博士論文

Essays on Structural Estimation of Auction Models

(オークションモデルを用いた構造推定に関する考察)

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Introduction

1.1 Literature Review

Auctions are an important market mechanism for determining prices and allocating goods in many markets. Every day, many kinds of foods and flowers are sold via auctions. Governments, as buyer of public goods, often choose suppliers using procurement auctions. The percentages of governments' expenditures for procurement auctions cannot be ignored in the gross domestic product (GDP). Some web search engine companies such as Google and Yahoo! have some advertising spaces on their own web pages and determine the locations of individual advertisements through auctions. Such an auction is called an internet advertisement auction, which is an important revenue source for web search engine companies. Clearly, auctions play an essential role in our economy. Furthermore, auctions became familiar as a place where we actually participate through the emergence of online auction markets (e.g., eBay and Yahoo!). In the online auction market, many people can become buyers or sellers and trade items easily.

It is natural that economists should take an interest in an important and universal market mechanism such as auctions. In addition to the importance of auctions from a practical perspective, there are several reasons auctions have been a fruitful research field for economists. In theoretical literature, economists often focus on how equilibrium prices are determined. While we cannot observe the process of equilibrium price determination in the general market, we can recognize the specific price determination process by observing competitive bids in auctions. Therefore, auction offer an attractive market arena in order to study the equilibrium price determination process. In particular, in the general equilibrium model, the fictional Walrasian auctioneer plays an essential role in the *tâtonnement* process, which is one of the most famous model that explains how equilibrium prices are determined.

From a game-theoretic perspective, auction theory is one of the most successful applications of incomplete information games. The second-price, sealed-bid auction demonstrated in Vickrey (1961) and Vickrey-Clarke-Groves (VCG) auction in general are typical examples of a strategy-proof mechanism that plays an essential role in the theory of mechanism design. Such studies are useful for designing the institutions of auctions in the real world.

From the empirical perspective, auction models are also an attractive research field for econometricians. Hendricks and Porter (2007) emphasized that auction data sets are often better than typical data sets in industrial organization. They describe two reasons why the quality of auction data is often relatively high as follows. First, the auction game is relatively simple, with well-specified rules. Second, the actions of the participants are observed directly, and payoffs can sometimes be inferred. In addition to the availability of high-quality data, there are several reasons auctions have been attractive to many empirical researchers: the structural econometrics, the Bayesian econometrics, and the nonparametric (or semiparametric) econometrics. These three econometric methodologies have succeeded in contributing to the econometrics of auction data. Below, we briefly review these three econometric methodologies in the literature of empirical auctions.

1.1.1 Structural Econometrics

Structural estimation is an econometric model based on the economic theory. In particular, in the structural econometrics of auction data, the econometric models are described by auction theory. In auction theory, economists consider the auction as a game, and often compute the equilibrium bidding strategy from each bidder's private type. In the literature of the structural econometrics of auction data, econometricians regard the auction theory model as the data generating process and interpret the observable auction data, such as bids, as the result of equilibrium behavior.[1] In most studies of the structural econometrics of auction data, by observing the equilibrium bids, we estimate the structural parameters, such as bidders' private types.

Paarsch (1992) is a seminal paper in this literature. After his pioneering work, structural econometrics has joined the mainstream of empirical auction literature. There are several reasons why many important studies that contribute to the empirical research of auctions adopt the methodology of structural estimation. First, the equilibrium transaction prices in auctions are considered to depend on both the valuation of goods and the magnitude of competition. Usually, both factors that help determine auction prices are not observable individually. Therefore, we cannot estimate the individual effects in the usual manner. However, auction theory tells econometricians the complex relationship between the valuation of goods and the magnitude of competition. Therefore, applying structural estimation enables econometricians to estimate both factors individually.

Second, since the bidder's decision is modeled explicitly in structural econometrics, we can conduct counterfactual simulations easily using estimated structural parameters. For example, we could simulate how the distribution of the transaction price changes if an auctioneer changes the auction format from English auction to Dutch auction.

[1] There are several exceptions. For example, Haile and Tamer (2003) made two assumptions that are weaker than the usual assumptions to derive the dominant strategy equilibrium, and they estimated the parameters in accordance with their assumptions. Aradillas-Lopez and Tamer (2008) dropped the Nash equilibrium assumption and used rationalizability as the basis for strategic play. Under their assumption, they studied identification in first-price auctions within the independent private values paradigm (IPVP).

Paarsch and Hong (2006), Athey and Haile (2007), and Hendricks and Porter (2007) provide excellent surveys of this literature.

We briefly review the methodology of structural estimation using the example of second-price, sealed-bid auctions within the independent private values paradigm (IPVP). Note that Chapter 2 is an application of the structural estimation of second-price, sealed-bid auction within the IPVP.

Consider that in a second-price, sealed-bid auction, each bidder submits a bid simultaneously. The bidder who bids the highest bid among participants wins the object. However, the price that the winner pays is not her own bid, but is equal to the second-highest bid among participants. There are N potential bidders with risk-neutral preferences indexed by $i = 1, ..., N$. Each bidder's willingness to pay is denoted by V_i, which is an independent and identically distributed random variable from the distribution, $F(\cdot)$. The realization of bidder i's willingness to pay, v_i, is her own private information, and she does not know others'. However, the distribution of V_i, $F(\cdot)$, is common knowledge to all participants. Under these assumptions, the equilibrium bid of bidder i with a willingness to pay, $V_i = v_i$, is denoted by $b_i = v_i$. In other words, it is a dominant strategy for the bidder to tell her true willingness to pay.

Note that although the distribution of valuation, $F(\cdot)$, is common knowledge among participants, $F(\cdot)$ is unknown for econometricians; our purpose is to identify and estimate $F(\cdot)$ from the observed bids. Econometricians accept these results of auction theory. We assume that observed bids are equal to their valuations for items in this example. Furthermore, the econometric model inherits all of the settings of auction theory. Under these settings, we can estimate the distribution of valuation, $F(\cdot)$, from the observed bids. Observe T, independent and identical auctions of an identical item, with the identical number of bidders, N, indexed by $t\{1, ..., T\}$. Let B_{it} be bidder i's bid at auction t which is observable for econometricians. Then, since each of bidder i's bids equals her willingness to pay (i.e., $B_{it} = V_{it}$), we gain the sample from the distribution,

$F(\cdot)$. Therefore, the structural parameters of $F(\cdot)$ can be easily estimated from the observed bid, B_{it}. For example, using the empirical distribution function, we have the sample analogue of $F(\cdot)$, $\hat{F}(\cdot)$ as

$$\hat{F}(v) = \frac{1}{TN} \sum_{t=1}^{T} \sum_{i=1}^{N} \mathbf{1}(B_{it} \leq v).$$

1.1.2 Bayesian Econometrics

Econometricians often deal with models too complex to compute. In particular, frequentist approaches, such as maximum likelihood estimation and least squares estimation, require maximization or minimization of object functions; such optimizations often cause computational difficulties. In contrast, Bayesian methods do not require maximization procedures. Therefore, Bayesian approaches can contribute to the progress of econometrics in various fields, especially when the econometric model is too complex. The structural econometrics of auction data is no exception. Several studies use Bayesian methods to circumvent difficulties in the literature of the structural estimation of auction data.

As described above, the structural econometrics of the second-price, sealed-bid auctions within the IPVP is made simply since the dominant strategy equilibrium bidding functions are identity functions of bidders' valuations and, hence, trivial functions for econometricians. However, since the Bayesian Nash equilibrium bidding functions in the first-price, sealed-bid auctions are complex and nonlinear functions of bidders' valuations, the structural econometrics of the first-price, sealed-bid auctions is not made simply.

Perhaps, one of the most famous problems in the literature of structural econometrics of first-price, sealed-bid auctions is that a standard regularity condition that ensures the asymptotic properties of the maximum likelihood estimator no longer holds. We review this problem briefly.

Consider that at first-price, sealed-bid auctions, each bidder submits a bid simultaneously. The participant with the highest bid wins the object and pays her own bid. There are N potential bidders with risk-neutral preferences indexed by $i = 1, ..., N$. Each bidder's valuation for the object is denoted by V_i, which is an independent and identically distributed random variable from the distribution, $F(\cdot; \boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is a vector of parameters. The realization of bidder i's willingness to pay is her own private information. However, the distribution of V_i, $F(\cdot; \boldsymbol{\theta})$, is common knowledge among all participants. Under these assumptions, the equilibrium bidding functions for bidder i with a valuation of $V_i = v$, $\beta(v)$, is

$$\beta(v) = v - \frac{\int_{\underline{v}}^{v} [F(u; \boldsymbol{\theta})]^{N-1} du}{[F(v; \boldsymbol{\theta})]^{N-1}},$$

where $\underline{v}$ denotes the lower bound of support of v.

