma^2}{\lambda_J^4} (1+\rho), \tag{2.57}$$

Here, γ is the polarizability anisotropy (Table 1) [34], $\rho = 0.69$ is the value of the depolarization [35]

Table 1: Several parameters to calculate RRS cross section for nitrogen.

B_0 [m^{-1}]	D_0 [m^{-1}]	H_0 [m^{-1}]	g_J (for even J)	g_J (for odd J)	γ^2 [m^3]
1.99×10^2	5.74×10^{-4}	3.70×10^{-8}	6	3	0.51×10^{-60}

in the TST-2 double-pass Thomson system (discussed in Sec. 3.4.6). ψ is the polarization angle of scattered light. Value of ψ is defined in Fig. 3.48.

Fig. 2.2 shows the effective cross section of RRS; $w_J\sigma_{a,sJ}(\lambda_J)$ (black lines) and the spectral responsivity of each filter f_i (blue). The red line represents λ_0.

Figure 2.2: Effective cross section of RRS for nitrogen; $w_J\sigma_{a,sJ}(\lambda_J)$ (black lines) and normalized spectral responsivity of each filter (blue).

2.4 Propagation of Gaussian Beam

For Thomson scattering measurement, a laser is used to excite Thomson scattering light. Then, calculation of the laser beam propagation is required in order to design a proper optical system. We use a laser of which the main transverse mode is Gaussian mode (TEM_{00}). In this section, propagation of Gaussian and realistic multimode beam is described.

From the Maxwell equation in a free space, the electric field $\hat{\mathbf{E}} = \mathbf{E}(x, y, z)\exp(i\omega t)$ satisfies

$$\nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0. \tag{2.58}$$

Here, we assume a plane wave written as

$$E(x, y, z) = \psi(x, y, z)\exp(-ikz). \tag{2.59}$$

z is in the traveling direction and x and y are perpendicular to z. We assume the profile of amplitude ψ is Gaussian, expressed as

$$\psi(x, y, z) = A\exp\left[-i\left\{p(z) + \frac{k}{2q(z)}(x^2 + y^2)\right\}\right]. \tag{2.60}$$

A is a constant and $p(z)$ and $q(z)$ are parameters depends only on z. When we suppose

$$\frac{\partial^2\psi}{\partial x^2}, \frac{\partial^2\psi}{\partial y^2} >> \frac{\partial^2\psi}{\partial z^2}, \tag{2.61}$$

we can solve the Eq. (2.59) with Eq. (2.60) like that

$$\psi(r, z) = A\frac{w_0}{w(z)}\exp\left[-\left(\frac{1}{w^2(z)} + i\frac{k}{2R(z)}\right)r^2 + i\phi(z)\right], \tag{2.62}$$

where $r^2 = x^2 + y^2$ and

$$w(z) = w_0\sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2} \tag{2.63}$$

$$R(z) = z\left\{1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right\} \tag{2.64}$$

$$\phi(z) = \tan^{-1}\left(\frac{\lambda z}{\pi w_0^2}\right). \tag{2.65}$$

Therefore, the electric field is written as

$$E(r,z) = A\frac{w_0}{w(z)}\exp\left\{-i(kz-\phi(z)) - \left(\frac{1}{w^2(z)} + i\frac{k}{2R(z)}\right)r^2\right\}. \tag{2.66}$$

When $r = w(z)$, the amplitude of laser is $1/e$ times as much as the maximum ($1/e^2$ in power). The $w(z)$ is called as beam spot size at $z = z$. $w_0 = w(z = 0)$ is the minimum beam spot size in the propagation, called as waist size of beam. The beam divergence angle θ can be written as

$$\theta = \frac{w(z)}{z} \simeq \frac{\lambda}{\pi w_0} \qquad \left(\text{at } \frac{\lambda z}{\pi w_0^2} >> 1\right) \tag{2.67}$$

Expression of propagation with a lens is useful. The distance from the lens to beam waist d_1 is written as

$$d_1 = \frac{\left(\frac{\pi w_0^2}{\lambda}\right)^2 \frac{1}{f} - d_0\left(1 - \frac{d_0}{f}\right)}{\left(\frac{\pi w_0^2}{\lambda}\right)^2 \left(\frac{1}{f}\right)^2 + \left(1 - \frac{d_0}{f}\right)^2}. \tag{2.68}$$

λ is wavelength of the beam, f is the focal length of the lens and w_0 is the waist size before entering the lens. d_0 is the distance from the lens to beam waist before entering the lens. And, the beam waist size after entering the lens w_1 is expressed as

$$\left(\frac{w_1}{w_0}\right)^2 = \frac{1}{\left(\frac{\pi w_0^2}{\lambda}\right)^2 \left(\frac{1}{f}\right)^2 + \left(1 - \frac{d_0}{f}\right)^2}. \tag{2.69}$$

Figure 2.3: Schematic diagram of one lens system.

Realistic laser is not a pure Gaussian beam but a multimode beam. Recently, a method to calculate realistic beam propagation from ideal Gaussian beam propagation calculation by introducing a

constant called as M^2 factor has been proposed and verified. When we measure the beam waist size (w_{0M}) and divergence angle (θ_M) of actual laser beam, M^2 factor is defined as

$$M^2 = \frac{\pi w_{0M}\theta_M}{\lambda}. \tag{2.70}$$

Defining $\lambda_M = M^2\lambda$, Eq. (2.68) and Eq. (2.69) are modified as

$$d_1 = \frac{\left(\frac{\pi w_{0M}^2}{\lambda_M}\right)^2 \frac{1}{f} - d_0\left(1 - \frac{d_0}{f}\right)}{\left(\frac{\pi w_{0M}^2}{\lambda_M}\right)^2 \left(\frac{1}{f}\right)^2 + \left(1 - \frac{d_0}{f}\right)^2}. \tag{2.71}$$

and

$$\left(\frac{w_1}{w_{0M}}\right)^2 = \frac{1}{\left(\frac{\pi w_{0M}^2}{\lambda_M}\right)^2 \left(\frac{1}{f}\right)^2 + \left(1 - \frac{d_0}{f}\right)^2}. \tag{2.72}$$

For multimode laser, beam profile is not Gaussian. In this case, spot size is evaluated by calculating second moment of electric field intensity, expressed as

$$w_x(z) = 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 \left|E(x,y,z)\right| \mathrm{d}x\mathrm{d}y, \tag{2.73}$$

$$w_y(z) = 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y^2 \left|E(x,y,z)\right| \mathrm{d}x\mathrm{d}y. \tag{2.74}$$

2.5 Temperature Anisotropy Measurement

As introduced in Sec. 2.1, plasma pressure of the direction parallel to the $\boldsymbol{k}$, written as

$$\boldsymbol{k} = \boldsymbol{k}_{\mathrm{s}} - \boldsymbol{k}_{\mathrm{i}} \tag{2.75}$$

is measured in Thomson scattering diagnostic. In the previous study shown in 1.4.1, they measured the difference between temperatures in the direction parallel and perpendicular to the magnetic field. In this thesis, the two directional temperatures are measured simultaneously by the double-pass configuration. Figure 2.4 shows the method of double-pass measurement.

As sketched in Fig. 2.4, the incident laser makes a round trip by a mirror. We call the forward pass as ”first-pass” and backward pass as ”second-pass”. The directions of $\boldsymbol{k}_{\mathrm{i}}$ in first- and second-pass are almost opposite. Since the direction of measurement is the same, the direction of $\boldsymbol{k}_{\mathrm{s}}$ is common in first- and second-pass. In this way, the directions of $\boldsymbol{k}$ in first- and second-pass can be almost perpendicular each other. In the TST-2 configuration, the direction of $\boldsymbol{k}$ is perpendicular to the magnetic field in the first-pass and parallel in the second-pass. The timing of measurement in first- and second-pass is almost simultaneous because the distance between the measurement point and mirror is short enough, about 3 m.

Figure 2.4: Schematic diagram describing geometry of double-pass Thomson scattering.

3 Double-Pass Thomson Scattering System

The double-pass Thomson scattering diagnostic system was developed on the TST-2 device. This is the first direct diagnostic to measure two directional broadening of velocity distribution function simultaneously [36, 37]. In this chapter, detail of the system is described.

3.1 Overview

The Thomson scattering diagnostic is based on a number of elements. Firstly, we describe some overview of the system. Figure 3.1 is a schematic to show total image of the double-pass Thomson scattering geometry. The diagnostic consists of laser injection optics, collection optics and detection system. The laser optics was designed by using the Gaussian propagation theory with M^2 factor. The incident laser goes through a plasma twice by a reflection mirror. When the laser comes on, Thomson scattering light is excited in a plasma. The scattered light is collected by a collection spherical mirror, travels through a fiber and is detected in a spectrometer called as a polychromator. In a standard scheme of Thomson scattering diagnostic, plasma parameters of electron density and temperature are calculated by comparing the measured spectrum with the theoretical prediction Eq. (2.32).

Figure 3.1: Schematic diagram of the double-pass TS laser path. Percentages denote relative power of the incident laser.

Measurement error arises in several part of the system. Displacement of the laser path or opti-

cal elements affects the total transmission. Variation of wavelength dependence of measurement is caused by variation of environmental temperature around detection device or variation of incident angle of scattered light or contamination of optical elements or, etc. These are causes of systematic error. In order to reduce such error, it is necessary to perform a robust calibration for absolute and relative sensitivity. The procedure and result of calibrations are described in the sections 3.3 and 3.4. Moreover, statistical error affects measurement, since the Thomson scattering cross section is small and the signal is week. The Discussion about errors are described in Sec. 3.5. The error is simulated with Monte Carlo Simulation (MCS). With this simulation, we discuss how large the error is and how much the error affects the measurement. The principle of MCS is described in Sec. 3.6.

A typical result of electron density (n_e) and temperature (T_e) profile measurement is shown in Fig. 3.2 with error bars. Typical errors for density and temperature are about 10%. The line integrated electron density on the mid-plane (Z=0) is also shown. They mostly agree each other.

3.1.1 Design of Laser Beam Propagation

The laser injection optics were designed by using calculation of Gaussian beam propagation with M^2 factor (see Sec. 2.4). First of all, we checked performance of the actual laser beam. Figure 3.3 shows the actual laser beam profile measured by a beam profiler (BGP-USB-SP620, OPHIR). The intensity profile of the beam is almost flat. Getting such snapshots of beam profile on several positions, actual beam waist size (w_{0M}) and divergence angle (θ_M) were estimated. The result is shown in Fig. 3.4. The spot size was evaluated by second moment of the intensity profile, written as Eq. (2.74). Fitting the result to the following expression

$$w_M = w_{0M}\sqrt{1 + \frac{z\lambda_M}{\pi w_{0M}^2}}, \tag{3.1}$$

values were estimated as $w_{0M} = 4.8$ mm and $\theta_M = 0.55$ mrad with M^2 factor is 7.8. We assumed the beam waist is at the laser rod (Amp, see Fig. 3.9).

For the Thomson scattering measurement, important optical components, which determine the beam size, are a convex lens and a concave mirror. The convex lens is used to focus a beam on a plasma. And the concave mirror is used to reflect back the laser and focus on the plasma again. These correspond to the lens and the spherical mirror in Fig. 3.12. The beam spot size must be reduced less than the diameter of fiber image on the measurement point to increase optical efficiency. In addition,

Figure 3.2: Left: A typical result of electron density and temperature profile measurement provided by double-pass Thomson scattering diagnostic system. Right: Time slice of line integrated electron density n_eL on the mid-plane (Z=0) measured by an interferometer. The measurement of Thomson scattering and interferometer was performed for the same plasma. The line integrated electron density measured as $n_eL = 5.7\times10^{18}$ m^{-2} by Thomson scattering, and $n_eL = 4.9\times10^{18}$ m^{-2} by interferometer.

the beam must not be focused on optical components such as a plane mirror to avoid damage of the components. Therefore, design of beam propagation is important. By using the Eqs. (2.71), (2.72) and (3.1), the propagation of beam was calculated shown as Fig. 3.5. The beam travels left to right in this figure. The first lens in this figure corresponds to the lens focusing the beam on a plasma. The second lens in this figure corresponds to the concave mirror to reflect back the beam. In order to focus the beam on proper position, focal length of the lens and the concave mirror were decided as 2 m. The diameter of fiber image on measurement position is around 5 mm (green). It is seen from Fig. 3.5 that the spot size is proper in all range of measurement (from R = 197 mm to R = 571 mm).

We checked the actual laser beam propagation by using heat-sensitive paper. The heat-sensitive

Figure 3.3: Typical profile of the actual laser beam measured by a beam profiler measured at the point A in the figure 3.9.

Figure 3.4: Beam spot size measured by the beam profiler.

paper is sensitive to thermal power of Nd:YAG laser, and its color changes when a laser with power over a threshold injected into the heat-sensitive paper. The pattern of the heat-sensitive paper is larger when the profile of injected beam is broader. Therefore, it is useful to find out focal point of a laser beam. Note that the diameter of pattern of the heat-sensitive paper is not equal to spot size of the beam. Figure 3.6 shows the result from measurement of pattern diameters. Comparing the Figs. 3.5 and 3.6, the beam travels as designed.

Figure 3.5: Design of laser propagation on TST-2. The beam travels from left to right. R197 and R571 indicate the positions corresponding to measurement points of major radius R = 197 mm and R = 571 mm in TST-2 device.

Figure 3.6: Diameters of beam pattern on a heat-sensitive paper. Definition of abscissa axis is same as that of Fig. 3.5.

3.1.2 Laser Incident Optics

Figs 3.7 and 3.9 show the photograph and schematic of the laser incident optics. The optics consists of a laser unit and several optical components. Specifications of the laser is listed in Tab. 2. We use a Q-switched Neodymium-doped Yttrium Aluminum Garnet (Nd:YAG) laser. The laser is excited in the laser rod (called as an Oscillation rod) in a cavity in the laser unit. After it is triggered, the laser beam is amplified through an additional laser rod (called as a Amplifier rod) and travels through the exit port. After the beam exits from the laser unit, travels reflected by several plane mirrors, through a waveplate and a lens, and then injects into the TST-2 device (first-pass). The beam is reflected and turns back again (second-pass), and vanishes in the laser dump. The YAG laser is a invisible laser. Thus, we align the laser path by bringing the path in line with a guiding visible laser, shown as green lines in Fig. 3.9. For this reason, all the plane mirrors are dichroic mirror, which reflects light of 1064 nm and 532 nm.

The timing of laser injection into a plasma is detected by a Si photodiode, which is drawn as ”Laser Monitor” in Fig. 3.9. The photodiode detects transmitted laser light from a mirror. The detected pulse is input oscilloscopes which detect the Thomson scattering signal and is used to identify the measurement timing.

Figure 3.7: Photograph of the laser and incident optics. The red line represents the path of the Nd:YAG laser.

Figure 3.8: Photograph of the isolator.

