論文の内容の要旨

論文題目

Localization of Five Dimensional Super Yang-Mills Theory and 4D/2D Duality

(5次元超対称ヤンミルズ理論の局所化と4D/2D双対性)

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In this thesis, we investigate a 4D/2D duality and its relation to M5-branes by using the five-dimensional $\mathcal{N} = 2$ supersymmetric Yang-Mills theory.

The study of M2/M5-branes has been an important research area in M-theory, because M2/M5-branes are elementary objects. It is known that they have exotic statistical properties, and such statistical properties are studied by the AdS/CFT correspondence. The microscopic theory for M2-branes is described by ABJM model, then it may be possible to understand some properties of M2-branes by using ABJM model. In particular, the statistical property for ABJM model was studied, and this result is consistent with the prediction of the AdS/CFT correspondence. On the other hand, it is difficult to investigate M5-branes directly, because the world volume theory for them has not been completely formulated yet.

4D/2D duality has been an active research area to study M5-branes since its discovery. In order to explain the relation between 4D/2D duality and M5-branes, let us consider M5-branes on $M_4 \times \Sigma$ with a four-dimensional closed manifold M_4 and a Riemann surface Σ. By compactifying M5-branes on Σ, we obtain a four-dimensional $\mathcal{N} = 2$ gauge theory on M_4 by Gaiotto's argument. On the other hand, by compactifying M5-branes on M_4, we obtain a two-dimensional theory on Σ. There is a correspondence between them, and it is called 4D/2D duality. These two theories and the 4D/2D duality have to reproduce properties of M5-branes, then it is important for understanding M5-branes to study the 4D/2D duality.

When $M_4 = S^4$, the two-dimensional theory on Σ is expected to be the Toda-Liouville theory. The partition function of the four-dimensional $\mathcal{N} = 2$ gauge theory on S^4 corresponds to the correlation function of the Toda-Liouville theory. This 4D/2D duality is called AGT conjecture, which is the most famous example in 4D/2D duality.

In this paper, we consider the case that $M_4 = S^1 \times S^3$. In this case, it is conjectured that the two-dimensional theory on Σ is the two-dimensional q-deformed Yang-Mills theory. The two-dimensional theory is dual to the four-dimensional $\mathcal{N} = 2$ gauge theory on $S^1 \times S^3$. The superconformal index for the four-dimensional theory is equal to the partition function of the q-deformed Yang-Mills theory. We discuss the relation between the q-deformed Yang-Mills theory and the M5-branes on $S^1 \times S^3 \times \Sigma$.

The world volume theory for M5-branes is not completely formulated. However, it is believed that the five-dimensional $\mathcal{N} = 2$ supersymmetric Yang-Mills theory describes the M5-branes theory on S^1. Therefore, we investigate the five-dimensional $\mathcal{N} = 2$ supersymmetric Yang-Mills theory on $S^3 \times \Sigma$ and compute the partition function by using the localization method. It turns out that the partition function of the five-dimensional theory is equal to the partition function of the two-dimensional q-deformed Yang-Mills theory on Σ. Our result is agreement with the 4D/2D duality and the relation between the M5-branes on S^1 and the five-dimensional theory.