The main difficulty is that the support of bids depends on parameters to be estimated, $\boldsymbol{\theta}$. Let $\bar{v}$ be the upper bound of support of v. Since $\beta(v)$ is a strictly increasing function with respect to v, the upper bound of support of bids is $\beta(\bar{v})$. The upper bound of support of bids, $\beta(\bar{v})$, is computed by

$$\begin{aligned} \beta(\bar{v}) &= \bar{v} - \frac{\int_{\underline{v}}^{\bar{v}} [F(u; \boldsymbol{\theta})]^{N-1} du}{[F(\bar{v}; \boldsymbol{\theta})]^{N-1}} \\ &= \bar{v} - \int_{\underline{v}}^{\bar{v}} [F(u; \boldsymbol{\theta})]^{N-1} du. \quad (\text{since } F(\bar{v}; \boldsymbol{\theta}) = 1) \end{aligned}$$

Therefore, $\beta(\bar{v})$ depends on the unknown parameter, $\boldsymbol{\theta}$. Then, a standard regularity condition that ensures the asymptotic properties of the maximum likelihood estimator is violated.[2]

Donald and Paarsch (1993) and Donald and Paarsch (1996) proposed a pseudo max-

[2]Amemiya (1985) provides regularity conditions that ensure the consistency of extremum estimators. In our case, it is difficult to show that the likelihood function converges to a non-stochastic function that attains a unique global maximum value at true θ in probability uniformly over the parameter space.

imum likelihood estimation that overcomes this difficulty. However, these estimators are computationally burdensome. A simple alternative solution for this problem is to apply the Bayesian method. Bayesian econometrics does not require the asymptotic theory, since the Bayes rule justifies the inference using the Bayesian method. Furthermore, thanks to the development of the Markov Chain Monte Carlo simulation method, the computational burden of Bayesian inference can be relaxed considerably, in many cases. Bajari and Hortaçsu (2003) utilized the Bayesian method to circumvent this problem in the literature of structural estimation of second-price auctions within the common value paradigm. In Chapters 2, 3, and 4, we used the Bayesian method to estimate structural parameters.

1.1.3 Nonparametric (Semiparametric) Econometrics

One of the major criticisms of parametric models is that they are approximations of the real process and lead to concerns about potential misspecifications. In particular, since economic data, unlike natural science data, are usually not controlled, parametric specifications often may be strong assumptions in econometrics. Structural econometrics of auction data is no exception. In this literature, a typical structural parameter is the distribution of a bidder's valuation of an object. To the best of our knowledge, there is no consensus among economists as to what distribution the buyer's valuation should follow. Obviously, we need nonparametric methods to overcome this problem.

Fortunately, in this literature, studies that focus on nonparametric identification and nonparametric estimation methods are well developed. Athey and Haile (2007) provided an excellent survey of the nonparametric (and semiparametric) structural estimation of auction data. Most studies in this literature have been based on the following two papers: Athey and Haile (2002) and Guerre et al. (2000) (hereafter GPV). Athey and Haile (2002) discussed the nonparametric identification conditions in standard auctions. They showed that the distribution of bidders' valuations (viz., the structural parameters) can be identified from observed bids in the independent private values model. GPV provided a nonparametric estimation strategy for first-price, sealed-bid auctions. We briefly review GPV's estimation algorithm.

The settings are similar to the first-price, sealed-bid auction explained in the previous subsection. The only difference is that we do not impose any parametric specifications on the distribution of bidders' valuations, $F(\cdot)$. Recall that the equilibrium bidding function of bidder i with a valuation of $V_i = v$, $\beta(v)$ is

$$\beta(v) = v - \frac{\int_{\underline{v}}^{v} [F(u)]^{N-1} du}{[F(v)]^{N-1}}. \tag{1.1}$$

Then, if we have the inverse function of $\beta(\cdot)$, the distribution of bidders' valuations can be estimated from observed bids, since $\beta(v)$ is a strictly increasing function of v. In general, however, computing inverse function is very difficult.[3] Therefore, GPV gave up on trying to estimate the structural parameters from equation (1.1) directly.

Instead of equation (1.1), GPV used the following equation:

$$v = \beta(v) + \frac{1}{N-1} \cdot \frac{F(v)\beta'(v)}{f(v)}, \tag{1.2}$$

where $\beta'(\cdot)$ is the derivative of $\beta(\cdot)$, and $f(\cdot)$ is the probability density function of bidders' valuations. Equation (1.2) can be derived from optimization problem of bidder i.[4] Let $G(\cdot)$ and $g(\cdot)$ be the cumulative distribution function and the probability density function, respectively, of the bidder's bid, b. Since $b = \beta(v)$, we have $G(b) = F(v)$ and $g(b) = f(v)/\beta'(v)$. Substituting these equations into equation (1.2), we have

$$v = b + \frac{1}{N-1} \cdot \frac{G(b)}{g(b)}. \tag{1.3}$$

[3]Bierens and Song (2012) estimated the distribution of bidders' valuations directly using a sieve approach.

[4]Note that the equilibrium function, $\beta(\cdot)$, satisfies equation (1.2). One can derive equation (1.2) easily by differentiating both sides of equation (1.1), with respect to v.

Note that each element in the right-hand side of equation (1.3) can be observed or recovered from the observed data. Let $\hat{G}(\cdot)$ and $\hat{g}(\cdot)$ be the sample analogues of $G(\cdot)$ and $g(\cdot)$, respectively. Then, we obtain the *pseudo* values, $\hat{v}$, from

$$\hat{v} = b + \frac{1}{N-1} \cdot \frac{\hat{G}(b)}{\hat{g}(b)}. \tag{1.4}$$

For each observed bid, b_{it}, we gain the corresponding *pseudo* values, $\hat{v}_{it}$, from equation (1.4) for each $i \in \{1, ..., N\}$ and $t \in 1, ..., T$, where index i denotes the bidder's identity, and index t denotes the observed auctions. Then, the kernel density estimator, with trimming the boundary values of bids, enables us to gain the sample analogues of the probability density function of the *pseudo* values, $\hat{f}(\cdot)$. GPV showed the consistency of the estimators and derived the optimal uniform convergence rate, which is slower than the optimal rate when valuations v_{it} were observed.[5]

Since implementing their estimator is simple, many papers apply their estimation strategy. However, the nonparametric GPV estimator often faces two problems: the curse of dimensionality and identifiability. The first difficulty of nonparametric estimation like GPV's is the curse of dimensionality, which stems from the dimensionality of covariates. In particular, Hubbard et al. (2012) pointed out that even when there is no covariate, the nonparametric estimator proposed in Li et al. (2002), which is an extension of GPV's estimator of first-price auctions within the affiliate private values paradigm, suffers from the curse of dimensionality stemming from the number of bidders in the first-price auctions within the affiliated private values paradigm. Hubbard et al. (2012) proposed the semiparametric estimator, which is an extension of GPV and Li et al. (2002). They utilized the parametric copula to capture the dependency of valuations among bidders and succeeded in reducing the curse of dimensionality as it relates to the

[5]GPV did not derive the asymptotic distribution of their estimators. Marmer and Shneyerov (2012) proposed a quantile-based nonparametric estimator, which is similar to GPV's estimator. Their estimator attains the optimal rate of the GPV estimator and has asymptotic normality.

number of bidders.

The second problem of the nonparametric estimation is identifiability. As the econometric model becomes complex, the structural parameters (e.g., the distribution of valuations) often cannot be nonparametrically identified from the observed data. Typical examples are common value auctions and bidders with risk-averse preferences. Although the common value auction model is an important theoretical model in auction theory, little empirical research focuses on the common value model. The main reason is that Athey and Haile (2002) showed that structural parameters cannot recovered nonparametrically from the observed bids.[6] Therefore, empirical studies such as that of Bajari and Hortaçsu (2003) impose some parametric specifications in the literature of the structural estimation of auctions within the common value paradigm. For this same reason, in Chapters 3 and 4, we impose the parametric specifications on the distribution of bidders' types.

Considering bidders' risk aversion also makes nonparametric identification of structural parameters difficult.[7] The identification problem is simple, since the utility functions are linear functions of valuations when we assume a risk-neutral preference of bidders. When we consider risk-averse bidders, however, the identification problem becomes difficult. Since risk aversion only restricts the concave shape of the utility functions with respect to valuations, the utility functions are not uniquely determined from the observed bids, obviously. Lu and Perrigne (2008) and Campo et al. (2011) provided semiparametric models for first-price auctions with risk-averse bidders and made estimates assuming additional identification conditions. For a similar reason, Chapter 5 of this thesis proposes a semiparametric estimation that is an extension of GPV's for the

[6]Some papers have studied the identification condition of the common value auction model. For example, Li et al. (2000) showed the identification under the additive separability of the common value component. Février (2008) showed the identification of the common value auction model restricting the shape of the density function of the common value. d'Haultfoeuille and Février (2008) proposed the identification condition of the common value auction model, assuming the support of a private signal is finite and varies depending on the common value.

[7]Guerre et al. (2009) investigated the nonparametric identification strategy for first-price auctions with risk-averse bidders using exclusion restrictions.

scoring auctions.

1.2 Auctions Focused on in the Thesis

As discussed in Section 1.1, various kinds of auctions are indispensable for our social economy. However, in practice most auctions differ from the standard auction models considered in auction theory. In this thesis, we focused on two sorts of auctions: online auctions and scoring auctions. These two auction formats also differ from the typical auction models studied in auction theory. In this section, we discuss the features of online auctions and scoring auctions.

1.2.1 Online Auctions

With the emergence of online auction markets, auctions have become familiar places where we can actually participate. In the online auction market, many people can become buyers or sellers and trade items easily.

In online auctions, anonymous people can become both buyers and sellers. This may lead to greater potential for Internet fraud related to online auctions. A typical example of Internet fraud is that sellers do not send goods to winning bidders even though they have received payment. For this reason, many online auction sites allow winning bidders and sellers to leave feedback after each pair of users conducts a transaction. All of the recorded ratings of all users are public information. This system is called *the feedback system.* From this point of view, online auctions differ from the typical auctions in auction theory. In Chapter 2, we focused on this issue.

Another difference between online auctions and typical auctions is the option of "buy prices." In online auction sites such as eBay, in addition to an auction, a seller can set a fixed price and a bidder can purchase the object if she accepts the buy price. In online auctions, therefore, bidders have to participate in auctions while observing the fixed prices. In Chapter 3, we proposed an empirical model of auctions with buy prices.