An important factor to design the incident optics is the polarization. The angular distribution of differential cross section of the TS emission is written as

$$\left(\frac{dI}{d\Omega}\right)_{\text{Thomson}} = \frac{3}{8\pi}\sin^2\beta, \tag{3.2}$$

Table 2: Specifications of the Nd:YAG laser (YG981-E, Quantel).

Wavelength	1064 nm
Energy	1.6 J
Repetition Rate	10 Hz
Pulse Duration	8 – 11 ns
Polarization Ratio	≥ 95 %(Horizontal)

where β is the angle between the polarized direction of the incident laser and the scattering direction. The collection optics is on the midplane of TST-2 (i.e., z=0). Therefore, the injection laser must be vertically-polarized in order to maximize the detected scattering light. The polarization of output from the oscillation Amp is designed as horizontal. The horizontal polarization is rotated by $\pi/4$ when the laser travels through the isolator by Faraday effect. With the $\pi/2$ waveplate, the polarization of the laser is rotated to the vertical. Horizontally polarized component is reflected at the Brewster window and can not come into the vessel. The polarization is kept in vertical until the laser vanishes in the beam dump.

In the double-pass scheme, return laser beam can turns back and enter into the laser unit. The return laser can destroy the optics or crystals of laser rod in the laser unit. In order to avoid such accidents, we use an optical isolator (Fig. 3.8). Due to a strong magnet set inside this isolator, the polarization of an incident laser is rotated by 45 degrees. The isolator is also called by Faraday rotator because this process is known as Faraday rotation. When the return laser is injected into this isolator, the polarization angle of the return laser is rotated by 45 degrees again. This return laser, whose polarization is perpendicular to that of the initial forward light, is reflected by a polarized filter set in the isolator and damped.

The results of a performance test are shown in Figs 3.10 and 3.11. The tests are performed with a low power Nd:YAG laser (about 120 mJ), a polarizer and a power monitor. The transmission of a polarizer was measured by rotating the polarizer (Fig. 3.10). Next, Nd:YAG laser was injected to the isolator through the polarized filter. The transmission of the isolator was measured as a function of the rotational angle of the polarizer (Fig. 3.11) for forward direction of isolator. In addition, the reflectivity of isolator at the injection port and the stopping capability of the isolator were measured Both of them were 30 dB.

Fig. 3.12 shows components around the location of measurement. The lens in Fig. 3.12 is the same one in Fig. 3.9. Both of first- and second-pass are drawn in this figure. The spherical mirror set

Figure 3.9: Schematic of the laser incident optics.

on the right-side is the mirror to reflect back the laser beam. The Thomson scattered light is collected by the collection (spherical) mirror below.

The point of design around the location of measurement is to remove stray light. Stray light is light in an optical system, which was not intended in the design. For example, it is necessary that laser beam pass through a vacuum window because plasma is produced only in a vacuum. On and in the vacuum window, the beam is refracted and scattered. Such refracted and scattered light travels a path different from the main laser path, and can enter the detector from an unintended path. In particular, we connects the vacuum windows and vacuum vessel with long vacuum tubes in order to keep the distance between the windows and the location of measurement away, since the window is a strong source of stray light. The stray light has the same wavelength as that of the original laser source (hire, it is 1064 nm and reaches to the detector almost same timing as the Thomson scattering light. Therefore, stray light can be a string noise source. The noise level of stray light is discussed in

Figure 3.10: The transmission of the polarized filter.

Figure 3.11: The relationship between polarization of Nd:YAG laser and the transmission of the isolator (not open to the public).

Sec. 3.5.4.

In the TST-2 configuration, There are four paths through which stray light come into the detector (Fig. 3.13). (i) From the laser unit and the incident optics to the detector directly. (ii) From the incident optics to the fiber without entering the vacuum vessel. (iii) From the vacuum vessel to the fiber after several reflections. (iv) From the laser damp to the polychromator directly. In fact, (i), (ii) and (iv) are not so serious problem. Because, these kinds of stray light can be removed if we identify and interrupt the path with black material. Actually, the laser unit and other incident optics are covered by a black cloth. However, the path (iii) can be a serious problem. Because the path is the same as the pass of Thomson scattering light, thus it is difficult to interrupt the stray light only.

In order to remove as much stray light of path (iii) as possible, two kinds of components are installed to the incident and the exit tube as shown in Fig. 3.9. These are called as ”baffle plate” (Fig. 3.15) and ”absorption apparatus” (Fig. 3.16).

Fig. 3.14 shows the mechanism to remove stray light by the baffle plates. A set of baffle device consists of several baffle plates with hole of two different diameter one after the other. Stray light is captured in the space between the two baffles with the smaller diameter as shown in Fig. 3.14, and diminish after multiple reflections. In this way, the stray light is reduced. On the other hand, the absorption apparatus consists of a number of blades with sharp edge. Stray light diminishes in

Figure 3.12: Schematic of the laser path.

the same way of baffle device after multiple reflections. The absorption apparatus is fixed on a bar movable in the radial direction of the tube as shown in Fig. 3.17. The absorption apparatus is fixed at an optimum position. As shown in Fig. 3.12, there are horizontally movable and vertically movable absorbers.

Actually, these devices reduced stray light successfully. In particular, it was verified by some experiments that the smaller the hole diameter is, and the more the number of these devices is, the less stray light was detected. However, stray light instead increases when the laser beam touches the edge of the baffle plates or absorption apparatus. For this reason, baffle plate with a larger hole as shown in Fig. 3.15 was used at locations where the laser beam spot is large.

Figure 3.13: Conceptual diagram of four types stray light.

Figure 3.14: Schematic of baffle plates. Stray light is captured and trapped between the two types of baffle plates.

Figure 3.15: Photograph of Baffle plates.

Figure 3.16: Photograph of an absorption apparatus.

Figure 3.17: Photograph of the inside of the guiding tube.

3.1.3 Collection Optics

The collection optics consists of a spherical collection mirror and fibers. The schematic of the side-view of the collection optics is shown in 3.18. Thomson scattered light comes outside through a vacuum window (made from quart). The scattered light is focused on fibers by a spherical collection mirror. A fiber is directly connected to a polychromator, which is described in Sec. 3.1.4. The surface of the collection mirror is coated by gold to improve the reflectivity. There are twelve fibers and each corresponds to a measurement position. The fiber is fixed by a fiber holder. The relationship between each measurement location and scattering angle for first- and second-pass is summarized in table 3.1.3. The scattering angle is measured by a method as shown in Fig. 3.19. The two red lines are the path of YAG laser and the line of sight of fiber. The line of sight of fiber is demonstrated by injecting red laser from the opposite side of fiber. These two lines crossing at the measurement point are recorded and the angle between them corresponds to the scattering angle.

Figure 3.18: Schematic of the horizontal view of collection optics.

The collection optics was designed based on a ray tracing calculation (Fig. 3.20). The incident scattered light into a fiber has an angular distribution. The designed value of distribution is plotted in Fig. 3.21. It is seen that the distribution at measurement point looking at plasma edge is narrower, because a part of photons are blocked by the vessel wall. In addition, a part of distribution is diminished because they are blocked by the TF coil. The measured scattering angles approximately fit to the averaged designed values. The specifications of fiber (APC2000/2150, Fiberguide) is that

Figure 3.19: Photograph showing the procedure of scattering angle measurement.

N.A. = 0.37, core diameter is 2000 μm, clad diameter is 2150 μm and refractive index of the core is $n = 1.449$ (quart). Incident light over a critical angle can not travel through the fiber. The critical angle is expressed by N.A., which is written as

$$\text{N.A.} = n \tan\theta, \tag{3.3}$$

where n is the refractive index of fiber core. θ is the angle shown in Fig. 3.18. Figure 3.22 shows the distribution of photons depends on incident angle into fibers. The red lines correspond the value of N.A. Most photons distribute inside the value of N.A., however, a part of them are injected outside the critical angle. The transimission efficiency of fiber over the critical angle strongly depends on wavelength. Therefore, it is important to calibrate the wavelength dependence of sensitivity for this optics.

The designed value of $L\Omega$ is plotted in Fig. 3.23, where L is the scattering length and Ω is the solid angle of measured scattered light. On the other hand, Fig. 3.24 shows the relative sensitivity measured by Raman scattering calibration described in Sec. 3.3. The trend of the relative sensitivity of second-pass is similar to the design. On the other hand, the trend of first-pass looks somewhat different. It seems that the line of sight sees the laser path of second-pass totally, but a part of first-pass outside. The sensitivity for the measurement point $R = 260$ mm is low, since the fiber for the point is cracking.

The laser path overlaps with the path of visible laser. In order to identify the location of measurement point, we use a movable ceramic plate. The ceramic plate inserted on a measurement point, and the visible laser is reflected by the plate. The fiber is fixed the location where the reflected visible

Table 3: Scattering angles of each measurement point for first- and second-pass.

Major Radius [mm]	First-pass [degree]	Second-pass [degree]
197	140.8	38.5
220	138.9	40.1
260	136.7	44.0
291	133.8	46.2
319	131.8	48.2
342	128.7	50.8
366	125.9	53.0
389	124.7	55.1
412	122.0	57.6
434	120.0	60.5
459	116.7	62.5
481	112.8	66.5
503	109.0	70.4
526	105.3	75.2
546	101.1	79.1
571	No Data	85.1

laser is focused on. The effect of alignment of the fiber location was measured (Fig. 3.25 and 3.26). The variation of Thomson scattering signal was plotted in Fig. 3.25 with scanning the location of fiber vertically by 1 mm. Each measurement was performed with plasmas of good reproducibility. Since the angular dependence of intensity of Thomson scattered light can be written as

$$\left(\frac{dI}{d\Omega}\right)_{\text{Thomson}} = \frac{3}{8\pi}\sin^2\beta, \tag{3.4}$$

the relationship is also plotted. It is seen that the location should be fixed to an accuracy of 0.1 mm for both of first- and second-pass vertically. Also, absolute sensitivity dependence on the fiber location in the direction parallel to the scattered light was measured (Fig. 3.26). As a light source of constant intensity, Raman scattered light was used. It is seen that the location should be aligned to an accuracy of 1 mm in the direction parallel to the injection light for both of first- and second-pass.

Figure 3.20: Ray tracing result for collection optics.

Figure 3.21: Calculated distribution of photons vs. scattering angle. $R = 190$ means the plot is calculated for the measurement point $R = 190$ mm, where R is major radius.

Figure 3.22: Calculated distribution of photons vs. incident angle to fiber. Red lines indicate the fiber N.A. = 0.37. R = 190 means the plot is calculated for the measurement point R = 190 mm, where R is major radius.

Figure 3.23: Design value of $L\Omega$.

Figure 3.24: Relative sensitivity of the optics on each measurement point. Left for first-pass and Right for second-pass. The fiber for the measurement point R = 260 mm is cracking.

Figure 3.25: Thomson scattering signal with scanning position of fiber in the direction perpendicular to the injection light.

Figure 3.26: Raman scattering signal with scanning position of fiber in the direction parallel to the injection light.

3.1.4 Polychromator and Data Acquisition

Thomson scattered light is detected by a polychromator, which is a multi-wavelength spectroscope. Figs. 3.27 and 3.28 show the appearance of polychromator. A polychromator consists of several components, relay lenses, interference filters, avalanche photodiodes (APDs) and signal amplifier circuits. The process of scattered light traveling in polychromator is as follows. Thomson scattered light coming from optical fiber expands on the surface of the fiber. The expanding light goes to parallel propagating light after a relay lens and reaches an interference filter. Interference filter transmits only in a certain range of wavelength and reflects the other. Transmitted light is focused on a detector, APD. The signal from APD is amplified and digitized by a oscilloscope. On the other hand, the reflected light goes to next channel.

Six channels are installed on each polychromator. Measurement wavelength range of each channel is different according to the pass-band of interference filters. There are two types of interference filter set. The pass-band of each filter set are shown in Fig. 3.29. In the one of set, all pass-band is shorter than the wavelength of YAG laser (the left in Fig. 3.29). In the other set, a pass-band of a filter is longer than the wavelength of YAG laser (Fig. 3.29). As shown in Sec. 4, we propose the shifted Maxwellian for the velocity distribution function. The accuracy to measure the wavelength shift is highly improved when there are pass-bands at both side of longer and shorter wavelengths. Therefore, the filter set shown in Fig. 3.29 is used in the all experiment in this thesis.

Figure 3.27: Photograph of inner structure of a polychromator.

The detection unit shown in Fig. 3.28 consists of APD and amplifier circuit. The unit is appropri-

Figure 3.28: Schematic of inner structure of a polychromator. Transferred Thomson scattering light from a fiber is expanded by a relay lens in front of the fiber and focused on each detection unit. Spectrum is divided into several wavelength ranges according to the pass-band of interference filters.

ately aligned on a location where the injected light is focused on. The alignment is performed with a visible laser by injecting from the edge region of N.A. A simplified schematic of the circuit is shown in Fig. 3.30. It is necessary to separate the signal pulses of first- and second-pass in the double-pass diagnostic. Therefore, time resolution should be much higher (e.g. less than 20 ns. In order to satisfy the demand, the circuit was appropriately designed [38]. Moreover, APD with high response was chosen (Si APD S8890-30, Hamamatsu Photonics). This made much higher time resolution as shown in Fig. 3.31. Typical waveform of the double-pass signal is shown in Fig. 3.32. The pulses of first- and second-pass are separated successfully.

The signals from APDs are recorded by an oscilloscope. We use one with the specifications: 1 GHz sampling rate, 500 MHz band-width.

Figure 3.29: Two types of the interference filter sets. Thomson scattering spectra, of which temperature is $T_e = 100$ eV and $T_e = 110$ eV in the left figure and $T_e = 100$ eV with wavelength shift $\Delta\lambda = 0$ and 1 nm are also plotted. In this thesis, only the right side of filter set was used.

Figure 3.30: Circuit diagram of the detection unit.

Figure 3.31: Comparison between a Si PIN PD and the APD circuit, which is installed in the polychromator, detecting a laser pulse. The pulse width of the laser is less than 10 ns [38].

Figure 3.32: Typical raw data (black line) and fitting waveform of TS signal (red line).