1.2.2 Scoring Auctions

Governments, as buyers of public goods, often choose suppliers using procurement auctions. The percentages of governments' expenditures for procurement auctions cannot be ignored in the gross domestic product (GDP). In many countries, governments tend to use the scoring auction format rather than the price-only auction.

The rules of price-only procurement auctions are simple. At price-only auctions, each bidder submits a bid in a sealed envelope, and at some predetermined time, all of the envelopes are opened. The participant with lowest bid wins the project contract, and she fulfills the contract for the price she bids. The amounts of the bid determines the winner in the price-only auctions.

In contrast, in scoring auctions, winners are determined not only by the amounts of their bids. Other factors help determine the winners in scoring auctions. At scoring auctions, each bidder submits a price-quality pair in a sealed envelope, and at some predetermined time, all of the envelopes are opened. The participant with the lowest score, which is calculated from the submitted price and the quality level, wins the project contract. The winner receives the payment she bids and fulfills the contract, providing the quality level she bids.[8] The scoring rules (i.e., the method of calculating scores from the submitted prices and qualities) have been published in advance. Thus, not only price but also quality determines the winners in scoring auctions. Examples of quality include noise level, completion time, and bidder experience.

A variety of forms of scoring rules are used in real-world public procurement. Some U.S. states' departments of transportation, for example, Delaware, Idaho, Oregon, Massachusetts, Utah, and Virginia, use *quasilinear scoring rules.* In quasilinear scoring rules, the score, s, is computed by the difference between the payment, p, and the qual-

[8]These are called *first score auctions* in Chapter 5. In Chapter 5, other auction formats are considered to compare the performance of the first score auctions.

ity level, q (viz., $s = p - q$). On the other hand, Alaska, Colorado, Florida, Michigan, North Carolina, and South Dakota use *price-over-quality ratio rules.* Price-over-quality ratio scoring rules are also used in most public procurement scoring auctions in Japan. In price-over-quality ratio scoring rules, the score, s, is computed by the dividing the payment, p, and the quality level, q (viz., $s = p/q$).

In Chapter 5, we proposed an empirical model that covers various kinds of scoring auctions, including quasilinear scoring rules and price-over-quality ratio scoring rules.

1.3 Organization of the Dissertation

1.3.1 Chapter 2: Inefficiency in Online Auctions

In Chapter 2, we estimated the inefficiency of online auctions. Online auctions can be inefficient due to Internet fraud. A typical example of Internet fraud is when sellers do not send goods to winning bidders, even though they have received payment. Therefore, bidders always bear a risk of fraud, and this risk may lead to transaction failure. To mitigate this risk, most online auction sites (e.g., eBay, and Yahoo!) utilize a feedback system. All winning bidders and sellers can leave feedback after each pair of users conducts a transaction. All of the recorded ratings are observable by all users before they participate.

Many studies have examined the effect of reputation on winning bids. However, such research mainly focuses on the effect that relates to the revenue of online auctions. We focus on the effects of reputation that relate to not only the revenue but also the efficiency of online auctions. Usually, a real-world auction is weakly efficient, in the sense that the bidder with highest willingness to pay always wins the item at the auction without reserve price. However, online auctions are not efficient because of Internet fraud. In other words, a bidder with the highest valuation may fail to win the item if she estimates the possibility of being defrauded to be relativity high. The main purpose of Chapter 2 is to estimate the magnitude and the frequency of inefficiency in online auction markets.

In our empirical example, we use eBay PlayStation 3 auctions held in 2009. We found that online auctions are inefficient with probability of more than 0.75. Namely, in more than 75% of online auctions, the objects (PlayStation 3) are not awarded to the bidders with the highest valuations. Besides, we found that the expected efficiency loss is about \$40. Since the market price of PlayStation 3 in 2009 was \$400, the value of estimated inefficiency is not small.

1.3.2 Chapter 3: Online Auctions with Buy Prices

One important difference between online auctions and the typical auction model in auction theory is the buy price option. Most online auction sites allow sellers to set a fixed buy price. In an auction with a buy price, the seller sets a fixed price, and a bidder can purchase the item if he or she accepts the buy price. In other words, in auctions with buy price options, buyers can receive the goods without going through an auction. In online auctions with buy prices, bidders must participate in auctions while observing the buy prices.

While many empirical studies have focused on the online auction market, most of these studies ignore the buy prices. When we estimate the structural parameters in the online auction model, ignoring the buy prices, the estimators may be incorrect. In Chapter 3, we constructed a structural econometric model of online auction models with buy prices.

Our empirical example is eBay mint coin auctions in 2013. We found that when we ignore the buy prices, we underestimate the mean of bidders' signals corresponding to the value of an item and the effect on the signals of sellers' positive rating. We computed the optimal buy price that maximizes the sellers' expected revenue using the estimated parameters. The estimated optimal buy price is \$53.20, which is higher than the average buy price. We also conducted a revenue comparison. We compared the revenue between auctions with buy prices and those without buy prices. We found that the mean of the revenue difference between auctions with optimal buy prices and auctions without buy prices is \$0.05.

1.3.3 Chapter 4: Bundling and Separate Sales in Online Auctions

In this paper, we focused on bundling auctions in the online auction market. Since there are many buyers and many sellers in the online auction market, various forms of sales are used in online auctions. In particular, some sellers often sell two or more items in bundling auctions. However, other sellers sell the separately.

Many studies focus on bundling sales in theoretical literature. In contrast, few empirical studies have researched bundling sales. In Chapter 4, we conducted an empirical study using both the data of bundling auctions and separate auctions.

Our empirical example is eBay mint coin set auctions in 2014. In our data set, there were two kinds of coin sets: 11-coin sets and 22-coin sets. We regarded 11-coin sets as the separate item and 22-coin sets as the bundled item. We conducted some simulations using the estimated parameters. We evaluated how aggressively bidders in separate auctions bid and compared the revenue of bundling auctions and separate auctions. We found that bidders in separate auctions will bid aggressively. Bidders in separate auctions will bid higher than in bundling auctions by \$2.4. For a revenue comparison, we considered two scenarios: the independent signals case and the identical signals case. In the independent signals case, we found that the expected revenue in bundling auctions was higher than that in separate auctions by \$0.37. In the identical signals case, we found that the expected revenue in separate auctions was higher than that in bundling auctions by \$5.30.

1.3.4 Chapter 5: Scoring Auctions

We established a structural estimation method of the scoring auction model that covers a broad class of scoring rules, including the quasi-liner scoring rule and the price-over-quality ratio rule.[9] In many countries, when governments decide the suppliers of public goods via procurement auctions, they often utilize scoring auctions rather than price-only auctions. In scoring auctions, the winners are determined not only by the prices but also by the quality that bidders bid. There are two typical scoring auction formats: the quasi-linear scoring rule and the price-over-quality ratio rule.

Since in scoring auction models bidders' types are often multi-dimensional, the model often becomes complex. The complexity of the econometric model often obstructs econometricians' study of scoring auctions. We propose a semiparametric model for identifying the joint distribution of biddersE multi-dimensional private signals from scoring auction data and conduct an empirical experiment to quantify the welfare impact of changing of formats and scoring rules for both bidders and the procurement buyer.

The data used in our empirical illustration contain the bid results of procurement auctions for civil engineering projects from 2010 to 2013 by the Ministry of Land, Infrastructure, and Transportation (MLIT) in Japan. We found that changing the auction format has a very small impact on welfare; under the price-over-quality ratio scoring rule, the procurement buyer has an approximately 0.003 to 0.004 percent lower utility (higher exercised score) when using first score auctions rather than the second score auctions, whereas the winning bidder earns a payoff greater by approximately 0.15 to 0.26 percent in first score auctions, as opposed to second score auctions. Furthermore, with a well-designed quasi-linear scoring rule, we found that the procurement buyer improves utility by approximately 0.29 percent, while bidders earn lower payoffs by 3.4 to 4.2 percent. In addition, the outcome of a price-only auction is compared with that of currently used price-over-quality ratio first score auctions. In simulated price-only

[9] This is a joint work with Jun Nakabayashi.

auctions, bidderspayoffs vary, ranging from -41.2 to 1.34 percent, whereas the procurement buyer 's utility is consistently 1 to 36 percent lower than with a price-over-quality ratio first score auction. These results suggest that a procurer can obtain an almost equivalent (slightly lower) gain with the use of a price-only auction with a well-designed fixed quality standard.

Chapter 2

Estimating Inefficiency in Online Auctions

2.1 Introduction

Many people use consumer-to-consumer electronic commerce sites to buy (or sell) goods. A common example of this is the online auction, in which a consumer posts an item for sale and other consumers bid to purchase it. In the third quarter of 2008, eBay, the largest online auction marketplace, hosted 700 million listings with $14 billion in goods trading and had 370 million registered users around the world.[1] In light of this, several studies have focused on online auctions. Examples include Melnik and Alm (2002), Livingston (2005), Houser and Wooders (2006) and Resnick et al. (2006) in reputation effects on sellers' revenue, Bajari and Hortaçsu (2003) in common value auction and winners' curse, Bapna et al. (2008) and Giray et al. (2009) in consumer surplus, Adams (2007) in demand in eBay, Hossain and Morgan (2005) in revenue equivalence theorem and Roth and Ockenfels (2002) in snipe bidding.

However, the growth of online auction markets also leads to greater potential for

[1] See eBay Inc. Reports Third Quarter 2008 Results (`http://investor.ebay.com`).

Internet fraud related to online auctions. A typical example of this is that sellers do not send goods to winning bidders even though they have received payment. Bidders always bear a risk of fraud, and this risk may lead to transaction failure. To mitigate this risk, many online auction sites (e.g., eBay, Amazon, and Yahoo!) allow winning bidders and sellers to leave feedback after each pair of users conducts a transaction. In addition, all the recorded ratings are observable by any users before they participate in an auction. This system is called the feedback system or the reputation system.

Many researchers have studied the effect of reputation on winning bids. Melnik and Alm (2002) applied the Tobit model and estimated the impact of the seller's reputation on the willingness of bidders to bid on items using data concerning coin sales as an example. They found that the seller's reputation has a positive but small impact on the price. Livingston (2005) examined the effect of the seller's reputation on the bidders' decision to participate and the willingness of bidders to bid on an item. Empirical results using data for golf clubs sold show that the seller's reputation has a positive impact on both the bidders' decision to participate and the willingness of bidders to bid on an item. Houser and Wooders (2006) assumed a log linear relationship between the bids and the reputation of the seller, and examined the effect of reputation on a winning bid. They reported that the seller's reputation has a statistically significant effect on the winning bid, but that the bidder's reputation does not. Resnick et al. (2006) conducted a controlled field experiment. In the experiment, the same honest seller sold to several bidders under his or her regular identity, which has a strong reputation, and under a new seller identities. Their results show that the established identity fared better; the difference in the bidder's willingness to pay was 8.1% of the selling price.

However, few studies have tried to examine the effect of reputation on the efficiency of the online auction. Usually, a "real-world" auction is a *weakly* efficient mechanism (i.e., the bidder with highest valuation always wins the item at auction without reserve price) within the independent private values (IPV) paradigm.[2] In an online auction, however, the bidder with the highest valuation can fail to win the item if he estimates the possibility of being defrauded to be relativity high. Therefore, an online auction can be inefficient.

Though efficiency is an important consideration in market (or auction) design (e.g., Maskin (2003)), few attempts have been made to estimate the efficiency losses in online auction markets. One possible explanation for the lack of attention is the difficulty of identifying the inefficiency. To identify the inefficiency, we usually need the data from efficient auctions to compare against that from online auctions. Unfortunately, these data are often unavailable.

In this paper, we estimate the inefficiency in online auctions using only online auction data. Dividing the private values in online auctions into the evaluation of risks and the (original) willingness to pay, we estimate the distributions of private values using only online auction data sets under our identification conditions. Consequently, we estimate the inefficiency without the data of counterfactual efficient auctions. To the best of our knowledge, this is the first empirical attempt to evaluate the inefficiency in online auctions.

Our empirical example is eBay PlayStation 3 auctions held in 2009. We found that bidders' confidence increases with the number of positive ratings and decreases with the number of negative ratings. These results are plausible for our intuition. Furthermore, using the values of estimated parameters, we estimate the inefficiency and the revenue difference. As discussed in Section 2.2, the efficiency loss can be computed by the difference between the total surplus of the efficient auctions and the total surplus of the online auctions. The inefficiency is estimated at \$43.5. The probability of the inefficient online auctions is estimated at 0.762. Thus, 76.2% online auctions are inefficient auctions. The revenue difference between the efficient auctions and the online auctions is estimated

[2]Generally, real-world auctions can be inefficient as well as online auctions. For example, auctions with asymmetric bidders may be inefficient (e.g., Maskin and Riley (2000)).

at \$83.0.

The rest of the paper is organized as follows. In Section 2.2, we present our inefficient online auction model. In addition, we discuss the relationship between the inefficiency and the total surplus in online auctions. In Section 2.3, we discuss how to estimate the structural parameters in our inefficient online auction model described in Section 2.2. In this paper, we impose parametric specifications for the structural model and use the Bayesian Markov Chain Monte Carlo (MCMC) method to estimate the structural parameters. After explaining the estimation strategy, we discuss how to estimate the inefficiency of an online auction market. Monte Carlo experiments are conducted in this section. In Section 2.4, we explain the eBay PlayStation 3 auction data used in our empirical example. We present the estimation results from the eBay PlayStation 3 auction data. In Section 2.5, we show the estimation result of inefficiency in eBay PlayStation 3 auctions. Section 2.6 concludes.

2.2 The Model

We develop a theoretical model that describes the *ex post* heterogeneity of bidders' evaluations for the risk of default. Furthermore, this model is useful for our empirical analysis to estimate the efficiency loss in online auction markets. Throughout this paper, we represent random variables in uppercase letters and their realizations in lowercase letters.

2.2.1 Theoretical Model

We observe L online auctions. In each auction $l \in \{1, 2, \cdots, L\}$, there are N_l risk neutral potential bidders and a seller. The number of potential bidders, N_l is a random variable and is an exogenous variable. At the beginning of auction l, seller l offers for sale a single item and sets a reserve (or starting) price r_l. Bidders submit their bids, and the bidder with the highest bid wins the object and pays the second highest bid. After auction l, seller l decides whether to cooperate or to deceive. If he cooperates, then the winning bidder receives the good from the seller. On the other hand, if he deceives, then the winning bidder receives nothing or a low-quality good (e.g., a defective good, a fake good and so on) from the seller. After the transaction, the winning bidder leaves a positive report if seller l cooperates and leaves a negative report if seller l deceives.[3]

We focus on the symmetric independent private values (IPV) model. Each bidder $i \in \{1, 2, \cdots, N_l\}$ has her private value v_{li} which represents her willingness to pay for the item at auction l and is the realization of a random variable V_{li}. These private values are i.i.d. random draws from a probability distribution $F(\cdot)$ with density $f(\cdot)$. The support of $F(\cdot)$ is denoted by $[\underline{v}, \bar{v}]$ where $\underline{v}$ is a positive number. Each bidder i knows the realization of her own valuation v_{li} but does not know that of others. Instead, the probability distribution $F(\cdot)$ is common knowledge among all bidders at auction l. Each bidder i discounts her willingness-to-pay v_{li} in online auction l because of the risk of Internet fraud. Observing the ratings of seller l, each bidder i at auction l estimates the risk that seller l deceives and discounts the willingness-to-pay v_{li}. Let $D_{li} \in [0, 1]$ denote the *risk-discount factor*. The risk-discount factor D_{li} is an i.i.d. random draw from a probability distribution $Q(\cdot|X_l)$ with density $q(\cdot|X_l)$, where X_l is the auction-specific covariate vector. The auction-specific covariate vector X_l contains the number of positive ratings and that of negative ratings of the seller l. The support of $Q(\cdot|X_l)$ is $[0, 1]$ for any auction-specific covariate vector X_l. Each bidder i knows the realization of her own risk-discount factor d_{li} but does not know that of others. Instead, the probability distribution $Q(\cdot|X_l)$ is common knowledge for any auction-specific covariate X_l among all bidders at auction l. We assume that bidder i's valuation V_{li} and her risk-discount factor D_{li} are mutually independent. In addition, we make an assumption on the risk-discount factor

[3] On eBay, winning bidders can leave either "Positive," "Neutral," or "Negative" reports. In addition, winning bidders can leave comments about sellers. For the sake of simplicity, we assume that winning bidders can leave only positive or negative reports. In our empirical example, the number of negative ratings is the sum of the number of neutral ratings and that of negative ratings.

D_{li}.

Assumption 1. *The risk-discount factor D_{li} converges in probability to a constant $d_* \in [0,1]$ as the number of ratings of seller l goes to infinity. Furthermore, the valued of d_* can be computed from the number of positive ratings and that of negative ratings.*

Assumption 1 implies when the number of ratings of seller l is sufficiently observed, all bidders' estimates for the risk of Internet fraud coincide. This assumption seems to be plausible since the number of ratings reflects the actions of seller l and the decisions of seller l are exogenously determined in our model.

We assume risk neutral bidders. Then, the utility function of bidder i is denoted by

$$u(b_{li}, b_{l-i}|v_{li}, d_{li}) = \begin{cases} d_{li}v_{li} - \max_{j\neq i} b_{lj} & \text{if } b_{li} = \max\{b_{l1}, \cdots, b_{lI_l}\} \\ 0 & \text{otherwise.} \end{cases}$$

where b_{li} is bidder i's bid at auction l and b_{l-i} represents the vector of bids at auction l except for bidder i's bid. Let $Z_{li} \equiv D_{li}V_{li}$, and let $G(\cdot)$ and $g(\cdot)$ be the cumulative distribution function and the probability density function of Z_{li}, respectively. Hereafter, Z_{li} is referred as bidder i's "risk-discounted" willingness to pay or bidder i's "risk-discounted" valuation. Then, ignoring the possibility of a tie, we have the following result.

Proposition 1. *A bidding strategy profile $\{z_{l1}, ..., z_{lN_l}\}$ is a Bayesian Nash equilibrium.*

This result is similar to Houser and Wooders (2006). While they considered the subjective probability instead of the risk-discount factor, and they assumed it is common among all bidders at auction l, the realization of bidders' risk-discount factor is different in our model. Under such an equilibrium, the bidder with the highest risk-discounted value $z_{l(1)}$ wins, and the winning bid w_l is given by $w_l = z_{l(2)}$, where $Z_{l(i)}$ is the i-th largest order statistic. That is, $Z_{l(1)} \geq Z_{l(2)} \geq \cdots \geq Z_{l(N_l)}$.

2.2.2 Efficiency Loss

In this subsection, we discuss why online auctions can be inefficient and discuss how to calculate the efficiency losses in online auctions.