3.2 Data Analysis

Figure 3.32 presents a typical waveforms of the double-pass Thomson scattering signal. Since the two peaks arising from the first and the second Thomson scattering overlaps in the time domain, it is necessary to separate the two scattering components. We resolve each magnitude of the two scattering signals by means of fitting to a template waveform obtained from pure single-pass Thomson scatterings. In order to avoid effects of noise, 28 single-pass waveforms were averaged (Fig. 3.33). We assume that the waveform of all wavelength channels are same, and channel 5 signal of which noise level is the lowest is used to make the template waveform. The functional expression $f(t)$ for this template waveform is,

$$f(t) = A\exp\left(-\frac{(t-t_1)^2}{2\tau_1^2}\right)\exp\left(-\frac{t-t_2}{\tau_2}\right)\left(\frac{1+\tanh(t-t_3)/\tau_3}{2}\right) + C. \tag{3.5}$$

This expression $f(t)$ is the product of Gaussian, general response of circuit $\exp(-(t-t_2)/\tau_2)$ and step function-like hyperbolic tangent. A and C are constants specifying the magnitude of signal and offset, τ_1, τ_2 and τ_3 are time constants, t_1, t_2 and t_3 are constant specifying the timings of rise and fall of signal. These constants were determined by means of fitting to the template data (Fig. 3.33). The parameters were determined as Table 4. In Fig. 3.33, red curve represents the waveform of the functional expression. The time-integrated functional expression agrees with time-integrated template waveform within 0.03% error. We use this functional expression for estimating magnitudes (i.e., time integral of signal) of double-pass Thomson scattering signal.

By using this expression $f(t)$, observed double-pass waveform $y(t)$ (with two peaks) can be expressed

Table 4: Constant values of the function $f(t)$.

A	t_1	τ_1	t_2	τ_2	t_3	τ_3	C
0.786	42.91	16.27	46.30	17.10	0.215	53.60	0.0596

as

$$y(t) = af(t-t_0) + bf(t-t_0-t_{\mathrm{delay}}) + c. \tag{3.6}$$

a and b are parameters specifying the magnitude of the first and the second scattering, respectively. c is a parameter specifying background light from the plasma, t_0 is a parameter specifying the timing of peak of the first scattering and t_{delay} is a constant specifying time delay between peaks of first- and second-pass. Figure 3.32 shows examples of the fitting to the actual double-pass waveforms. Two

Figure 3.33: The template waveform (black line) and fitting function (red line).

peaks correspond to the first- and second-pass.

3.3 Absolute Sensitivity Calibration

Figure 3.34 shows an example of the raw data of Raman scattering at the nitrogen pressure $p(N_2)$ = 100 Torr. The signals are averaged for 1024 shots of laser injection. The Raman scattering signal is much larger than that of stray light (more than 10 times) at channels 1, 2, 3 and 6 of polychromator. Especially, noise level on channel 1 is the lowest. Although the signal intensity increases when the pressure of nitrogen is increased, the frequency of electrical discharge at the focus point of laser. The relationship between the pressure of nitrogen and the signal intensity of Raman scattering.

In addition, calibration by channel 1 is most stable compared to other channels. Figure 3.36 shows a calculated variation of Raman scattering intensity when the wavelength of the filter sensitivity varies $\Delta\lambda$. The distribution of Raman scattering cross section is discrete. Thus, result of calibration can vary largely if the wavelength of sensitivity is different from the true value. From Fig. 3.36, the variation of channel 1 is the smallest. For these reasons, we used the Raman calibration data with channel 1. The calibration result is shown in Fig. 3.24. The result of reproducibility test of Raman calibration is shown in Fig. 3.37. The reproducibility is quite good.

Figure 3.34: Typical waveform of RRS signal at N_2 pressure of 100 Torr.

Figure 3.35: S_R/σ_R vs. N_2 pressure (black symbols). Red line represents the results of line fitting.

Figure 3.36: Change rate of Raman scattering signal when the standard of wavelength is shifted by $\Delta\lambda$.

3.4 Relative Sensitivity Calibration

By the Thomson scattering measurement, a part of plasma velocity distribution function is measured with several number of wavelength channels. In order to determine temperature and current from Thomson scattering signal, it is important to calibrate relative sensitivities of each channel, because the form of velocity distribution function is estimated by comparing relative intensities of each wavelength channel. Although the measuring instruments are carefully designed symmetric, sensitivities of each channel can be significantly different due to character of each detector (APD) or environmental effect, such as daily changes of environmental temperature. Therefore, it is necessary to calibrate relative sensitivities precisely. In the Thomson system on TST-2, Two kinds of calibration were performed. In this section, details of these calibration is described.

3.4.1 Calibration Procedure

In this section, the procedure of relative calibration is described. On the TST-2 Thomson system, the procedure of relative calibration is separated two steps. In the first step, sensitivity dependence of each channel on wavelength is scanned with a light source and a monochromator on a optics different

Figure 3.37: Reproducibility test for Raman calibration. The horizontal axis denotes number of trials (1-14).

from the optics for Thomson scattering measurement. Figure 3.38 shows the outline of the first step calibration. Monochromatic light generated by a light source and a monochrometor is injected into a polychromator, and output signal is recorded. This process repeated with scanning the wavelength of monochromatic light. The spectrum of the light source is measured beforehand (Fig. 3.39). In this process, sensitivity dependences on wavelength of each channel of polychromator is calibrated like Fig. 3.40. Note that optics and environment of this calibration is different from those of Thomson scattering measurement.

Figure 3.38: Schematic diagram of the first step calibration of polychromator.

Figure 3.39: Intensity of the standard light measured by a power meter located at the exit slit of a monochromator.

Figure 3.40: Spectral responsivity of the channels 1-6 in the polychromator.

The second step of calibration is performed in order to calibrate relative sensitivity with same condition of Thomson scattering measurement. Since the shape of sensitivity dependence on wavelength hardly affect on diagnostics, we should consider only the relative value of integrated sensitivity on all range of sensitivity. Figure 3.41 shows the outline of the second step relative calibration. In order to use same optics as the Thomson scattering measurement, imitation of scattering right is injected into the polychromator through the same path as the Thomson scattering light. The imitation light is generated by a light source (Labsphere, Flux Standard, SCL-1400) and white diffuser panel. Any light injected into the diffuser panel diffuses uniformly in all directions. The spectrum of the light source is measured beforehand (Fig. 3.42).

Figure 3.41: Schematic diagram of the second step calibration of polychromator.

Figure 3.42: Power flux of standard light for relative calibration.

Figure 3.43: Time evolution of standard light intensity. (i) Immediately after turning on the power of standard light. (ii) After (i), stopping off the current of standard light one hour, and starting again. (iii) After (ii), turning off the power of standard light, and turning on again. Blue, red and green lines indicate the wavelength of 920, 1000 and 1064 nm.

3.4.2 Sensitivity Dependence on Room Temperature and Reverse Bias Voltage

The sensitivity of detection device to measure Thomson scattering light, APD, depends on environmental temperature and bias voltage. If the condition evolves during experiment or changes depending on season, that causes systematic error. The sensitivity dependence on environmental temperature and bias voltage was measured. In order to scan the dependence, the calibration method of second step described in Sec. 3.4.1 was used. That is, the dependence was measured by scanning environmental temperature and bias voltage and recording the output voltage with injection of continuous standard light into the polychromator. The results are shown in Figs. 3.44 and 3.45. With the change of environmental temperature about 6 °C or bias voltage of about 15 V, the sensitivities linearly depend on these parameters in all wavelength channels. Typical evolution of condition do not deviate this range in a day of experiment. These measurement shows that it is enough to calibrate in this process before an experiment once, and to monitor the environmental temperature and bias voltage. If these parameters evolves largely, it is needed to calibrate again.

Figure 3.44: Relative sensitivity of polychromator vs. room temperature. The sensitivities are normalized by the sensitivity at 31.6 °C of room temperature.

Figure 3.45: Relative sensitivity of polychromator vs. bias voltage. The sensitivities are normalized by the sensitivity at 440 V of bias voltage at which the multiplication factor of APD is about 100 for this polychromator (# 2).

3.4.3 Reduction of Sensitivity Caused by Continuous Light

The sensitivity of APD depends on reverse bias voltage on the APD, shown as Fig. 3.45 . If some continuous light injects to the APD, the sensitivity should be decreased, since the reverse voltage is decreased slightly. This effect is discussed in this section.

After photons injects to an APD, current flows in the circuit of polychromator amplified by a gain of APD G_{APD}. Because the gain G_{APD} depends on the reverse voltage on the APD V_{APD}, the current flows from the APD I_{APD} is written as

$$I_{\mathrm{APD}} = I_{\mathrm{photon}} G_{\mathrm{APD}}\left(V_{\mathrm{APD}}\right). \tag{3.7}$$

From Fig. 3.30, the reverse voltage V_{APD} is

$$V_{\mathrm{APD}} = V_{\mathrm{HV}} - R_{\mathrm{HV}} I_{\mathrm{APD}}. \tag{3.8}$$

That is, the reverse voltage is decreased by $R_{\mathrm{HV}} I_{\mathrm{APD}}$ from the original voltage V_{HV}. We consider the decrease of the sensitivity. Neglecting less than second order terms, the gain G_{APD} is expressed as

$$G_{\mathrm{APD}}\left(V_{\mathrm{APD}}\right) \simeq G_{\mathrm{APD}}\left(V_{\mathrm{HV}}\right) + \frac{\mathrm{d}G_{\mathrm{APD}}(V_{\mathrm{HV}})}{\mathrm{d}V}(-R_{\mathrm{HV}} I_{\mathrm{APD}}), \tag{3.9}$$

where V_{APD} denotes the actual reverse voltage. Figure 3.45 shows the dependence of the gain of APD on reverse voltage is about

$$\frac{\mathrm{d}G_{\mathrm{APD}}}{\mathrm{d}V} \simeq \frac{0.01 G_{\mathrm{APD}}\left(V_{\mathrm{HV}}\right)}{1[\mathrm{V}]}. \tag{3.10}$$

Therefore, Eq. (3.7) is written as

$$I_{\mathrm{APD}} \simeq I_{\mathrm{photon}} G_{\mathrm{APD}}(V_{\mathrm{HV}})\left(1 - 0.01 R_{\mathrm{HV}} I_{\mathrm{APD}}\right). \tag{3.11}$$

Because I_{APD} is amplified by a two stage amplifier (see Fig. 3.30), the output voltage V_{out} detected by a oscilloscope is given as

$$V_{\mathrm{out}} = G_{\mathrm{amp}} I_{\mathrm{APD}} R_0 \frac{R_{\mathrm{osc}}}{R_{\mathrm{osc}} + R_1}. \tag{3.12}$$

From Eqs. (3.11) and (3.12), true output voltage $V_{\mathrm{true}} = I_{\mathrm{photon}} G_{\mathrm{APD}}(V_{\mathrm{HV}}) R_0 G_{\mathrm{amp}} R_{\mathrm{osc}}/(R_{\mathrm{osc}} + R_1))$ is written by V_{out} as

$$V_{\text{true}} \simeq V_{\text{out}} \left(1 + 0.01 \frac{R_{\text{HV}}}{G_{\text{amp}} R_0} \frac{R_{\text{osc}} + R_1}{R_{\text{osc}}} V_{\text{out}} \right). \tag{3.13}$$

If chopping the continuous light, the current I_{APD} is half compared to no chopping case. Then the expression modified as

$$V_{\text{true,chopper}} \simeq V_{\text{out}} \left(1 + 0.005 \frac{R_{\text{HV}}}{G_{\text{amp}} R_0} \frac{R_{\text{osc}} + R_1}{R_{\text{osc}}} V_{\text{out}} \right). \tag{3.14}$$

Note that if the charge stored in the capacitor C_0 (see Fig. 3.30) remains enough, sensitivity of the APD does not decrease. In order to estimate timescale of the charge loss, we assume $V_{\text{HV}} = 600$ V, $I_{\text{APD}} = 1\ \mu$A as typical value. The time during which the charge in the capacitor losses 1% Δt is written as

$$\Delta t = C_0 V_{\text{HV}} / I_{\text{APD}} / 100 = 0.6 \text{ s}. \tag{3.15}$$

Therefore, radiation from Ohmic heated plasma does not cause for sensitivity reduction, because the duration of discharge is less than 40 ms.

3.4.4 Reproducibility of Calibration

The reproducibility of the relative calibration of second step (the calibration with collection optics for Thomson scattering measurement) was checked. We define the sensitivity of i-th channel is f_i. The calibration was performed 5 times. The average and standard deviation of f_i are defined as μ_i and σ_i. Then, the relative sensitivity of channel i and 1 is written as

$$\frac{f_i}{f_1} = \frac{f_{i,0} + \sigma_i}{f_{1,0} + \sigma_1} \simeq \frac{f_{i,0}}{f_{1,0}} \left(1 - \frac{\sigma_1}{f_1} + \frac{\sigma_{\text{i}}}{f_{\text{i}}} \right). \tag{3.16}$$

From the error propagation rule,

$$\left. \frac{f_i}{f_1} \right|_{\text{true}} = \frac{f_i}{f_1} \left(1 + \sqrt{\left(\frac{\sigma_1}{f_1} \right)^2 + \left(\frac{\sigma_i}{f_i} \right)^2} \right) = \frac{f_i}{f_1} \left(1 + \delta_{\text{i}} \right), \tag{3.17}$$

where

$$\sigma_{\text{i}} = \sqrt{\frac{1}{N-1} \sum_i (f_{\text{i}} - \mu_{\text{i}})^2}. \tag{3.18}$$

δ_i is the error of relative calibration. The results are summarized in Table 5 and Fig. 3.46. As shown in the table and figure, reproducibility is quite good because $\delta < 1\%$ for all channels.

Table 5: Reproducibility of relative sensitivity calibration for polychromator #2.

	ch1	ch2	ch3	ch4	ch5	ch6
μ_i	5.0828	4.0336	5.2488	24.9556	44.7918	5.2614
σ_i	0.007362	0.006189	0.008643	0.056474	0.168693	0.014741
δ_i [%]	0.204839	0.210997	0.219303	0.268682	0.403507	0.3154

Figure 3.46: Reproducibility of relative sensitivity calibration for polychromator #2. f_i and μ_i indicate each and mean output value of i-th channel, respectively. The calibration was performed 5 times (1st-5th).

3.4.5 Effect of Refraction in Nitrogen Gas

The windows of the inject and exit port are quartz which is composed of pure SiO_2. Laser beam drifts because of refraction at the window. Usually, inside of the device is maintained as a vacuum on laser path. In the process of Raman or Rayleigh scattering calibration, inside of the device is filled with nitrogen gas. Therefore, the laser path during Raman or Rayleigh calibration should move over a little from usual laser path because of the refractive index of nitrogen gas. The drift of laser path is evaluated as follows.

Refractive index of gas depends on the pressure of gas, environmental temperature and wavelength of injected laser. That is expressed as Edlén equation like

$$(n-1)_{tp} = \frac{(p/\mathrm{Torr})\,(n-1)_s}{720.775}\,\frac{\left[1+(p/\mathrm{Torr})\,(0.817-0.0133t/\,^\circ\mathrm{C})\times 10^{-6}\right]}{(1+0.0036610t/\,^\circ\mathrm{C})}, \tag{3.19}$$

and

$$(n-1)_s \times 10^8 = 8342.13 + 2406030\left[130-\left(\sigma/\mu\mathrm{m}^{-1}\right)^2\right]^{-1} + 15997\left[38.9-\left(\sigma/\mu\mathrm{m}^{-1}\right)^2\right]^{-1}, \tag{3.20}$$

where p is gas pressure, t is room temperature, σ is wavelength of laser, n_s is refractive index of air and n is refractive index of the filled gas. Refractive index of the window (quartz) is 1.449.

Figure 3.47: Displacement of laser path when the nitrogen pressure in the vacuum vessel is increased. Top: Laser path (300Torr). Bottom: The displacement (shift) of laser path at the location of 5 m from the Brewster window.

3.4.6 Effect of Polarization

Polarization of Raman scattering light is, in contrast to the case of Thomson and Rayleigh scattering, different from the polarization of incident laser. Eq. (2.57) includes this effect as depolarization ρ. However, if sensitivity of the measurement system depends on polarization, effective depolarization ρ_{eff} must be modified from ideal value of $\rho = 0.75$ in nitrogen gas.

The modified ρ_{eff} was measured by using a polarizer (Fig. 3.49). A polarizer was placed facing to the optical fiber. Raman scattering signal was measured with scanning the angle of polarizer. Transmissivity and extinction ratio of the polarizer are 66.4 % and 8.5%, respectively. The angular dependence of Raman cross section is expressed as

$$\sigma_{\mathrm{measured}} = \sigma_{\mathrm{Raman}}[(1-\rho_{\mathrm{eff}})\cos^2\psi + \rho_{\mathrm{eff}}], \tag{3.21}$$

where σ_{Raman} $\sigma_{\mathrm{measured}}$ and are ideal and effective cross section of Raman scattering, respectively. ψ is defined in Fig. 3.48. By fitting to Eq. (3.21), the effective depolarization $\rho_{\mathrm{eff}} = 0.69$.

Figure 3.48: Definition of angles to describe depolarization. ϵ_1, ϵ_2 denote polarization direction of incident laser and polarization of Raman scattered light, respectively. Axis z is parallel to ϵ_1 and axis x is perpendicular to axis z and in the plane of determined by ϵ_1 and ϵ_2. ψ is the angle between ϵ_1 and ϵ_2.

Figure 3.49: Polarization dependence of Raman scattering.

3.5 Estimation of Errors

There are several kinds of noise source. Estimation of errors is important to predict final measurement error Classification and quantitative estimation are described in this section.

3.5.1 Thermal Noise

Brownian motion of free electrons in a resistor results in thermal noise. Amplitude of this noise depends on temperature of resistor as below,

$$\delta V_{\text{thermal}} = \sqrt{4k_{\text{B}}TR\Delta f}, \tag{3.22}$$

where V_{thermal} is detected voltage caused by thermal noise, $k_{\text{B}} = 1.38 \times 10^{-23}$ m^{2}kgs^{-2}K^{-1} is the Boltzmann constant, T is temperature of resistor, R is resistant value of resistor and Δf is bandwidth. In our measurement system, main source of thermal noise arises at the resistor $R_0 = 470\ \Omega$ in the Fig. 3.30. The thermal noise is amplified by a factor of $G \simeq 104$, the gain of the amplifier. Therefore, the output fluctuation voltage is

$$\delta V_{\text{out,thermal}} = G_{\text{amp}} \times \delta V_{\text{thermal}} \simeq 6\ \text{mV}, \tag{3.23}$$

where we assume $T = 300$ K and $\Delta f = 500$ MHz. Figure 3.50 shows a sample waveform of thermal noise only. And Fig. 3.51 is the mean and standard deviation of thermal noise only. These values calculated by getting 1024 times. It is clear that the calculation above ($\delta V_{\text{out,thermal}} \simeq 6$ mV) is good estimation by comparing with the standard deviation in Fig. 3.51.

3.5.2 Shot Noise

In the case of Thomson scattering photon counting, number of trials is quite large and probability is quite low. In such case, the number of photon follows Poisson distribution. Probability that event occurs ν times in Poisson distribution is written as

$$P_{\mu}(\nu) = \mathrm{e}^{-\mu}\frac{\mu^{\nu}}{\nu!}, \tag{3.24}$$

where μ is expected mean value. The standard deviation σ^2 is equal to μ in Poisson distribution. Therefore, statistical error of photon counting is written as

Figure 3.50: Sample signal of thermal noise.

Figure 3.51: Mean (black) and standard deviation (red) of thermal noise for 1024 count signals.

$$\sigma_{\text{shotnoise}} = \sqrt{\mu} = \sqrt{\eta' \bar{N}}, \tag{3.25}$$

where η' is the conversion efficiency from photon number to signal level, and $\bar{N}$ is averaged detected photon number. This statistical error is called as shot noise. The comparison with actual experimental data is performed in Sec. 3.5.7.

3.5.3 Conversion Efficiency from Detected Photon Number to Signal

It is convenient to calculate the conversion efficiency from detected photon number to signal for estimation of statistical error such as shot noise. As shown in Fig. 3.30, N photons detected in APD is amplified by the multiplication factor $M \simeq 100$ and converted to current $i = eMN/\Delta t$. The voltage $v = iR_0$ is amplified by the amplifier $G \simeq 104$. The time integrated signal S_{rmout} is the final output signal from one pulse of Thomson scattering. That is

$$S_{\text{out}} = eMNR_0G \simeq N \times 7.8 \times 10^{-4} \text{ nVs}. \tag{3.26}$$

Therefore the conversion efficiency from detected photon number to signal $\eta = S_{\text{out}}/N \simeq 7.8 \times 10^{-4}$.

3.5.4 Fluctuation of Stray Light

The stray light is measured every time when the TS measurement is carried out. The averaged stray light is extracted from the TS signal to obtain the net TS signal in the following way. Stray signals for channels of polychromator were measured without plasma discharge. The stray signals are averaged for 32 shots of laser injection. The averaged stray signals are extracted from the TS signals measured with plasma discharge. The timing of stray signals of pure stray measurements and stray signals of TS signals with plasma discharge are synchronized by a laser monitor. The laser monitor consists of Si photodiode and detects the Nd:YAG laser split from laser path for TS measurement.

3.5.5 Fluctuation of Plasma Light

Plasma radiates over wide range of wavelength because there are charged particles with wide range of energy. The radiation is also detected by polychromator. In most cases, the time-scale of the evolution of the intensity, such as caused by MHD instability, is longer than the timescale of Thomson

Figure 3.52: Stray signal (black), shot noise of stray light (red error bar) and thermal noise (blue lines). Ratio of shot noise of stray light and thermal noise ($\sigma_{\text{stray}}/\sigma_{\text{thermal}}$ green) is also plotted. Right figure is enlarged view of the left figure.

Figure 3.53: Weak stray light case. Color of lines are same as Fig. 3.52

scattering measurement (e.g., $\Delta t < 100$ ns as shown in Fig. 3.54. However, Poisson noise of plasma light can be a problem when the signal intensity is high.

Figure 3.54: Signal of polychromator channels 1-6. The vertical dotted line indicates timing of laser injection. Large peak around 0.52 ms is caused by IRE event. The thin peaks around 0.46 ms are caused by X-ray.

3.5.6 Fluctuation of Laser Power

Fluctuation level of incident laser power and its effect to signal intensity are evaluated by using the signal of Raman scattering. Multiple laser pulses are injected into constant pressure nitrogen gas.

Laser monitor shown in Fig. 3.9 was used as a monitor of incident laser power. Figure 3.55 shows the averaged laser monitor signal over 256 laser injection. The time-integrated signal for each injection of 256 times incident laser is shown in Fig. 3.56. The standard deviation $\sigma \simeq 1.7$ %, where σ is written as

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i}^{N}\frac{(y_i-\bar{y})^2}{\bar{y}^2}}. \tag{3.27}$$

Furthermore, the correlation between the laser power above and the Raman scattering signal in that time is shown in Fig. 3.57. The correlation $r \simeq 1.6$ %, where r is written as

$$r = \frac{\sum_i^N (x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_i^N (x_i-\bar{x})^2}\sqrt{\sum_i^N (y_i-\bar{y})^2}} \tag{3.28}$$

Therefore, it is concluded that the fluctuation level of laser power is enough small and the correlation with scattering intensity is also enough small. The fluctuation of scattering intensity is caused other reason, such as fluctuation of nitrogen density.

Figure 3.55: A waveform of laser monitor signal. Power of Nd:YAG laser pulse is monitored by a photodiode.

Figure 3.56: Fluctuation of laser power. S_{out} is time-integrated value of laser monitor output.

Figure 3.57: Correlation between laser power and Raman scattering signal intensity.

3.5.7 Total Error

We can express resultant error from square-root of sum of squares of above-mentioned errors. That is

$$\sigma_{\text{total}} = \sqrt{\sigma^2_{\text{thermal}} + \sigma^2_{\text{strshot}} + \sigma^2_{\text{TSshot}}}, \tag{3.29}$$

where σ_{total} is resultant error, and σ_{thermal}, σ_{strshot} and σ_{TSshot} are errors caused by thermal noise, shot noise of stray light and shot noise of Thomson scattering light, respectively. From Eqs. (3.22) and (3.25), resultant error is expressed as

$$\sigma_{\text{total}} = \sqrt{\eta' \bar{N} + \alpha'^2} = \sqrt{\eta\mu + \alpha^2}, \tag{3.30}$$

where $\bar{N}$ is mean value of detected photon. η is a constant relates to conversion efficiency from photon to signal intensity. α is also constant relates to thermal noise.

The constants η and α were measured on double-pass Thomson scattering measurement system. We used Raman scattering as a constant signal source. As long as the pressure of nitrogen is constant, the mean signal from Raman scattering is constant. Therefore, we can measure the error σ_{total} and determine the η and α by fitting the σ_{total} to the model equation (3.30). We used polychromators #2, #9 and #12, and data with nitrogen pressure of $25, 50, 75, 100$ Torr. In each pressure, Raman scattering signals were measured 1024 times. Then mean μ and standard deviation σ are calculated from the data. The relations between σ/μ and μ are plotted in Fig. 3.58 for each polychromators. Because the amplifier gain of polychromator #12 is 1.77 times larger than that of others, μ for polychromator #12 is divided by 1.77 in this plot. 4 datum of same symbol and same color correspond to 4 kind of nitrogen pressure (μ is proportional to the nitrogen pressure). And calculated α and η are summarized for each channel of polychromator #9 in the table 6. As expected, α of channel 6 is larger because intense stray light is detected at this channel.

The effect of thermal noise α is smaller than value estimated in Eq. (3.23). It is caused from fitting to the waveform of Thomson scattering signal (see Sec. 3.2). Fitting reduces the effect of random fluctuation.

Figure 3.58: Mean vs. relative error of 1024 times Raman scattering signals (double logarithmic plot). μ and σ represent mean and standard deviation. Diamond($\diamond$), square($\square$), triangle($\triangle$) and cross($\times$) represent CH1, 2, 3, 6 of polychromator, respectively. Black, red and blue lines represent index of polychromator #2, #9, #12, respectively.

Table 6: Summary of η and α for polychromator #9

poly#9	ch1	ch2	ch3	ch6
$\eta \times 10^3$	1.80	1.68	2.96	1.12
$\alpha^2 \times 10^3$	0.682	0.798	0.589	1.37

3.6 Monte Carlo Simulation

In this thesis, Monte Carlo algorithm is used to simulate measurement error. Monte Carlo algorithm is a numerical method with random number. The main noise source of Thomson scattering measurement is random noise such as shot noise and thermal noise (see Sec. 3.5.2 and Sec. 3.5.1). The signal of random noise is different on each measurement, but follows a probability distribution. In order to express the distribution, random number is used in the Monte Carlo Method.

On the double-pass Thomson scattering measurement, 12 signals are detected on each measurement. Each polychromator has 6 wavelength channels. 12 signals are consists of 6 signals by first-pass measurement and 6 signals by second-pass measurement. We label the 12 signals as

$$\boldsymbol{y} = [\boldsymbol{y}_{\mathrm{first}}, \boldsymbol{y}_{\mathrm{second}}] \tag{3.31}$$

where

$$\boldsymbol{y}_{\mathrm{first}} = [y_{\mathrm{first},1}, y_{\mathrm{first},2}, y_{\mathrm{first},3}, y_{\mathrm{first},4}, y_{\mathrm{first},5}, y_{\mathrm{first},6}] \tag{3.32}$$

$$\boldsymbol{y}_{\mathrm{second}} = [y_{\mathrm{second},1}, y_{\mathrm{second},2}, y_{\mathrm{second},3}, y_{\mathrm{second},4}, y_{\mathrm{second},5}, y_{\mathrm{second},6}] \tag{3.33}$$

Thomson scattering spectrum is a function of electron density n_{e}, electron temperature T_{e}, scattering angle θ and wavelength where spectrum is peaked λ_0, that is

$$S_{\mathrm{TS}}(\lambda) = S_{\mathrm{TS}}(n_{\mathrm{e}}, T_{\mathrm{e}}, \theta, \lambda_0; \lambda), \tag{3.34}$$

where S_{TS} indicates a Thomson scattering spectrum. The i-th $\boldsymbol{y}$ is product of the Thomson scattering spectrum S_{TS} and sensitivity of i-th channel f_{i} like as Fig. 3.40.

$$y_{\mathrm{i}} = S_{\mathrm{TS}}(\lambda) \times f_{\mathrm{i}}(\lambda). \tag{3.35}$$

In the actual measurement, the signal $\boldsymbol{y}$ fluctuates caused by random error following a normal distribution with standard deviation σ_{random}. The σ_{random} is written as Eq. (3.30).

Following is the process of the Monte Carlo simulation. Firstly, we assume true values of electron density (n_{e0}) and temperature (T_{e0}), and central wavelength (λ_0). Assuming a scattering angle θ, true

Thomson scattering spectrum $S_0(\lambda)$ is given by

$$S_0(\lambda) = S_{\mathrm{TS}}(n_{\mathrm{e0}}, T_{\mathrm{e0}}, \theta, \lambda_0; \lambda). \tag{3.36}$$

Then, true values of signal of i-th channel y_{rmi0} is specified as

$$y_{\mathrm{i0}} = S_0(\lambda) \times f_{\mathrm{i}}(\lambda). \tag{3.37}$$

Secondly, we add random noise. The dependence of noise on signal intensity was measured by using Raman scattering written as Eq. (3.30). by using this expression, signal of i-th channel $y_{\mathrm{i,trial}}$ is expressed as

$$y_{\mathrm{i,trial}} = N\left[\mu = y_{\mathrm{i0}}, \sigma = \sqrt{\eta y_{\mathrm{i0}} + \alpha^2}\right], \tag{3.38}$$

where $N[\mu, \sigma]$ is a normal distribution of which average is μ and standard deviation is σ. Fitting the signal Eq. (3.38) to Eq. (3.35), estimated values n_{es}, T_{es} and λ_{0s} are determined. We make enough number of trials to try over almost all range of normal distribution. Finally, histogram of each values of n_{es}, T_{es} and λ_{0s} are made. Fitting the histogram to a normal distribution, we can determine the measurement error, which is the standard deviation of the histogram. Figure 3.59 shows an example of the histogram.