Suppose all buyers and a seller in an online auction use a real-world auction to trade the object instead of an online auction. Notice that since all the participants trade in a public and face-to-face situation, none of the buyers must worry about the risk of being defrauded by the seller. Thus, each bidder's risk-discount factor D_{li} $(i \in \{1, \cdots, N_l\})$ equals one with probability one in real-world auctions. Then, the bidder with the highest private value $v_{l(1)}$ wins, and the winning bid w_l is given by $w_l = v_{l(2)}$ where $V_{l(i)}$ is the i-th largest order statistic and $v_{l(i)}$ denotes the realization of $V_{l(i)}$. Therefore, real-world auctions are efficient.

In contrast to a real-world auction, the bidder with the highest risk-discounted private value always wins in an online auction. This does not always imply that the bidder with highest private value wins. Let $\mathrm{Id}(\cdot)$ denote the mapping from the bidder's private value to her identity. That is,

$$\mathrm{Id}(a_{li}) = i, \quad (a_{li} = v_{li} \text{ or } z_{li}).$$

Then, an online auction l is inefficient if and only if $\mathrm{Id}(v_{l(1)}) \neq \mathrm{Id}(z_{l(1)})$. Furthermore, we characterize the inefficiency using the valuations v. Let v_{l*} denote the "original" private value of bidder $\mathrm{Id}(z_{l(1)})$ (i.e., $v_{l*} = v_{l\mathrm{Id}(z_{l(1)})}$). Then, the efficiency loss occurs only when $v_{l(1)} \neq v_{l*}$ holds. In particular, since $v_{l(1)} \geq v_{l*}$ always holds, we gain the following result:

$$\begin{cases} v_{l(1)} - v_{l*} > 0 & \text{if efficiency loss occurs, and} \\ v_{l(1)} - v_{l*} = 0 & \text{otherwise.} \end{cases}$$

Actually, since we cannot observe the realized value of valuations and risk-discount factors; therefore, we must estimate the value of $v_{l(1)} - v_{l*}$.

2.2.3 Relation between Efficiency and Surplus

Online auction l becomes inefficient if and only if $v_{l(1)} > v_{l*}$ holds. However, the interpretation of $v_{l(1)} - v_{l*}$ itself is not clear. We discuss the relation between efficiency loss and total surplus and show that $v_{l(1)} - v_{l*}$ can be interpreted as the difference of total surplus.

First, we consider the total surplus of real-world auctions. Since the winning bid of real-world auction equals the second highest private value $v_{l(2)}$, the surplus of the winning bidder is $v_{l(1)} - vl(2)$. Analogously, the surplus of the seller is $v_{l(2)} - v_{l0}$, where v_{l0} is seller l's valuation for the object. Therefore, the total surplus of real-world auction l, $TS_{l\mathrm{RA}}$ is denoted by

$$TS_{l\mathrm{RA}} = v_{l(1)} - v_{l0}. \tag{2.1}$$

We gain the total surplus of online auctions in the same manner. Since the original valuation of winning bidder in online auction l is denoted by v_{l*}, the surplus of the winning bidder is given by $v_{l*} - z_{l(2)}$. Note that the winning bid of online auction equals to the second highest risk-discounted valuation. Analogously, the surplus of the seller is $z_{l(2)} - v_{l0}$. Therefore, the total surplus of online auction l, $TS_{l\mathrm{OA}}$ is denoted by

$$TS_{l\mathrm{OA}} = v_{l*} - v_{l0}. \tag{2.2}$$

Then, from equation (2.1) and equation (2.2), we have the difference of total surplus as follows:

$$\begin{aligned} TS_{l\mathrm{RA}} - TS_{l\mathrm{OA}} &= v_{l(1)} - v_{l0} - (v_{l*} - v_{l0}) \\ &= v_{l(1)} - v_{l*}. \end{aligned} \tag{2.3}$$

The seller's private value v_{l0} is difficult to estimate. Fortunately, however, the seller's private value v_{l0} is eliminated in equation (2.3). Consequently, the value of $v_{l(1)} - v_{l*}$

can be interpreted as that of the difference in total surplus.

2.3 Estimation

2.3.1 Identification

Though efficiency is an important consideration in auction theory, few attempts have been made to estimate efficiency losses in online auction markets. One possible explanation for this lack of attention is the difficulty of identifying the inefficiency. In our model, to identify the inefficiency, we must identify the distribution of willingness to pay, V_{li}.

However, neither the distribution of valuations, $F(\cdot)$, nor the distribution of risk-discount factors, $Q(\cdot)$, is identified nonparametrically. Recall that $Z_{li} = D_{li}V_{li}$. Let $D'_{li} \equiv \epsilon D_{li}$ and $V'_{li} \equiv \epsilon^{-1} V_{li}$ where $\epsilon \in (0, 1)$. Then, we have

$$\begin{aligned} Z_{li} &= D_{li}V_{li} \\ &= (\epsilon^{-1}D'_{li})(\epsilon V'_{li}) \\ &= D'_{li}V'_{li}. \end{aligned}$$

Therefore, even if the distribution of risk-discounted valuation, $G(z)$, is identified, neither V nor D is identified. Therefore, we need additional assumptions to identify the distribution of valuations, $F(\cdot)$, and the distribution of beliefs, $Q(\cdot)$, separately.

Assumption 1 is the key assumption of our identification strategy. Since both X_l and Y_l are observable for econometricians, the valued of d_* is also known to econometricians. If d_* is known, and if the distribution of risk-discounted valuation, $G(\cdot)$, is identifiable, since $Z_{li} = d_* V_{li}$, the distribution of "original" valuation, $F(\cdot)$, is also identifiable nonparametrically as the number of ratings goes to infinity.

Once $F(\cdot)$ is recovered, then the distribution of risk-discount factor, $Q_{X_l,Y_l}(\cdot)$, is also identified for every finite X_l and Y_l. Let $\tilde{Z}_{li}$, $\tilde{D}_{li}$, and $\tilde{V}_{li}$ denote the logarithm of Z_{li},

D_{li} and V_{li} respectively. Then, since $Z_{li} = D_{li}V_{li}$, we have $\tilde{Z}_{li} = \tilde{D}_{li} + \tilde{V}_{li}$. That is, $\tilde{Z}_{li}$ is the sum of two independent random variables. Therefore, we have

$$\psi_{\tilde{Z}} = \psi_{\tilde{D}}\psi_{\tilde{V}}, \tag{2.4}$$

where $\psi_{\tilde{Z}}$ is the characteristic function of $\tilde{Z}_{li}$, $\psi_{\tilde{Z}}$ is the characteristic function of $\tilde{D}_{li}$, and $\psi_{\tilde{Z}}$ is the characteristic function of $\tilde{V}_{li}$. Since the distribution of $\tilde{Z}_{li}$ is identified from the standard result when the number of potential bidders is known, the characteristic function of $\tilde{Z}_{li}$, $\psi_{\tilde{Z}}$ is also recovered from the data set. Since $F(\cdot)$ is recovered, the characteristic function of $\tilde{V}_{li}$, $\psi_{\tilde{V}}$ is also recovered. Therefore, the characteristic function of $\tilde{D}_{li}$, $\psi_{\tilde{D}}$ is recovered from equation (2.4). Since the characteristic function of $\tilde{D}_{li}$ is recovered, the distribution of $\tilde{D}_{li}$ is identified. As a result, the distribution of risk-discount factor D_{li}, $Q(\cdot|X_l, Y_l)$, is identified for every $X_l < \infty$ and $Y_l < \infty$. Therefore, we have the following result.

Proposition 2. *Suppose $Q(\cdot|X_l = 0, Y_l = 0)$ is a non-singular distribution whose support is $[0, 1]$. Suppose that the distribution of discounted willingness to pay, $G(\cdot)$ is identifiable. Then, both the distribution of willingness to pay $F(\cdot)$, the distribution of belief $Q(\cdot|X_l, Y_l)$ and efficiency loss $V_{l(1)} - V_{l*}$ are identified at infinity (i.e., $T_l \to \infty$).*

So far, we assume that the distribution of risk-discounted valuation, $G(\cdot)$, is identifiable. If the number of potential bidders, N_l is observable, then $G(\cdot)$ is identified in our model. [4] Unfortunately, the number of potential bidders, N_l is not observed in eBay auctions. When the number of potential bidders, N_l is not known, the distribution of risk-discounted valuation, $G(\cdot)$, is not identified from only winning bids w_l nonparametrically.

Several papers consider the identification of the auction model with an unknown number of bidders. First, Guerre et al. (2000) assume the number of potential bidders,

[4] For example, Donald and Paarsch (1996) consider identification of auction model in parametric setting and Athey and Haile (2002) study identification in nonparametric setting.

N, is unknown for econometricians but is constant among all auctions. Under their assumption, they show that the distribution of valuations (i.e., $G(\cdot)$ in our model) is identifiable. Song (2004) considers the identification and estimation of eBay auctions with an unknown number of bidders. She shows that the distribution of valuations (i.e., $G(\cdot)$ in our model) is identifiable from observation of any two valuations for which rankings from the top are known. Using the second and third highest bids in eBay university yearbook auctions, she estimates the distribution of valuations by the semi-nonparametric maximum likelihood estimation method proposed by Gallant and Nychka (1987). An et al. (2010) study nonparametric identification and estimation of first-price auction models with an unknown number of bidders. They develop a nonparametric procedure for recovering the distribution of bids using instruments that exogenously affect the number of potential bidders. Shneyerov and Wong (2011) consider nonparametric identification of first-price and Dutch auction models when the number of potential bidders is unobservable. Although Song (2004) and An et al. (2010) focus on symmetric independent private values models, they study identification of asymmetric IPV models. In this paper, we apply Guerre et al. (2000)'s identification strategy to recover the distribution of risk-discounted valuation, $G(\cdot)$. That is, assuming that the number of potential bidders, N is constant among auctions, we recover the distribution of risk-discounted valuation, $G(\cdot)$. Then the identification of efficiency loss follows from Proposition 2.