Figure 3.59: Typical histogram of result from Monte Carlo simulation for current density measurement. Number of trials is 2000 times. Random numbers following normal distribution with standard deviation σ_{random} are generated on each trial. The histogram is fitted by a normal distribution (red curve). True wavelength shift is also plotted (blue).

4 Models of Velocity Distribution Function

In Sec. 5, experimental results of velocity distribution function measurement in an Ohmic heated plasma is described. It is necessary to build some models simulating actual velocity distribution function by using the knowledge of preceding theoretical studies and calculations. In this chapter, we propose models to simulate an Ohmic heated plasma and estimate several important plasma parameters to confirm which model is appropriate.

In the conventional Thomson scattering analysis, plasma is assumed in thermal equilibrium and the electron velocity distribution function is the Maxwell-Boltzmann distribution. In the relativistic region, the distribution function is written as

$$f(\beta) = n_{\mathrm{e}}\gamma^5 \frac{1}{2\pi}\frac{\alpha}{\mathrm{K}_2(2\alpha)}\exp(-2\alpha\gamma), \tag{4.1}$$

where $\alpha = m_0c^2/2k_{\mathrm{B}}T$, $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. K_2 is a modified Bessel function of the second order. In this thesis, temperature of target plasmas is less than 500 eV, which is non-relativistic region. Therefore, Maxwell-Boltzmann distribution function is approximated as

$$f(v) = n_{\mathrm{e}}\sqrt{\frac{m}{2\pi k_{\mathrm{B}}T}}\exp\left(-\frac{mv^2}{2k_{\mathrm{B}}T}\right) \tag{4.2}$$

However, the simple Maxwell-Boltzmann distribution function is not the case with an Ohmic heated plasma in TST-2 device as shown in section 5. There must be some temperature anisotropy in the plasmas. The distortion from Maxwell distribution is not so large because relaxation time is short (several μs) due to high collision frequency. Therefore, we approximate the actual distribution function by a slightly modified function from Maxwell distribution.

4.1 Model

The target plasma is produced in Ohmic discharge. In this section, we suppose three kinds of models to express actual plasma electron velocity distribution function for Ohmic heated plasma. The Thomson scattering spectrum is reflected only on electron thermal motion in the region of plasma parameters in the TST-2 device. Motion of ion and motion of fast electron does not affect the spectrum. Therefore, we consider only thermal electron.

Toroidal magnetic field plays an important role. Electrons are trapped and their moves are restricted within a diffusion length in the direction perpendicular to the magnetic field. On the other hand, electrons move freely along the magnetic field.

Plasma is heated by electric field induced by time evolution of current flowing in the Ohmic coil placed in the center of the torus. The electric field is applied along the toroidal direction. In addition, plasma current is driven in the toroidal direction (i.e., parallel to the magnetic field) due to the induced electric field. Therefore, there should be difference between the motion of electrons along the direction perpendicular and parallel to the magnetic field. Then, we separate the velocity distribution function $f_{\mathrm{e}}(v)$ into two directions as

$$f_{\mathrm{e}}(v_{\perp}, v_{\parallel}) = n_{\mathrm{e}} f_{\mathrm{e},\perp} f_{\mathrm{e},\parallel}, \tag{4.3}$$

where $f_{\mathrm{e},\perp}$ and $f_{\mathrm{e},\parallel}$ represent perpendicular and parallel components of velocity distribution function.

Typical electron temperature and density of plasma are about $T_{\mathrm{e}} \simeq 100$ eV and $n_{\mathrm{e}} \simeq 10^{19}$ m^{-3}. From the Eq. (1.6), collision time of electrons is about several μs. Therefore, distorted distribution function is relaxed to Maxwell distribution in several μs. Also, relaxation between velocity distributions along perpendicular and parallel direction is achieved within several μs.

Since no external force is applied to plasma along perpendicular direction, the distribution along perpendicular direction can be represented as Maxwell distribution. Therefore, the distribution function $f_{\mathrm{e},\perp}$ is written as

$$f_{\mathrm{e},\perp}(v_{\perp}) = \frac{m}{2\pi k T_{\mathrm{e},\perp}} \exp\left(-\frac{m v_{\perp}^2}{2k T_{\mathrm{e},\perp}}\right). \tag{4.4}$$

In the parallel direction, we should consider the effect of electric field. The first moment of velocity distribution corresponds to the plasma current. Note that the first moment for thermal electrons is not necessarily equal to torodal current, because fast electron can contribute to the current. In this thesis, we propose three kinds of models for the parallel distribution function.

The first model is a distorted form from the normal Maxwellian, which is caused by acceleration of electric field. In this case, shape of the distribution function over $v_{\parallel} \geq 0$ is different from that over $v_{\parallel} < 0$. In order to simplify the expression, we approximate the distribution by combining two Maxwellians with temperatures T_{co} and T_{ctr}. Here, T_{co} and T_{ctr} represent the co-directed (i.e., the direction of electron acceleration) and counter-directed (i.e., the opposite direction of electron

acceleration) parallel temperatures, respectively. Then, the distribution function is expressed as

$$f_{\mathrm{e},\parallel}(v_\parallel \geq 0) = \sqrt{\frac{m}{2\pi k T_{\mathrm{e,co}}}} \exp\left(-\frac{m}{2kT_{\mathrm{e,co}}} v_\parallel^2\right), \tag{4.5}$$

$$f_{\mathrm{e},\parallel}(v_\parallel < 0) = \sqrt{\frac{m}{2\pi k T_{\mathrm{e,ctr}}}} \exp\left(-\frac{m}{2kT_{\mathrm{e,ctr}}} v_\parallel^2\right). \tag{4.6}$$

In this model, the drift velocity is assumed as $v_{\mathrm{d}} = 0$. Then, current density is calculated as

$$j_{\mathrm{e}} = \int_{-c}^{c} e n_{\mathrm{e}} f(v_\parallel) v_\parallel dv_\parallel \tag{4.7}$$

$$= e n_{\mathrm{e}} \left(\sqrt{\frac{kT_{\mathrm{e,co}}}{2\pi m}} - \sqrt{\frac{kT_{\mathrm{e,ctr}}}{2\pi m}} \right). \tag{4.8}$$

We call this model as ”three temperature model”.

The second model is the shifted-Maxwellian model. The shape is the Maxwell-Boltzmann distribution but shifted by the drift velocity v_{d} in the parallel direction. In this case, the distribution function is written as

$$f_{\mathrm{e},\parallel}(v_\parallel) = \sqrt{\frac{m}{2\pi k T_{\mathrm{e}}}} \exp\left[-\frac{m}{2kT}(v_\parallel - v_{\mathrm{d}})^2\right]. \tag{4.9}$$

Here, we assume $T = T_\perp = T_\parallel$ and $T_{\mathrm{e,co}} = T_{\mathrm{e,ctr}}$. Then, current density is written as

$$j_{\mathrm{e}} = \int_{-c}^{c} e n_{\mathrm{e}} f(v_\parallel) v_\parallel dv_\parallel \simeq e n_{\mathrm{e}} v_{\mathrm{d}}. \tag{4.10}$$

We call this model as ”shifted model”.

The third model is the combination of the distortion and the shift described above. That is

$$f_{\mathrm{e},\parallel}(v_\parallel \geq 0) = \sqrt{\frac{m}{2\pi k T_{\mathrm{e},+}}} \exp\left(-\frac{m}{2kT_{\mathrm{e},+}}(v_\parallel - v_{\mathrm{d}})^2\right), \tag{4.11}$$

$$f_{\mathrm{e},\parallel}(v_\parallel < 0) = \sqrt{\frac{m}{2\pi k T_{\mathrm{e},-}}} \exp\left(-\frac{m}{2kT_{\mathrm{e},-}}(v_\parallel - v_{\mathrm{d}})^2\right). \tag{4.12}$$

In this case, the current density is written as

$$j_{\mathrm{e}} = \int_{-c}^{c} e n_{\mathrm{e}} f(v_\parallel) v_\parallel dv_\parallel \tag{4.13}$$

$$\simeq e n_{\mathrm{e}} \left(v_{\mathrm{d}} + \sqrt{\frac{kT_{\mathrm{e},+}}{2\pi m}} - \sqrt{\frac{kT_{\mathrm{e},-}}{2\pi m}} \right). \tag{4.14}$$

Figure 4.1: Conceptual sketches of the velocity distribution function in the three temperature model and the shifted model.

We call this model as ”hybrid model”.

Here, we calculate temperature anisotropy $\Delta T \equiv T_{\mathrm{e,co}} - T_{\mathrm{e,ctr}}$ by assuming the three temperature model, and all plasma current consisting of the thermal electron. We assume the anisotropy is so small that $\Delta T << T_{\mathrm{e}}$. Then, Eq. (4.8) is solved by ΔT as

$$\frac{\Delta T}{T} \simeq \sqrt{\frac{8\pi m}{kT}}\frac{j_{\mathrm{e}}}{en}. \tag{4.15}$$

4.2 Estimation of Collision Frequency of Electron with Ions and Neutral Particles

In the previous studies, collision frequency of electron with neutral particles makes an important role on the formation of velocity distribution function applied an electric field to as shown in 1.4.3. As an order estimation of the collision frequency, the calculation shown in Sec. 1.1.8 was performed with the diagnostics results on TST-2 plasma. We assume the profiles of ion and neutral are

$$n_{\mathrm{i}}(x) = n_{\mathrm{i}\infty}\tanh(x/x_0) \tag{4.16}$$

$$n_{\mathrm{n}}(x) = n_{\mathrm{n}0}\,\mathrm{sech}^2(x/x_0) \tag{4.17}$$

Electron densities on minor radii $r = 0$ m and $r = 0.17$ m are measured as $n_e(r = 0) \simeq 1 \times 10^{19}$ m^{-3}, $n_e(r = 0.17) \simeq 4\times10^{18}$ m^{-3} by Thomson scattering diagnostic on a typical Ohmic heated plasma. The penetration distance $x_0 \simeq 0.18$ m. Deuterium gas is injected into the vacuum vessel before plasma discharge as a fuel of plasma. The amount of deuterium is about 4×10^{-5} Torr. We assume the same amount of gas flows from vessel wall into plasma. Then, the effective filling pressure is assumed as $p_{\rm fill} = 8\times10^{-5}$ Torr. From the total amount of particles $N_{\rm all} \equiv N_{\rm n,all} + N_{\rm i,all} + N_{\rm e,all} \simeq N_{\rm n,all} + 2N_{\rm i,all}$, the neutral density at $r = 0$ m is estimated as $n_{\rm n0}(r = 0) \simeq 1.1 \times 10^{-18}$ m^{-3}. By using these parameters, profiles of ion density, neutral density, temperature and collision frequencies of electron with ions and neutrals were calculated as shown in Fig. 4.2. We see that the value of $\nu_{\rm en}/\nu_{\rm ei}$ is less than 0.02, which is much smaller than the value in which velocity distribution function becomes shifted-Maxwellian form ($\nu_{\rm en}/\nu_{\rm ei} \simeq 100$). When the Ref. [18] (described in Sec. 1.4.3) is assumed to be correct, the velocity distribution function should be distorted form because the collision frequency between electron and neutral is low enough. Therefore, this estimation argues the three temperature model is most suitable.

Figure 4.2: Profiles of ion density, neutral density, temperature and collision frequencies of electron with ions and neutrals. Calculation is shown in 1.1.8.

4.3 Estimation of Fast Electron Contribution

The first moment of velocity distribution function corresponds to current. In Ohmic heated plasmas, electric field is applied to plasma and velocity distribution strains in a direction. This strain generates the plasma current. Since the total plasma current is measured by Rogowski coil precisely, estimation of current helps to understand velocity distribution function. In the Thomson scattering measurement of the TST-2 system, measurable region of velocity is no more than 1 keV, which is converted to thermal velocity. Therefore, the measured plasma current by Thomson scattering is caused by only thermal electrons. On the other hand, fast electrons, such as explained in section 1.1.7, can play the main role in plasma current in some case. In this section, contribution of the high energy fast electrons to plasma current is estimated.

The fraction of fast electron is roughly estimated by the Eqs. (1.35), (1.38) and (1.39). These depends on magnitude of electric field $E_{\|}$, total current I_{p}, plasma density n_{e} and temperature T_{e}, major radius R_0 and minor radius a. In the TST-2 device, typical values of major radius and minor radius are $R_0 = 0.38$ m and $a = 0.25$ m. Magnitude of electric field $E_{\|} = 4/2\pi a$ [V/m]. In a typical target plasma, total current $I_{\mathrm{p}} \simeq 70$ kA. Fraction of fast electron increases with temperature. Figure 4.3 shows the fraction of fast electron vs. density, assuming temperature $T_{\mathrm{e}} = 200$ eV. In this case, the current fraction of fast electron f_{J} is less than 1% in the typical range of Ohmic discharge (e.g. $n_{\mathrm{e}} < 4\times10^{18}$ m^{-3}). Therefore, the estimation argue the contribution of fast electron to plasma current is much smaller in the target plasmas in this thesis.

Figure 4.3: Fractions of runaway electron estimated by Eqs. (1.35), (1.38) and (1.39). We assumed temperature $T = 200$ eV, total current $I_{\mathrm{p}} = 70$ kA, major radius $R = 0.38$ m and minor radius $a = 0.25$ m.

4.4 Calculation of Velocity Distribution Function by CQL3D

In order to simulate the velocity distribution function of the Ohmic heated plasma on the TST-2 device, calculation with CQL3D Fokker-Planck code [39] was performed. As the plasma parameters in the calculation, measured values in the actual plasma experiments shown in Sec. 5 was used. Temperature and density profile was assumed by using Thomson scattering data as shown in Fig. 4.4. The magnetic configuration is calculated by EFIT code [40] as shown in Fig. 4.5. Major radius and minor radius are $R_0 = 0.38$ m and $a = 0.29$ m, respectively. The strength of toroidal electric field is assumed to be $E_{\|} = 5/(2\pi R_0) \simeq 2$ V/m (constant in space and time). The pulse length is 6 ms.

The resultant total plasma current is $I_{\mathrm{p}} = 72$ kA. As shown in Figs. 4.6 and 4.7, widths of the distribution function for $v_{\mathrm{e},\|} > 0$ and $v_{\mathrm{e},\|} < 0$ are different while there is no shift of distribution function. Therefore, the calculation argue the three temperature model is most suitable for thermal electron. Moreover, we compared the contribution of thermal electron and fast electron to the plasma current. The ratio $J_{\mathrm{thermal}}/J_{\mathrm{fast}} \simeq 1.05$. Therefore, the calculation argue the contributions are almost equal.

Note that spatial transport such as turbulent transfer is not included in this code.

Figure 4.4: Temperature and density profiles used in the calculation of CQL3D. Squares are the data measured by Thomson scattering on the TST-2 device of shot number 99471. The temperature profile is assumed as $T(r) = 124 - 1984r^2$ [eV] (parabolic). The density profile is assumed $n(r) = 38 - 152r$ [10^{18}m^{-3}] (linear).

Figure 4.5: Tokamak equilibrium in the TST-2 device plasma calculated by EFIT.

4.5 Plasma Equilibrium

Tokamak plasma equilibrium was calculated for the TST-2 plasmas [41] by considering the current flowing the wall of device (i.e., Eddy current). The calculation result for the plasma shot number 99471 is shown in Figs. 4.9 and 4.10. The timing of plasma discharge is 24 ms in this calculation. The fitting is poor as shown in Fig. 4.10 because of the large Eddy current.

Figure 4.6: Up: Velocity distribution function calculated by CQL3D. Bottom left: Contribution factors in Fokker-Planck equation. Bottom right: Velocity distribution function of the electron with $v_{e,\parallel} > 0$ (blue) and $v_{e,\parallel} < 0$ (red) at $v_{e,\perp} = 0$.

Figure 4.7: Velocity distribution function of the electron with $v_{e,\parallel} > 0$ (blue) and $v_{e,\parallel} < 0$ (red) at $v_{e,\perp} = 0$ calculated by CQL3D.

Figure 4.8: Velocity distribution function of the electron with $v_{e,\parallel} > 0$ (blue) and $v_{e,\parallel} < 0$ (red) at $v_{e,\perp} = 0$ calculated by CQL3D.

Figure 4.9: Calculation result of the plasma equilibrium in TST-2, shot number of 99471.

Figure 4.10: Fitting to the measured values.

5 Experimental Results

The double-pass Thomson scattering diagnostic system was developed as described in Sec. 3 on the TST-2 device. By using this system, experiments to investigate temperature anisotropy in plasma was performed. In this chapter, setups and results of the experiments are summarized.

5.1 Ohmic Discharge

The target plasma is produced by an Ohmic discharge. Plasma is heated by electric field along the toroidal direction induced by time varying Ohmic coil current. Electrons are accelerated by the electric field in a single direction. Therefore, electron velocity distribution function in the direction parallel to the magnetic field must be distorted and be asymmetric when we compare the regions $v_{\|} \geq 0$ and $v_{\|} < 0$. The asymmetry is the source of plasma current. On the other hand, distribution function in perpendicular direction is not distorted and is symmetric. The difference between parallel and perpendicular direction causes plasma temperature anisotropy. The anisotropy was measured by the double-pass Thomson diagnostics.

As shown in Fig. 3.29, measurement range of wavelength is concentrated in lower side compared to the wavelength of incident laser. It means that only one side of velocity distribution ($f(v \geq 0)$ or $f(v < 0)$) is measured. Since no force is applied along perpendicular direction, distribution is symmetric. Therefore, all range of distribution can be obtained by measurement only on one side. On the other hand, distribution function is asymmetric in parallel direction. The distributions $f(v_{\|} \geq 0)$ and $f(v_{\|} < 0)$ are different. Therefore, we can measure the temperature only on the range of $f(v_{\|} \geq 0)$ or $f(v_{\|} < 0)$.

In order to investigate the all range of distribution, measurement of plasma with normal and reversed plasma current was performed (Fig. 5.1). Normally, we produce a plasma with electric field which is directed to clockwise along toroidai direction. Also, we can reverse the electric field (counterclockwise direction). The distribution parallel to the toroidal magnetic field of the plasma produced by the reversed electric field should be in reverse about the axis $v_{\|} = 0$ Then, the difference of the distribution in the range of $f(v_{\|} \geq 0)$ and $f(v_{\|} < 0)$ by comparing the spectrum measured by normal direction of electric field to the spectrum by reversed direction.

The distortion of distribution, which is driven by the Ohmic electric field, corresponds to the plasma current. Therefore, plasma current is also reversed in the plasma with reverse of electric

field. Then, we call the discharge with normal direction of electric field as ”Normal plasma current (I_p) discharge”, and call the discharge with reversed direction of electric field as ”Reversed plasma current discharge”.

The laser pass and the directions of measurement are described in Fig. 5.1. $k_{i,first}$ and $k_{i,second}$ denote the laser incident direction of first- and second-pass, respectively. k_{first} and k_{second} represent the direction of electron motion measured by first- and second-pass, respectively. The blue and red vector with the word ”I_p” show the direction of plasma current I_p in normal and reversed plasma current discharge, respectively. Energy provided by Ohmic electric field to electrons is as much as that to ions. However, momentum provided by the Ohmic field to electrons is much higher than that to ions, because mass of electron is much smaller than that of ion. Actually, $m_i \simeq 3600 m_e$ for deuterium, where m_i and m_e denote mass of ion and electron, respectively). Therefore, almost all plasma current consists of net electron motion. The blue and red vector with the word ”v_e” represent the direction of net electron motion in normal and reversed plasma current discharge, respectively. These motion are along to toroidal direction, which is nearly parallel to the direction of k_{second}.

Detailed value of angle between the toroidal direction and the measurement direction (k_{first} and k_{second}) on each measurement point is discussed in Sec. 6.2 and in Fig. 6.8.

Typical time evolution of operational parameters in the experiment are shown in Fig. 5.2. The loop voltage in the plasma V_{loop} is the voltage along the toroidal direction for one loop. The voltage is induced by time evolution of current flowing the Ohmic solenoid coil. V_{loop} can be reversed by reversing the Ohmic coil current (I_{OH}). Then, the direction of plasma current (I_p) is also reversed. The external vertical magnetic field must be also reversed for plasma equilibrium and force balance $\nabla \boldsymbol{p} = \boldsymbol{j} \times \mathbf{B}$.

As shown in Eq. (1.4), collision frequency of electrons and ions increases with density and decreases with temperature in the plasma. On the other hand, temperature anisotropy to generate plasma current decreases with density and temperature as shown in Eq. (4.15). The plasma density and temperature varies in time on a discharge. In order to investigate the dependence of anisotropy on the density and temperature, the measurement of Thomson scattering was performed in several timings of discharge.

Spatial profile of plasma current plays an important role on the plasma confinement for fusion plasmas [43]. Diffusion of induced electric field and penetration of the field to plasma are essential features to control plasma current profile, In order to measure plasma current profile and estimate

Figure 5.1: Schematic of experimental geometry for normal and reversed I_p experiments.

penetration of electric field, double-pass measurement was performed at two positions in plasma, which are plasma center (R = 389 mm) and plasma edge (R = 220 mm). See Fig. 3.2. Here, R denotes the major radius.

5.2 Fitting

Three kinds of model about electron velocity distribution function in Ohmic heated plasma was proposed in the Sec. 4: three temperature model, shifted model and hybrid model. These models represent plasma temperature anisotropy caused from difference of temperature between the range of $f(v \geq 0)$ and $f(v < 0)$, or shift of distribution. Data of Thomson scattering spectrum measured by the double-pass Thomson system is fitted to the velocity distribution function of these models. Then, the form of distribution is determined and degree of anisotropy is estimated from the result of the fitting.

On the doule-pass Thomson scattering diagnostics, two kinds of spectrum are obtained from the scattering of first- and second-pass. In the following, $\boldsymbol{y}_{\mathrm{first}}$ and $\mathbf{y}_{\mathrm{second}}$ denote the signals measured

Figure 5.2: Time evolution of operational parameters and several kinds of emission. (a) Total plasma current in a cross section measured by a Rogowski coil. (b) Current flowing in the Ohmic coil. (c) Loop voltage induced by the time varying Ohmic current. (d) Current flowing in the toroidal field coil. (e) H_α emission. (f-i) X-ray emission measured by AXUV and SBD detector. The sensitive energy regions of the detectors are shown in Fig. 5.3. SBD(Al) denotes the SBD detector without any filter (i.e., sensitive region is wide).

by first- and second-pass, respectively. In the TST-2 configuration, $\boldsymbol{y}_{\mathrm{first}}$ and $\boldsymbol{y}_{\mathrm{second}}$ reflect mainly the velocity distribution function perpendicular and parallel to the magnetic field, respectively. The fitting parameters in the three models are defined as

Three temperature model: $(\boldsymbol{y}_{\mathrm{first}} : n_{\mathrm{e,first}}, T_{\mathrm{e,first}})$ $(\boldsymbol{y}_{\mathrm{second}} : n_{\mathrm{e,second}}, T_{\mathrm{e,second}})$

Shifted model: $(\boldsymbol{y}_{\mathrm{first}} : n_{\mathrm{e,first}}, T_{\mathrm{e,first}})$ $(\boldsymbol{y}_{\mathrm{second}} : n_{\mathrm{e,second}}, \Delta\lambda)$

Hybrid model: $(\boldsymbol{y}_{\mathrm{first}}, \boldsymbol{y}_{\mathrm{second}} : n_{\mathrm{e,first}}, T_{\mathrm{e,first}}, n_{\mathrm{e,second}}, T_{\mathrm{e,second}}, \Delta\lambda)$

n_{e}, T_{e} and $\Delta\lambda$ denote electron density and electron temperature and shift of distribution, respectively. The suffix ”first” and second” represent values obtained from first- and second-pass measurement, respectively. In the shifted mode, $T_{\mathrm{e,second}}$ is assumed to be equal to $T_{\mathrm{e,first}}$, and $T_{\mathrm{e,second}}$ is not a fitting parameter.

Figure 5.3: Left: Responsibility of AXUV (Absolute UV Silicon Photodiodes) detector. Right: Transmissivity of filters covered on the SBD (Surface Barrier Diode) detector. Relatively low energy X-rays penetrate the P.P. (Polypropylene) filter, while the Be (Beryllium) filter is opaque to low energy X-rays [42].

The Thomson scattering spectrum is written as Eq. (2.32), i.e.,

$$S_R^{(2)}(\epsilon) = S_N(\epsilon)\left[1 - \frac{7}{2}\epsilon + \frac{\alpha\epsilon^3}{2(1-\cos\theta)} - \frac{1}{8\alpha}\left(\frac{39}{4} - 5\cos\theta\right) + \frac{1}{8}\left(29 + \frac{5}{1-\cos\theta}\right)\epsilon^2 \right.$$
$$\left. - \frac{1}{16(1-\cos\theta)}\left(28 - \frac{1}{1-\cos\theta}\right)\alpha\epsilon^4 + \frac{\alpha^2\epsilon^6}{8(1-\cos\theta)^2}\right], \tag{5.1}$$

$$S_N(\epsilon) = \left(\frac{\alpha}{\pi}\right)^{1/2} \frac{1}{\sqrt{2(1-\cos\theta)}} \exp\left\{\frac{-\alpha\epsilon^2}{2(1-\cos\theta)}\right\}, \tag{5.2}$$

$$\epsilon = \frac{\Delta\lambda}{\lambda_i} \qquad \Delta\lambda = \lambda_s - \lambda_i \qquad \alpha = \frac{m_0 c^2}{2T_e}. \tag{5.3}$$

Typical signals of the Thomson scattering are shown in Fig. 5.4 as black curves. The measurement point is the center of plasma (R = 389 mm). The blue lines represent error bars estimated as shown in Sec. 3.5.1. The signal includes two pulses which correspond to the scattering light of first- and second-pass. In order to distinguish the two pulses, the signal is fitted to two template single waveforms as described in Sec. 3.2. The fitted two template waveforms are plotted as red curves in Fig. 5.4. The plasma density is high enough and signal noise ratio is quite high in these experiments.

Figures 5.5, 5.6 and 5.7 are the data of Thomson scattering spectrum and the fitting results by assuming the three temperature model, the shifted model and the hybrid model, respectively. The spectrum corresponds to the signals shown in Fig. 5.4. The measurement point is the center of plasma (R = 389 mm). Signal of each channel is the integrated value of scattering spectrum over the sensitive range of the each channel, as seen in Fig. 3.29. The fitting method is the least squares

method. On the three temperature model, two parameters ($n_{\mathrm{e,first}}$, $T_{\mathrm{e,first}}$) are estimated by six signals of first-pass, and two parameters ($n_{\mathrm{e,second}}$, $T_{\mathrm{e,second}}$) are estimated by six signals of second-pass. On the shifted model, two parameters ($n_{\mathrm{e,first}}$, $T_{\mathrm{e,first}}$) are estimated by six signals of first-pass, and two parameters ($n_{\mathrm{e,second}}$, $\Delta\lambda$) are estimated by six signals of second-pass. On the hybrid model, five parameters ($n_{\mathrm{e,first}}$, $T_{\mathrm{e,first}}$, $n_{\mathrm{e,second}}$, $T_{\mathrm{e,second}}$, $\Delta\lambda$) are estimated by twelve signals of first- and second-pass. Estimated fitting parameters are also shown in Fig. 5.5, Fig. 5.6 and Fig. 5.7. Δ represents error of the fitting parameter. For example, $\Delta n_{\mathrm{e,first}}$ is the error of $n_{\mathrm{e,fitst}}$ estimated by the fitting procedure. Note that the wavelength shift is written as λ in these figures to distinguish the wavelength shift from that of error $\Delta\lambda$.

Figure 5.4: Signal waveform of Thomson scattering measurement. Black, red and blue curves represent raw signal of Thomson scattering, fitting curve of template waveform (described in Sec. 3.2) to the raw signal, error bar estimated by the procedure described in Sec. 3.5.7, respectively.

Figure 5.5: Thomson scattering spectrum measured by the double-pass Thomson scattering diagnostic system (Black squares). Red curves represent the fitting result by assuming the three temperature model. Black bars represent the error bar estimated by the fitting. The purple lines denote the wavelength of the incident YAG laser (1064 nm).

Figure 5.6: Thomson scattering spectrum and fitting result by assuming the shifted model. The definition of symbols and lines are same as in Fig. 5.5.

Figure 5.7: Thomson scattering spectrum and fitting result by assuming the hybrid model. The definition of symbols and lines are same as in Fig. 5.5.

5.3 Results

Experiment of plasma measurement with double-pass Thomson scattering were performed. The numbers of experiments with normal and reversed direction of electric field (Normal and Reversed plasma current discharge) are 315 and 256, respectively. Fitting parameters are calculated for each experiment by assuming the three kinds of model as described in Sec. 5.2. The results of fitting are shown in Figs. from 5.8 to 5.11. Measurement points are the plasma center (R = 389 mm) and the plasma edge (R = 220 mm). The blue and red symbols represent the result from experiments with normal and reversed plasma current, respectively. The ”negative” temperatures seen in Figs. 5.9, 5.10 and 5.11 represent measured temperature in the experiments with reversed plasma current. The notation is just for the sake of convenience.