2.3.2 Estimation Procedure

From Proposition 2, if $G(\cdot)$ is identified, both the distribution of original valuations, $F(\cdot)$, and the distribution of beliefs, $Q(\cdot)$, are also identified as the number of ratings goes to infinity. Actually, the number of auctions seller l has held, the number of ratings is finite for each $l \in \{1, ..., L\}$. Therefore, we must assume alternative conditions. Instead of the identification condition described above, we assume each bidder i does not discount her original valuation V_{li} if seller l is a PowerSeller. In other words, $D_{li} = 1$ if seller l

is a PowerSeller. PowerSellers receive many positive ratings but few negative ratings. Almost all PowerSellers in our data set gain more than 500 positive ratings.[5]

In this paper, we impose a parametric specification on the distribution of valuation, $F(\cdot)$, and the distribution of belief, $Q(\cdot)$.[6] We apply the Bayesian Markov Chain Monte Carlo (MCMC) simulation method to estimate the parameters. We assume that valuation V_{li} follows the gamma distribution. That is, we assume

$$V_{li} \sim i.i.d.\, \mathrm{Ga}(\alpha, \beta).$$

Furthermore, given auction-specific covariates, we assume that risk-discount factor D_{li} follows the truncated normal distribution with support $[0, 1]$. That is,

$$D_{li}|X_l \sim i.i.d.\, \text{Trunc-N}_{[0,1]}(\mu_l, \sigma_l^2)$$

where X_l is the auction-specific covariates and $\mu_l = \boldsymbol{\gamma}' X_l$ and $\sigma_l = \exp(\boldsymbol{\delta}' X_l)$. As described in Section 2.2, bidder l's risk-discount factor D_{li} depends on the positive and negative ratings of seller l. Therefore, the number of positive ratings and the number of negative ratings are plausible covariates.

First, we estimate the parameters of gamma distribution, α and β. Notice, since we assume that $D_{li} = 1$ for each bidder i if seller l is a PowerSeller, and since $Z_{li} = D_{li} V_{li}$, $g(\cdot)$ and $G(\cdot)$ are equal to $f(\cdot)$ and $F(\cdot)$, respectively, if seller l is a PowerSeller. Therefore, the likelihood of winning bids is

$$L(w_1, ..., w_L|\alpha, \beta) = \prod_{l=1}^{L} \begin{pmatrix} & N & \\ N-2 & 1 & 1 \end{pmatrix} [1 - F(w_l|\alpha, \beta)] f(w_l|\alpha, \beta) [F(w_l|\alpha, \beta)]^{N-2}, \tag{2.5}$$

[5] In Section 2.4, we explain "PowerSeller" in detail.

[6] The distributions $F(\cdot)$ and $Q(\cdot)$ can be estimated nonparametrically using the decomposition technique. However, nonparametric estimation can be computationally burdensome.

where N is the number of potential bidders. Note that letting n_l be the number of active bidders, which is observable for econometricians, we can estimate N consistently by $\hat{N} = \max n_1, ..., n_L$. Therefore, without too much loss of generality, we can assume that N is given.

For auctions in which sellers are not PowerSellers, belief $D_{li} \neq 1$. Using the distribution of risk-discounted valuations, $G(\cdot)$, we have the likelihood function

$$L(w_1, ..., w_L|\alpha, \beta, \mu_l, \sigma_l) = \prod_{l=1}^{L} \begin{pmatrix} & N & \\ N-2 & 1 & 1 \end{pmatrix} [1-G(w_l|\alpha, \beta, \mu_l, \sigma_l)] g(w_l|\alpha, \beta, \mu_l, \sigma_l) \times [G(w_l|\alpha, \beta, \mu_l, \sigma_l)]^{N-2}. \tag{2.6}$$

Unfortunately, the probability density function $g(\cdot)$ has no closed-form expression. Using $q(\cdot)$ and $f(\cdot)$, $g(\cdot)$ can be described as

$$g(w) = \int_0^1 \frac{1}{d} f\left(\frac{w}{d}\right) q(d) \mathrm{d}d.$$

Similarly, using $q(\cdot)$ and $F(\cdot)$, $G(\cdot)$ can be described as

$$G(w) = \int_0^1 F\left(\frac{w}{d}\right) q(d) \mathrm{d}d.$$

Therefore, we can compute the probability density function, $g(\cdot)$, and the probability distribution function, $G(\cdot)$, by numerical integration.

The goal of this study is to estimate the efficiency loss of online auctions. The efficiency loss of online auctions can be computed from equation (2.3). The distribution of original valuation V_{li}, $F(\cdot)$, can be estimated from the procedure described above. Therefore, the distribution of the largest valuation $V_{l(1)}$ can be recovered from the estimated distribution of original valuation V_{li}, $\hat{F}(\cdot)$. The distribution of risk-discounted valuation Z_{li}, $G(\cdot)$, can also be obtained. As explained above, the distribution function of risk-discount factor, $Q(\cdot)$, can be recovered from the estimated distribution function of risk-discounted valuation Z_{li}, $\hat{G}(\cdot)$, and the estimated distribution function of original valuation V_{li}, $\hat{F}(\cdot)$. Since the distribution of risk-discount factor P_{li}, $Q(\cdot)$, and the distribution of risk-discounted valuation Z_{li}, $G(\cdot)$, can be estimated, the distribution of V_{l*} can be recovered from the estimated distributions $\hat{Q}(\cdot)$ and $\hat{G}(\cdot)$. Concretely, the distribution of efficiency loss defined in equation (2.3) can be obtained by the Monte Carlo method. First, generate random draws $v_1^{(s)}, ..., v_N^{(s)}$ and $p_1^{(s)}, ..., p_N^{(s)}$ from the estimated distributions $\hat{F}$ and $\hat{Q}$. Then compute the maximum value $v_{(1)}^{(s)} \equiv \max\{v_1^{(s)}, ..., v_N^{(s)}\}$. Similarly, compute $z_{(1)}^{(s)} \equiv \max\{p_1^{(s)} v_1^{(s)}, ..., p_N^{(s)} v_N^{(s)}\}$. Calculate the realization of efficiency loss $v_{(1)}^{(s)} - v_{\mathrm{Id}(z_{(1)}^{(s)})}^{(s)}$. Iterate this procedure until s becomes a large number, S.

2.3.3 Simulation Experiment

We estimate the parameters of our model using simulation data. The number of observed auction markets is $L = 750$. The number of observed auctions with a PowerSeller is $L_1 = 300$ and the number of auctions with a non-PowerSeller is $L_2 = 450$. In our simulation experiments, the number of potential bidders, N is equal to 5 for all auctions. We draw the valuations V from gamma distribution with parameters 6 and 2.

Analogously, we draw the risk-discount factor D from the truncated normal distribution, Trunc-$\mathrm{N}_{[0,1]}(\mu_l, \sigma_l)$. The parameters of truncated normal distribution are given by $\mu_l = \gamma_1 + \gamma_2 \cdot \mathrm{Pos.Rep.}_l + \gamma_3 \cdot \mathrm{Neg.Rep.}_l$ and $\sigma_l^2 = \exp(\delta_1 + \delta_2 \cdot \mathrm{Pos.Rep.} + \delta_3 \cdot \mathrm{Neg.Rep.})$. The true values of $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are

$$\gamma_1 = 0.5, \gamma_2 = 0.1, \gamma_3 = -0.15,$$

$$\delta_1 = -1.0, \delta_2 = 0.17, \text{ and } \delta_3 = -0.3.$$

The prior distributions of gamma parameters α and β are

$$\alpha \sim \mathrm{N}(0, 1000) \text{ and } \beta \sim \mathrm{N}(0, 1000).$$

The prior distributions of the parameters of truncated normal distribution, $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are

$$\boldsymbol{\gamma} \sim \mathrm{N}(\mathbf{0}, 1000\boldsymbol{I}) \text{ and } \boldsymbol{\delta} \sim \mathrm{N}(\mathbf{0}, 1000\boldsymbol{I}).$$

We apply the random walk-based MH algorithm to compute the posterior distribution of parameters. The number of iteration is 70000, and burn-in period is 10000.

Table 2.1 shows p-values of the convergence diagnostics for the MCMC (CD) and inefficiency factors.[7] All p-values of the convergence diagnostics are more than 0.01. Furthermore, the values of the inefficiency factor values are sufficiently low. The inefficiency factors are 63.06 to 228.63, which implies that we would obtain the same variance of the posterior sample means from 300 uncorrelated draws, even in the worst case. Hence, we conclude that the sample paths of estimated parameters converge to posterior distributions.

Parameter	Covariate (Coefficient Parameter)	CD	Inefficiency factor
α	–	0.04	228.63
β	–	0.04	228.42
μ	Const (γ_0)	0.30	98.96
	Pos.Rep. (γ_1)	0.98	126.66
	Neg.Rep. (γ_2)	0.51	133.84
σ^2	Const (δ_0)	0.27	104.88
	Pos.Rep. (δ_1)	0.65	63.06
	Neg.Rep. (δ_2)	0.20	64.67

Table 2.1: The convergence diagnostics for the MCMC (CD) and the inefficiency factors for the simulation data

[7] The CD test statistic tests the equality of the means of the first part and last part of the sample path. The definition of inefficiency factor is $1 + 2\sum_{k=1}^{\infty} \rho(k)$, where $\rho(k)$ is the sample autocorrelation at lag k.

The result is shown in Table 2.2. Our estimator contains the true values in a 95% credible interval. We conclude that our estimator performs well.

Parameter	Covariate (Coefficient Parameter)	True	Mean	SD	95% credible interval
α	–	6.00	5.89	0.50	$(4.90, 6.93)$
β	–	2.00	1.92	0.16	$(1.61, 2.25)$
μ	Const (γ_0)	0.50	0.45	0.05	$(0.34, 0.53)$
	Pos.Rep. (γ_1)	0.10	0.09	0.01	$0.07, 0.12)$
	Neg.Rep. (γ_2)	−0.15	−0.13	0.02	$(-0.17, -0.11)$
σ^2	Const (δ_0)	−1.00	−1.12	0.27	$(-1.63, -0.58)$
	Pos.Rep. (δ_1)	0.17	0.20	0.04	$(0.13, 0.29)$
	Neg.Rep. (δ_2)	−0.30	−0.34	0.04	$(-0.43, -0.26)$

Table 2.2: Posterior inferences for the simulation data

2.4 Empirical Examples

2.4.1 Data

Our empirical example is auctions of PlayStation 3 held on eBay in 2009. Data were collected from 730 completed eBay auctions from June 10 through August 26, 2009. Auctions with fewer than two actual bidders were dropped, since there are no competitions with no bidder or one bidder. Since our model does not account for the use of the "Buy-It-Now" option, auctions in which the item was sold with the Buy-It-Now option were dropped. Consequently, 520 auctions were used to estimate the inefficiency of eBay PlayStation 3 auctions.

A PlayStation 3 auction is an excellent example for estimating the inefficiency of online auctions for at least two reasons. First, PlayStation 3 is a relatively high-value item. In the United States, the market price of PlayStation 3 was about \$400 in 2009. Therefore, bidders would care about the risk of being defrauded by sellers. Second, PlayStation 3 is a homogeneous item. For example, the color of almost all PlayStation 3s is black. Furthermore, we collected only auctions in which the condition of the

PlayStation 3 was new. Auctions for used PlayStation 3s are excluded. Therefore, we can estimate the distribution of valuation, $F(\cdot)$, with few covariates.

Table 2.3 provides the summary statistics. The first column describes variables. "Winning bid" is the transaction price and second highest bid in the eBay auction. Note that the market price of PlayStation 3 was about \$400 in 2009. Therefore, winning bidders could get a PlayStation 3 in eBay auction for \$70 less than the market price on average. "Starting price" is the price a seller sets at the beginning of an auction. All bids must be higher than the starting price. "Positive ratings" denotes the number of positive ratings a seller receives. "Negative ratings + Neutral ratings" is the sum of the number of negative ratings and the number of neutral ratings a seller receives. Usually, the number of neutral ratings and the number of negative ratings are small (see the mean value). We regard neutral ratings as negative ratings. "Number of actual bidders" is the number of participants who actually bid at auction l. Since bidders whose valuation is lower than the starting price cannot bid, the number of actual bidders is less than that of potential bidders. "Days" denotes the duration that auction l was held.

	Mean	Median	Std	Max	Min
Winning bid	328.60	330.00	28.59	405.01	177.50
Starting price	79.92	15.99	98.37	325.00	0.01
Positive ratings	250.60	26.00	1135.97	21752.00	0.00
Negative ratings + Neutral ratings	2.94	0.00	13.19	263.00	0.00
Number of actual bidders	10.31	10.00	3.83	21.00	3.00
Days	3.54	3.00	2.41	17.00	1.00

Table 2.3: Descriptive Statistics (# of obs. = 520)

Next, we present descriptive statistics of PowerSellers in Table 2.4. PowerSeller status is an award for sellers on eBay. Only sellers who have sold many items and receive mostly positive ratings can become PowerSellers. Most PowerSellers receive more than one hundred positive ratings. Indeed, the mean value of positive ratings is 796.83. To compare PowerSellers with non-PowerSellers, we provide the summary statistics of a data set excluding PowerSellers in Table 2.5. The mean value of positive ratings of non-PowerSellers is 39.81. Therefore, PowerSellers receive more positive ratings than non-PowerSellers. The winning bids in PowerSellers' auctions tend to be higher than those in non-PowerSellers' auctions. Most notably, the minimum value of transaction prices in PowerSellers' auctions is much higher than that in non-PowerSellers' auctions. There are few differences in the number of actual participants. However, since the starting prices in PowerSellers' auctions tend to be lower than those in non-PowerSellers' auctions, the number of potential bidders in PowerSellers' auctions may be lower than that in non-PowerSellers' auctions.

	Mean	Median	Std	Max	Min
Winning bid	336.04	335.00	23.51	405.01	266.99
Starting price	35.50	15.95	63.76	313.00	0.01
Positive ratings	796.61	306.00	2041.94	21752.00	12.00
Negative ratings + Neutral ratings	8.66	5.00	23.89	263.00	0.00
Number of actual bidders	11.11	11.00	3.33	21.00	3.00
Days	2.50	1.00	2.09	7.00	1.00

Table 2.4: Descriptive Statistics (PowerSellers, # of obs. = 144)

	Mean	Median	Std	Max	Min
Winning bid	325.72	329.65	29.86	395.00	177.50
Starting price	97.09	49.95	103.94	325.00	0.01
Positive ratings	39.87	14.00	160.17	2892.00	0.00
Negative ratings + Neutral ratings	0.73	0.00	2.02	31.00	0.00
Number of actual bidders	10.00	10.00	3.96	21.00	3.00
Days	3.94	3.00	2.40	17.00	1.00

Table 2.5: Descriptive Statistics (Non-PowerSellers, # of obs. =376)

New entrants are the users who have registered with eBay within one month. Descriptive statistics of new entrants are provided in Table 2.6. Both the mean of"Positive ratings" and the mean of "Negative ratings + Neutral ratings" are less than one.[8] The

[8]The definition of new entrant is the users who have registered with eBay within a month. Thus, some new entrants have sold items and gained ratings.

mean of the winning bids of new entrants' auctions is 309.14. Since the mean value of winning bids using all data is 328.60, new entrants earn relatively small profits.

	Mean	Median	Std	Max	Min
Winning bid	309.14	310.00	27.85	395.00	235.00
Starting price	93.61	65.00	91.66	300.00	0.99
Positive ratings	0.28	0.00	1.20	9.00	0.00
Negative ratings + Neutral ratings	0.02	0.00	0.13	1.00	0.00
Number of actual bidders	9.71	9.00	3.50	18.00	3.00
Days	3.66	3.00	1.69	10.00	3.00

Table 2.6: Descriptive Statistics (New Entrants, # of obs. =61)

2.4.2 Estimation Results

We estimate the structural parameters using eBay PlayStation 3 data. We assume the valuation V_{li} follows the gamma distribution. That is,

$$V_{it} \sim i.i.d.\, \text{Gamma}(\alpha, \beta),$$

where both α and β are the parameters of gamma distribution. Similarly, the distribution of each bidder's risk-discount factor D is assumed to be the truncated normal distribution. That is,

$$D_{it} \sim i.i.d.\, \text{N}_{[0,1]}(\mu_l, \sigma_l^2),$$

where $\mu_l = \gamma_0 + \gamma_1 \cdot \text{Pos.Rep.}_l + \gamma_2 \cdot \text{Neg.Rep.}_l$ and $\sigma_l^2 = \exp(\delta_0 + \delta_1 \cdot \text{Pos.Rep.}_l + \delta_2 \cdot \text{Neg.Rep.}_l)$.

We generate random samples from the posterior distributions by the random walk-based MH algorithm. The number of iteration is 800000, and the burn-in-period is 10000. Figure 2.1 presents sample paths of the estimated parameters.

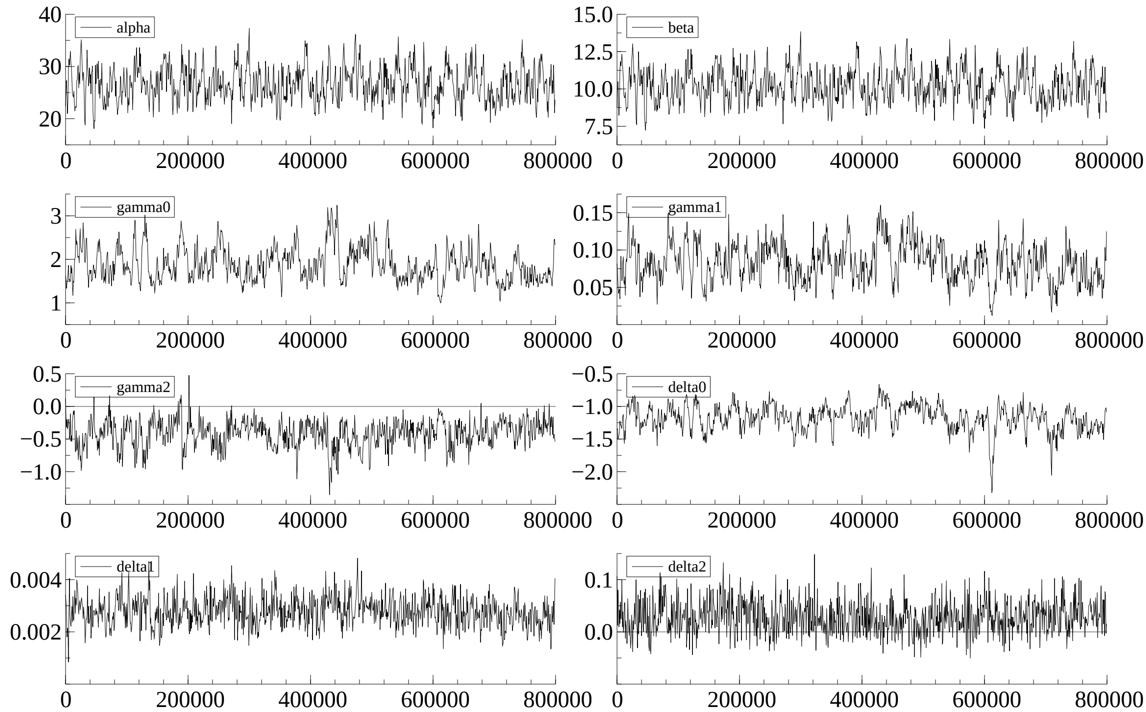

Figure 2.1: Sample path of the gamma parameters (α and β) and the truncated normal parameters ($\boldsymbol{\gamma} = (\gamma_0, \gamma_1, \gamma_2)$ and $\boldsymbol{\delta} = (\delta_0, \delta_1, \delta_2)$)

Table 2.7 shows p-values of the convergence diagnostics for the MCMC (CD) and inefficiency factors. All p-values of the convergence diagnostics are more than 0.01. Moreover, the values of the inefficiency factor values are sufficiently low. The inefficiency factors are 187.7 to 1220.6, which implies that we would obtain the same variance of the posterior sample means from 650 uncorrelated draws, even in the worst case.