Fig. 5.8 shows the results by assuming velocity distribution is a Maxwellian with three temperatures $T_{\mathrm{e,co}}$, $T_{\mathrm{e,ctr}}$ and $T_{\mathrm{e},\perp}$. Here, the direction of net electron velocity, which is the source of plasma current, is defined as co-direction. The opposite direction is defined as counter-direction. The co- and counter-direction are parallel to the toroidal magnetic field. On the other hand, $\perp$ denotes the perpendicular direction. $T_{\mathrm{e,co}}$, $T_{\mathrm{e,ctr}}$ and $T_{\mathrm{e},\perp}$ denote the temperatures of co-, counter- and perpendicular direction. $T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$ denote temperatures measured by first- and second-pass, and correspond to temperatures perpendicular and parallel to the toroidal magnetic field, respectively. The black lines in the Figs represent the isotropic temperature. There is clear difference in ratio of $T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$ in the experiments of normal and reversed current. The relationship is $T_{\mathrm{e,ctr}} < T_{\mathrm{e},\perp} < T_{\mathrm{e,co}}$ for the most of data. $T_{\mathrm{e,co}}$ and $T_{\mathrm{e,ctr}}$ correspond to the blue and the red symbol of $T_{\mathrm{e,second}}$ in Fig. 5.8. $T_{\mathrm{e},\perp}$, which is temperature perpendicular to the magnetic field, corresponds to the $T_{\mathrm{e,first}}$ The temperature anisotropies were around 20% at the plasma center, while around 50% at the plasma edge. The temperature anisotropy at the plasma edge is larger than at the plasma center. The relationship $T_{\mathrm{e,co}}$, $T_{\mathrm{e,ctr}}$ and $T_{\mathrm{e},\perp}$ is qualitatively reasonable to explain that the anisotropy of velocity distribution is the source of plasma current . While the angles of measurement are precisely perpendicular and parallel to the magnetic field in the plasma center, angles are different in the plasma edge. Therefore, actual anisotropy must be larger than the measured anisotropy in the plasma edge. The effective anisotropy estimated by considering the angle between measurement direction and magnetic field is discussed in Sec. 6.2.

Fig. 5.9 shows the results by assuming velocity distribution is a shifted-Maxwellian (parallel

shift) with $T_{e,\parallel} = T_{e,\perp}$. There is also clear difference in wavelength shift in the experiments of normal and reversed current. The center of spectrum is shifted to negative compared to the incident laser wavelength in the normal current experiment, while shifted to positive in the reversed current experiment. The sign of shift is also qualitatively reasonable to explain the shift of velocity distribution is the source of plasma current. The shifts are clearly divided to positive and negative in the plasma edge, while the shifts are smaller and variance is relatively large in the plasma center.

Figs. 5.10 and 5.11 show the results by assuming velocity distribution follows the hybrid model measured at the plasma edge and plasma center, respectively. T_{co}, T_{ctr} and $T_{\perp}$ can be different and velocity distribution can be shifted to the parallel direction in the model. There are also differences in the ratio of $T_{e,first}$ and $T_{e,second}$ and in wavelength shift in the experiments of normal and reversed current. The variance of data is larger than that with the three temperature model and the shifted model because the degree of freedom of fitting is larger. The difference between the experiments with normal and reversed plasma current is relatively small. The temperature anisotropies in the plasma edge and center are qualitatively reasonable to explain the anisotropy is the source of plasma current. While the sign of wavelength shift in the plasma edge is also reasonable, the sign is unreasonable in the normal plasma current experiments at plasma center.

Fig. 5.12 shows the electron densities measured by first- and second-pass. The densities at the plasma center is larger than that of at the plasma edge for the most of plasma discharge.

Figure 5.8: Fitting results of double-pass Thomson scattering measurement with normal (blue) and reversed (red) current plasma discharge by assuming the three temperature model. $T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$ denote the temperatures measured by first- and second-pass, respectively. Left: plasma edge (R = 220 mm) Right: plasma center (R = 389 mm).

Figure 5.9: Fitting results of double-pass Thomson scattering measurement with normal (blue) and reversed (red) current plasma discharge by assuming the shift model. $T_{\mathrm{e,first}}$ denote the temperature measured by first-pass. $\Delta\lambda$ represents the wavelength shift measured by second-pass. Left: plasma edge (R = 220 mm) Right: plasma center (R = 389 mm).

Figure 5.10: Fitting results of double-pass Thomson scattering measurement with normal (blue) and reversed (red) current plasma discharge by assuming the hybrid model at the plasma edge (R = 220 mm). $T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$ denote the temperatures measured by first- and second-pass, respectively. $\Delta\lambda$ represents the wavelength shift measured by second-pass. Left: Relationship between $T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$. Right: Relationship between $T_{\mathrm{e,first}}$ and $\Delta\lambda$

Figure 5.11: Fitting results of double-pass Thomson scattering measurement with normal (blue) and reversed (red) current plasma discharge by assuming the hybrid model at the plasma center (R = 389 mm). $T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$ denote the temperatures measured by first- and second-pass, respectively. $\Delta\lambda$ represents the wavelength shift measured by second-pass. Left: Relationship between $T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$. Right: Relationship between $T_{\mathrm{e,first}}$ and $\Delta\lambda$

Figure 5.12: Fitting results of double-pass Thomson scattering measurement with normal (blue) and reversed (red) current plasma discharge by assuming the three temperature model at the plasma center (R = 389 mm). $n_{e,first}$ and $n_{e,second}$ denote the densities measured by first- and second-pass, respectively. Left: plasma edge (R = 220 mm) Right: plasma center (R = 389 mm).

6 Discussion and Conclusions

6.1 Comparison of Fitting χ^2 for Each Model

In order to estimate which model is the more suitable for the measured spectra, the chi-squared values of fittings are calculated and compared. The chi-squared value χ^2 is defined as

$$\chi^2 = \sum_i \frac{(y_i - y_{\mathrm{fit},i})^2}{\sigma_i^2} \tag{6.1}$$

where y_i is the signal measured by Thomson scattering, and $y_{\mathrm{fit},i}$ is the resultant value of fitting. χ^2 should follows the chi-square distribution and the averaged value $\langle \chi - 2 \rangle$ should be the value of degree of freedom of the measurement. σ_i is the error estimated as in Sec. 3.5.7.

Figs. 6.1 and 6.2 show the probability distribution function (PDF) of measured χ^2 for the three temperature model and the shifted model at the plasma edge and center, respectively. The spectra detected by first- and second-pass are fitted independently in these model. The PDF in Figs. 6.1 and 6.2 includes only the measured χ^2 values for second-pass. The number of wavelength channels is 6. Then, the number of data is 6 for both of the first- and second-pass. The number of fitting parameters are 2 for the three temperature model ($n_{\mathrm{e,second}}$ and $T_{\mathrm{e,second}}$) and for the shift model ($n_{\mathrm{e,second}}$ and $\Delta\lambda$). Therefore, the degree of freedom is 4 for both of the models.

The values of χ^2 are summarized in Tabs. 7 and 8. The averaged values of χ^2 are 6.20 for the three temperature model and 8.02 for shifted model at the plasma edge. The averaged values of χ^2 are 7.36 for the three temperature model and 11.3 for shifted model at the plasma center. The total numbers of measurements are 571 and 382 for the plasma center and plasma edge, respectively. The probabilities that χ^2 are these values are less than 1%. Therefore, both of the models is not enough to explain the data. There are two reasons as possible causes. Firstly, the number of parameters can be inadequate. We supposed the parameters of anisotropic temperature and shift to express the distorted velocity distribution function. There can be other parameters which is required to express the distribution. Secondly, the estimated error can be too small. We used the error estimated as in Sec. 3.5.7 as σ_i. The true value of σ_i can be larger. When σ_i is larger, χ^2 becomes smaller and comes close to the ideal value 4. However, the error must be 50% larger than that of estimated value to modify the value of χ^2 to 4. Therefore, the second case is unlikely to occur.

In order to compare the three temperature model and shifted model, we estimate the standard

deviation of averaged χ^2. The distribution of χ^2 should become close to the ideal chi-squared distribution with increasing the number of measurement. Here, we separate the measured χ^2 values into groups. The number of element of each group is N. Then, χ^2 values are averaged in each group, which are defined as $\mu_N = (\langle\chi^2\rangle_1, \langle\chi^2\rangle_2, \cdots \langle\chi^2\rangle_N)$. The standard deviation of μ_N, which is defined as $\Delta\langle\chi^2\rangle_N$, should decrease with increasing N. The relationships between N and $\Delta\langle\chi^2\rangle_N$ are shown in Figs. 6.3 and 6.4. The values of $\Delta\langle\chi^2\rangle_N$ with actual value of N are calculated by extrapolation in the figures (red curves). The values of $\Delta\langle\chi^2\rangle_N$ are 0.415 for the three temperature model and 1.80 for the shifted model in the plasma edge. The values of $\Delta\langle\chi^2\rangle_N$ are 0.509 for the three temperature model and 1.76 for the shifted model in the plasma edge. Therefore, the three temperature model is better than shifted model with a finite difference about 1σ in the plasma edge and larger than 2σ in the plasma center. σ represents the standard deviation of χ^2.

Figure 6.1: Probability distribution function (PDF) of χ^2 for the fitting to the three temperature model (top) and the shifted model (bottom) at the plasma edge (R = 220 mm) for all data including the normal and reversed plasma current discharges. Chi-squared distribution with which degree of freedom is 4 is also plotted (blue curve). The black lines represent the averaged value of χ^2.

Figure 6.2: Probability distribution function (PDF) of χ^2 for the fitting to the three temperature model (top) and the shifted model (bottom) at the plasma center (R = 389 mm) for all data including the normal and reversed plasma current discharges. Chi-squared distribution with which degree of freedom is 4 is also plotted (blue curve). The black lines represent the averaged value of χ^2.

Figure 6.3: Relationship between N and $\Delta\langle\chi^2\rangle_N$ at the plasma edge.

Figure 6.4: Relationship between N and $\Delta\langle\chi^2\rangle_N$ at the plasma center.

Table 7: χ^2 values of fitting for each model at the plasma edge (R = 220 mm). n and m represent the numbers of data and fitting parameters, respectively. AVE denotes the averaged value of χ^2. $n - m$ is the ideal value of χ^2.

Models	n	m	n-m	AVE	Data number
3T	6	2	4	6.20	382
Shift	6	2	4	8.02	382
Hybrid	12	5	7	9.14	382

Table 8: χ^2 values of fitting for each model at the plasma center (R = 389 mm). n and m represent the numbers of data and fitting parameters, respectively. AVE denotes the averaged value of χ^2. $n - m$ is the ideal value of χ^2.

Models	n	m	n-m	AVE	Data number
3T	6	2	4	7.36	571
Shift	6	2	4	11.3	571
Hybrid	12	5	7	11.6	571

6.2 Angle between $\boldsymbol{k}$ and $\boldsymbol{B}$

Strictly, measurement angles of first-pass and second-pass are not purely perpendicular and parallel to the magnetic field $\boldsymbol{B}$, since the geometrical relation between $\boldsymbol{k}$ and $\boldsymbol{B}$ depends on the location. We estimate the effect in this section.

Here, we define the symbols of angles as shown in Fig. 6.5. $\boldsymbol{B}_\mathrm{t}$, $\boldsymbol{B}_\mathrm{p}$ and $\boldsymbol{B}$ are toroidal field, poloidal field and total field, respectively. The $\boldsymbol{B}_\mathrm{t}$ and $\boldsymbol{B}_\mathrm{p}$ are produced by toloidal field coil and plasma current, respectively. The plane formed by $\boldsymbol{B}$ and $\boldsymbol{B}_\mathrm{t}$ is orthogonal to the plane formed by $\boldsymbol{B}_\mathrm{t}$ and k. Therefore, the relationship between the angles are written as

$$\delta = \cos^{-1}(\cos(\theta)\cos(\phi)). \tag{6.2}$$

ϕ is uniquely determined from the geometrical configuration. The related geometry is shown in Fig. 6.7. Since $k_\mathrm{i,first} = k_\mathrm{i,second} = k_\mathrm{s}$, ϕ_first and ϕ_second are written as

$$\phi_\mathrm{first} = \frac{\pi}{2} + \sin^{-1}\left(\frac{l_0}{R}\right) - \frac{\theta_{0,\mathrm{s}}}{2}, \tag{6.3}$$

$$\phi_\mathrm{second} = \sin^{-1}\left(\frac{l_0}{R}\right) - \frac{\theta_{0,\mathrm{s}}}{2}. \tag{6.4}$$

Because $\phi_\mathrm{second} = \phi_\mathrm{first} - \pi/2$, ϕ_second is uniquely determined by ϕ_first. Then, we define $\phi_\mathrm{first} = \phi$. On the other hand, θ and δ depend on the magnitude of plasma current. Profiles of ϕ, θ and δ are shown in Fig. 6.8 with typical parameters of I_p, $\boldsymbol{B}_\mathrm{t}$, R_0 and a.

The direction of electron velocity, which is measured by Thomson scattering, is parallel to the $\boldsymbol{k} = \boldsymbol{k}_\mathrm{s} - \boldsymbol{k}_\mathrm{i}$. By assuming the velocity distribution is three Maxwellian, it is written as

$$f_\mathrm{e}(v_\perp, v_\parallel > 0) = n_\mathrm{e}\left(\frac{m}{2\pi k T_{\mathrm{e},\perp}^2 T_\mathrm{e,co}}\right)^{3/2} \exp\left[-\frac{m}{2k}\left(\frac{v_\perp^2}{T_{\mathrm{e},\perp}} + \frac{v_\parallel^2}{T_\mathrm{e,co}}\right)\right]. \tag{6.5}$$

The effective temperatures, which are actually measured by Thomson scattering, T_eff are expressed as Fig. 6.6, i.e.,

$$\frac{1}{T_\mathrm{eff,first}} = \frac{\sin^2\delta}{T_\perp} + \frac{\cos^2\delta}{T_\parallel}, \qquad \frac{1}{T_\mathrm{eff,second}} = \frac{\cos^2\delta}{T_\perp} + \frac{\sin^2\delta}{T_\parallel}. \tag{6.6}$$

$T_{\mathrm{e},\perp}$ and $T_{\mathrm{e},\parallel}$ are the temperatures in the directions purely parallel and perpendicular to the field,

respectively. Then, $T_{e,\perp}$ and $T_{e,\parallel}$ are written as

$$T_{e,\perp} = \frac{T_1 T_2(\sin^2\delta - \cos^2\delta)}{T_2 \sin^2\delta - T_1 \cos^2\delta}, \qquad T_{e,\parallel} = \frac{T_1 T_2(\sin^2\delta - \cos^2\delta)}{T_1 \sin^2\delta - T_2 \cos^2\delta} \tag{6.7}$$

where we defined $T_1 \equiv T_{\mathrm{eff,first}}$ and $T_2 \equiv T_{\mathrm{eff,second}}$. Here, the temperature anisotropy is written as

$$\frac{T_{e,\parallel}}{T_{e,\perp}} \equiv r_0 = \frac{\cos^2\delta - (T_2/T_1)\sin^2\delta}{(T_2/T_1)\cos^2\delta - \sin^2\delta} = \frac{\cos^2\delta - r\sin^2\delta}{r\cos^2\delta - \sin^2\delta}, \tag{6.8}$$

where $r = T_2/T_1$. For example, the values of δ are 2.24 rad and 1.59 rad at $R = 220$ mm and $R = 389$ mm, respectively. Therefore, $T_{e,\perp} \simeq T_1$ and $T_{e,\parallel} \simeq T_2$ at $R = 389$ mm. On the other hand, $r_0 = 1.5$ and 0.62 when $r = 1.1$ and 0.9, respectively. It is seen that the measurement around plasma center reflects purely perpendicular and parallel direction to the magnetic field, while the measurement around plasma edge reflects their mixture. Moreover, true values of temperature anisotropy must larger than measured at plasma edge.

As shown in Figs. 6.7 and 6.8, $T_{e,\perp}$ or $T_{e,\parallel}$ is not a finite value when $T_2/T_1 = \tan^2\delta$ or $T_2/T_1 = 1/\tan^2\delta$. In fact, the solution exists only when $1/\tan^2\delta < T_2/T_1 < \tan^2\delta$ as seen in 6.6. This is a limitation of the Maxwellian model. The region, where a solution exists with assumption of the three temperature model for the plasma edge measurement, is clearly specified in the Fig. 6.9 by green lines. There are many data beyond the lines. There are anisotropy in a form which can be expressed by the Maxwellian model in these data. On the other hand, all data are in the region where a solution exists for the plasma center.

Figure 6.5: Definition of angles θ, ϕ and δ.

Figure 6.6: Effectively measured temperature T_{eff}.

Figure 6.7: Schematic of scattering geometry. ϕ_{first} and ϕ_{second} are the angles between toroidal magnetic field and $\boldsymbol{k} = \boldsymbol{k}_{\mathrm{s}} - \boldsymbol{k}_{\mathrm{i}}$ for the first- and the second-pass. R is the major radius of measurement point. l_0 is the distance between the center of TST-2 device and the laser pass. $\gamma = \sin^{-1}(R/l_0)$. $\theta_{0,\mathrm{s}}$ is the scattering angle in the second-pass.

Figure 6.8: Profiles of ϕ, θ and δ along major radius R. θ is calculated with following assumptions: The total plasma current is 60 kA, current profile is flat, the current flowing in the toroidal coil is $24 \times 15.5 = 372$ kA, major radius of magnetic field is 0.38 m and minor radius is 0.25 m.

Figure 6.9: Fitting result for temperatures at plasma edge with assumption of the three temperature model. The green lines represents $T_2 = (\tan^2\delta)T_1$ and $T_2 = (1/\tan^2\delta)T_1$, where $\tan^2\delta \simeq 1.58$.

6.3 Estimation of Current Density from Experimental Data

Plasma current density can be calculated from the measured density and temperature by using the models described in Sec. 4.1. Spectrum measured by the Thomson scattering measurement is not affected by fast electron because density of fast electron is quite low. Thus, the plasma current density measured by Thomson scattering is limited to that caused by thermal electron. As shown in Sec. 4.4, contribution of fast electron to plasma current can be large in Ohmic heated discharge On the other hand, the fraction of fast electron in the plasma current should be small in the high density discharge of TST-2 device as calculated in Sec. 4.3. In order to validate the method of plasma current density measurement with the double-pass Thomson scattering system, and to investigate the contributions from thermal electron and fast electron to plasma current density, current density was calculated.

Here, we assume the three temperature model because the model is better than the shifted model as discussed in Sec. 6.1. In addition, we ignore the effect of angles between $\boldsymbol{k}$ and $\boldsymbol{B}$ described in Sec. 6.2. Therefore, $T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$ correspond to $T_{\mathrm{e},\perp}$ and $T_{\mathrm{e},\parallel}$ (=$T_{\mathrm{e,co}}$ or $T_{\mathrm{e,ctr}}$), respectively. In this model, the current density j_{e} is written as Eq. (4.8), i.e.,

$$j_{\mathrm{e}} = en_{\mathrm{e}}\left(\sqrt{\frac{kT_{\mathrm{e,co}}}{2\pi m}} - \sqrt{\frac{kT_{\mathrm{e,ctr}}}{2\pi m}}\right). \tag{6.9}$$

Concept of the calculation is described in Fig. 6.10. The temperatures measured in one plasma experiment is ($T_{\mathrm{e},\perp}$, $T_{\mathrm{e,co}}$) or ($T_{\mathrm{e},\perp}$, $T_{\mathrm{e,ctr}}$). The former is measured by the normal current discharge and the latter is measured by the reversed current discharge. We can not measure $T_{\mathrm{e,co}}$ and $T_{\mathrm{e,ctr}}$ simultaneously. On the other hand, $T_{\mathrm{e},\perp}$ can be measured in all experiment. Here, we assume when $T_{\mathrm{e},\perp}$s of plasmas are equal, then the velocity distributions of the plasmas are same. A set of ($T_{\mathrm{e,first}}$ and $T_{\mathrm{e,second}}$) is measured in each experiment. We divide the sets with $T_{\mathrm{e,first}}$ in every 10 eV ($0-10$ eV, $10-20$ eV, $20-30$ eV, $\cdots$). In each group, corresponding $T_{\mathrm{e,second}}$ are averaged, and defined as $\overline{T}_{\mathrm{e,second}}$. We define $\overline{T}_{\mathrm{e,first}} \equiv 5, 15, 25, \cdots$ eV, which corresponds to $\overline{T}_{\mathrm{e,second}}$.

The relationship between $\overline{T}_{\mathrm{e,first}}$ and $\overline{T}_{\mathrm{e,second}}$ is shown in Fig. 6.11. The symbols of blue and red represent the data measured from normal and reversed plasma current discharge, respectively. They correspond to T_{co} and T_{ctr}, respectively. The data set is same as shown in Fig. 5.8.

Electron current density j_{e} is calculated from the T_{co} and T_{ctr} by using Eq. 6.9 as shown in Figs. 6.12 and 6.13. Electron density n_{e} is calculated by averaging $n_{\mathrm{e,first}}$ and $n_{\mathrm{e,second}}$ in each group of $T_{\mathrm{e,second}}$. In typical plasmas in the experiments, total plasma current I_{p} is about 60 kA and plasma

minor radius a is about 0.25 m. Thus, spatial averaged plasma current density $j_{\mathrm{ave}} = I_{\mathrm{p}}/\pi a^2$ is about 300 kA/m^2. j_{ave} is also plotted in Figs. 6.12 and 6.13. Measured value of j_{e} is close to j_{ave} for the most of data at both of the plasma edge and center. The result suggests that the contribution of thermal electron to plasma current is quite large.

Plasma resistivity η, which is calculated by Eq. 1.13 should be related to plasma current density. η was calculated by assigning $\overline{T}_{\mathrm{e,first}}$ to T. Figs. 6.14 and 6.15 show the relationship between j_{e} and η at the plasma edge and center. The result is reasonable because the plasma current density increases with decreasing plasma resistivity.

Electric field along toroidal direction E can be estimated with j_{e} and η from the Ohm's law (Eq. 1.10). While the Ohmic induced electric field is applied in the experiment, penetration of the external electric field takes a certain time due to plasma resistivity (skin effect). Here, we estimate the current diffusion time τ_η with Eq. 1.17. We assign $\overline{T}_{\mathrm{first}}$ to T again. Figs. 6.16 and 6.17 show the relationship between E and τ_η. Typical applied voltage along a toroidal loop V_{loop}, which is induced by Ohmic current, is about 4 V (see Fig. 5.2). Thus, the Ohmic electric field $E_{\mathrm{OH}} = V_{\mathrm{loop}}/2\pi a$ is about 2.9 V/m and 1.6 V/m at $R = 220$ mm and $R = 389$ mm, respectively. E_{OH} is also plotted in Figs. 6.16 and 6.17. The results show electric field in plasma is lower with current diffusion time is longer, and the Ohmic electric field itself is applied to plasma when the current diffusion time is short. This is also reasonable.

Therefore, it is concluded that the result of plasma current density measurement is reasonable qualitatively and quantitatively from the analysis above.

Figure 6.10: A conceptual sketch of analysis to rebuild the velocity distribution function for calculation of plasma current density.

Figure 6.11: The relationship between $\overline{T}_{\mathrm{e,first}}$ and $\overline{T}_{\mathrm{e,second}}$ at the plasma center (R = 389 mm) and the plasma edge (R = 220 mm).

Figure 6.12: Plasma current density j_e vs. $T_{e,first}$ at the plasma edge.

Figure 6.13: Plasma current density j_e vs. $T_{e,first}$ at the plasma center.

Figure 6.14: Plasma current density j_e vs. plasma resistivity η at the plasma edge (double logarithm).

Figure 6.15: Plasma current density j_e vs. plasma resistivity η at the plasma center (double logarithm).

Figure 6.16: Electric field in plasma E vs. current diffusion time τ_η at the plasma edge.

Figure 6.17: Electric field in plasma E vs. current diffusion time τ_η at the plasma center.

6.4 Error Analysis

Figure 6.18 shows the expected error of wavelength shift with assumption of the hybrid model by using Monte Carlo simulation (see Sec. 3.6). The error depends on temperature, because the width of the spectrum expands with temperature. Electron density is assumed to be 10^{19} m^{-3}.

Figure 6.18: Error of wavelength shift measurement estimated by Monte Carlo simulation. Electron density is assumed to be 10^{19} m^{-3}.

6.5 Comparison with Fusion Plasma Parameter

The design values of ITER plasma in table 9 as parameters of typical fusion plasma. From Eq. (1.4), collision frequency of ITER plasma is1.6×10^3 s^{-1}. Collision frequency of the measured plasma in TST-2 is , 3.6×10^5 s^{-1} since plasma parameters are almost $T = 0.06$ keV, $n = 0.04 \times 10^{20}$ m^{-3}. Since the anisotropy of velocity distribution function appeared in the high collision frequency TST-2 plasma, anisotropy can also appear in the lower collision frequency fusion plasma.

Table 9: Design parameters for ITER [1].

I_p [MA]	B_0 [T]	a [m]	R_0 [m]	T_e [eV]	n_e [10^{20} m^{-3}]
15	5.3	2.0	6.2	19	1.0

6.6 Conclusions

In high temperature fusion plasmas, the velocity distribution function can be distorted and anisotropic in the velocity space. Such anisotropy results in deformation of plasma pressure from flux function and destabilization of the ballooning modes. Therefore, it is important to investigate the physics of plasma anisotropy. However, there was no diagnostics with which the anisotropy in the core of plasma can be measured.

A double-pass Thomson scattering diagnostic system was developed on the TST-2 device. This is the first diagnostics to measure two dimensional velocity distribution function of thermal electrons at the core of plasma. In our system, the velocity distribution functions parallel and perpendicular to the magnetic field can be measured. Thus, plasma anisotropy can be measured with the double-pass diagnostics.

The double-pass Thomson scattering system consists of a laser injection optics, a collection optics and a detection system. A Nd:YAG laser with a beam energy of 1.6 J and a pulse width of 10 ns was used. The laser is injected into the plasma, and reflected back by a spherical mirror. The Thomson scattering lights, which are excited by the double-passed laser, are collected by a collection spherical mirror and are transferred into a polychromator by a fiber with a numerical aperture of 0.37. There are six wavelength channels in each polychromator. Each channel consists of an interference filter, an avalanche photodiode and a two stage amplifier. The detected signal is digitized by an oscilloscope. The forward- and backward-scattering pulses are separated clearly with the fast and low-noise detection system.

Absolute and relative sensitivity calibrations are essential to estimate the electron density, the electron temperature and the electron current density. Particularly, it is important to evaluate and to reduce the error of the system in order to confirm finite temperature anisotropy. The systematic error of ”absolute” sensitivity calibration was suppressed below 10% by reducing the noise sources such as dependence on fiber location and by improving the reproducibility of the calibration. The absolute calibration was validated by comparing with the result from the microwave interferometer, by which line integrated electron density is obtained. In addition, the systematic error of ”relative” sensitivity calibration of each channel was suppressed below 10% by reducing the noise sources such as dependence on environmental temperature and by improving the reproducibility of the calibration. In addition to these systematic errors, random error was evaluated in detail. The result of evaluation was

used as a typical noise level for Monte Carlo simulation and the fitting procedure of the experiments.

Several calculations were performed to evaluate various effects causing the velocity distribution function deformation and resultant current under the Ohmic electric field. The collision frequency of electron with neutral particles for the present plasma is calculated to be much lower than the collision of electron with ions. The contribution of fast electron was also estimated, and it was found to be negligible. The last candidate situation is that the velocity distribution function is determined by the collisions between thermal electrons and ions under the electric field. This situation is calculated by a Fokker-Planck code CQL3D. It was found that 10% temperature anisotropy is expected without any wavelength shift. In contrast to the above estimation, the contribution of fast electrons is not negligible in this calculation.

Electron velocity distribution functions of Ohmic heated plasmas, where we expect deformations of electron velocity distribution function, were measured with the double-pass Thomson scattering system. Measurement was performed at the plasma center and the plasma edge. The scattering light spectrum was analyzed assuming the following three models: three temperature model, shifted model and hybrid model. Theoretically, any of these models can represent the velocity distribution function of electrons accelerated by the Ohmic electric field. The goodnesses of fitting for these three models were calculated and compared. As a result, the three temperature model is better than the shifted model at both of the plasma center and edge, while the measured velocity distribution was not expressed enough with both of the models. It suggests that other fitting parameter is required to represent the distribution.

We found finite difference between the perpendicular and the parallel temperatures. The co-directed (i.e., the direction of electron acceleration) parallel temperature is almost always higher than the perpendicular temperature, and the counter-directed parallel temperature is almost always lower than the others. Comparing the temperatures along the direction perpendicular and parallel to the magnetic field, there was temperature anisotropy of around 20% at the plasma center, while around 50% at the plasma edge. This is the first measurement of temperature anisotropy for thermal electrons in magnetically confined high temperature plasmas. The electron current density of the plasma was calculated with the assumption of the three temperature model. The values are close to the average current density estimated from the total current. The result suggests that the contribution of thermal electrons to the plasma current is relatively large in the Ohmic discharge. This is consistent with the calculations. It was also revealed that anisotropy of thermal electron temperature appears even in

plasmas with a relatively high collision frequency (which is about a hundred times higher than that in fusion plasmas).

The results of our investigation would contribute to the understanding of anisotropy of plasma velocity distribution function, and demonstrated the possibility of current density measurement with Thomson scattering.

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