Parameter	Covariate (Coefficient Parameter)	CD	Inefficiency factor
α	–	0.80	1051.70
β	–	0.80	1051.80
μ	Const. (γ_0)	0.43	1220.60
	Pos.Rep. (γ_1)	0.14	1139.0
	Neg.Rep. (γ_2)	0.34	994.80
σ^2	Const. (δ_0)	0.29	1186.00
	Pos.Rep. (δ_1)	0.01	632.10
	Neg.Rep. (δ_2)	0.92	187.70

Table 2.7: The convergence diagnostics for the MCMC (CD) and the inefficiency factors (IF) from the eBay data

All p-values of the convergence diagnostics are sufficiently high. In addition, the inefficiency factors are low. From Figure 2.1 and Table 2.7, we conclude that the random samples of estimated parameters converge to posterior distributions.

Table 2.8 shows the posterior means, the posterior standard deviations, and the 95% credible intervals. [9] Figure 2.2 presents the posterior densities of estimated parameters.

Parameter	Covariate (Coefficient Parameter)	Mean	SD	95% credible interval
α	–	26.75	3.29	$(20.74, 33.47)$
β	–	10.27	1.13	$(8.20, 12.57)$
μ	Const. (γ_0)	1.89	0.39	$(1.29, 2.75)$
	Pos.Rep. (γ_1)	0.08	0.03	$(0.04, 0.14)$
	Neg.Rep. (γ_2)	-0.41	0.21	$(-0.88, -0.05)$
σ^2	Const. (δ_0)	-1.18	0.20	$(-1.58, -0.84)$
	Pos.Rep. (δ_1)	0.003	0.001	$(0.002, 0.004)$
	Neg.Rep. (δ_2)	0.03	0.03	$(-0.03, 0.09)$

Table 2.8: Posterior inferences for the eBay data

[9] We divide winning bids by 100 when we estimate the parameters.

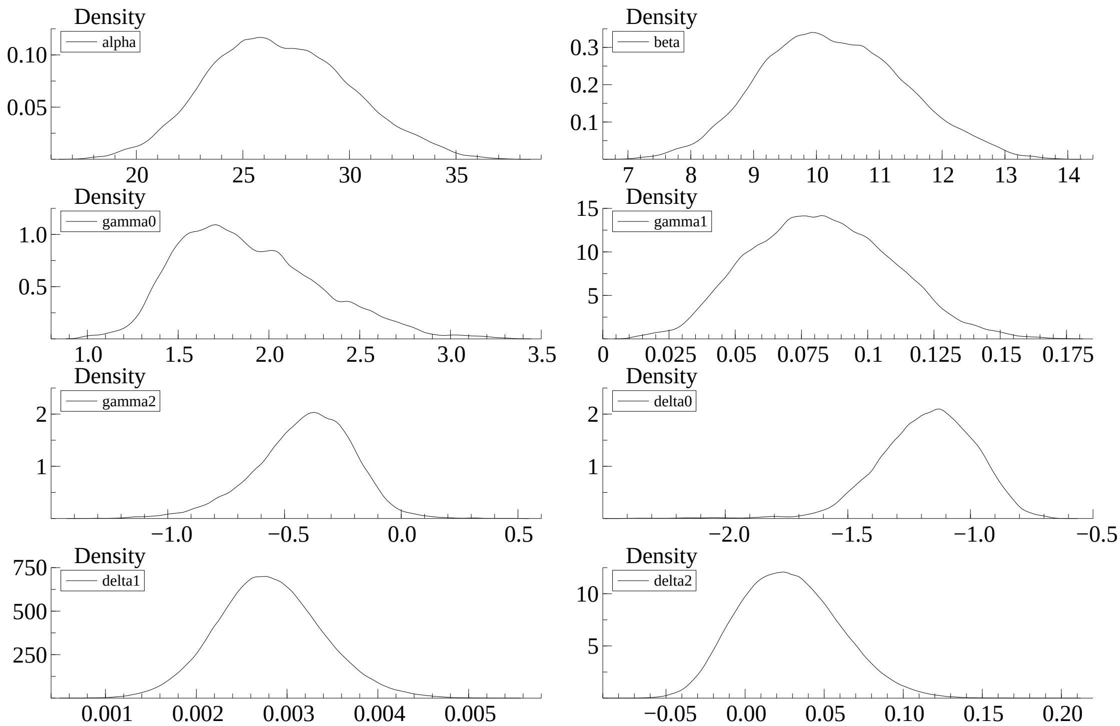

Figure 2.2: Posterior densities of the gamma parameters (α and β) and the truncated normal parameters ($\boldsymbol{\gamma} = (\gamma_0, \gamma_1, \gamma_2)$ and $\boldsymbol{\delta} = (\delta_0, \delta_1, \delta_2)$)

From Table 2.8 and Figure 2.2, the posterior means of gamma parameters, α and β are 26.75 and 10.27, respectively. That is, on average, bidders' willingness to pay for PlayStation 3 is estimated at about \$270. Since the mean of winning bids is about \$330 (Table 2.3), this result is plausible.

Note that the mode of the truncated normal random variable with parameters μ_l and σ_l^2 is given by μ_l. Since in our specification $\mu_l = \gamma_0 + \gamma_1 \cdot \text{Pos.Rep.} + \gamma_2 \cdot \text{Neg.Rep.}$, if the sign of γ_1 is positive (negative), the mode of risk-discount factor, D increases (decreases) with respect to the number of positive ratings. Similarly, if the sign of γ_2 is positive (negative), the mode of risk-discount factor, D increases (decreases) with respect to the number of negative ratings. From Table 2.8, the signs of the posterior means of γ_1 and

γ_2 are positive and negative, respectively. Thus, the mode of risk-discount factor, D, increases with respect to the number of positive ratings and decreases with respect to the number of negative ratings. Furthermore, intuitively, the bidders will trust the seller if the number of positive ratings increases and will not trust the seller if the number of negative ratings increases. These results are consistent with this intuition.

2.5 Inefficiency and Revenue Comparison

The goal of our paper is the estimation of inefficiency in online auction markets. As discussed in Section 2.2, the value of efficiency loss is the difference of total surplus between (counterfactual) efficient auctions and online auctions. In this section, we estimate the efficiency loss of eBay PlayStation 3 auctions using the parameters estimated in Section 2.4. Furthermore, from estimated structural parameters, the difference of revenue between efficient auctions and online auctions can be computed. We also present the revenue comparison in this section.

We estimate inefficiency in the eBay PlayStation 3 auction market using the estimated parameters in Section 2.4. Let $\bar{\alpha}$, $\bar{\beta}$, $\bar{\boldsymbol{\gamma}} = (\bar{\gamma}_0, \bar{\gamma}_1, \bar{\gamma}_2)$ and $\bar{\boldsymbol{\delta}} = (\bar{\delta}_0, \bar{\delta}_1, \bar{\delta}_2)$. be the posterior means of α, β, $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$. In addition, let $\overline{\text{Pos.Rep.}}$ and $\overline{\text{Neg.Rep.}}$ be the sample average of the number of positive and negative ratings, respectively, in "Non-PowerSeller" auctions. That is,

$$\bar{\alpha} = 26.75, \bar{\beta} = 10.27, \bar{\boldsymbol{\gamma}} = (1.89, 0.08, -0.41), \bar{\boldsymbol{\delta}} = (-1.18, 0.003, 0.03)$$

and

$$(\overline{\text{Pos.Rep.}}, \overline{\text{Neg.Rep.}}) = (39.814, 0.74).$$

We estimate efficiency loss in the eBay PlayStation3 auction market using $\bar{\alpha}$, $\bar{\beta}$, $\bar{\boldsymbol{\gamma}}$, $\bar{\boldsymbol{\delta}}$ and $(\overline{\text{Pos.Rep.}}, \overline{\text{Neg.Rep.}})$. The computation method is described in Section 2.3. We conducted 50000 random draws to compute the efficiency loss.

Table 2.9 presents the result of the estimated inefficiency and the difference of revenue. Figure 2.3 is a histogram of the estimated efficiency loss.

	Mean	Standard dev.	Min	5% quantile	95% quantile	Max
Efficiency loss	0.44	0.41	0.00	0.00	1.20	2.99
Difference of revenue	0.83	0.32	0.00	0.32	1.38	2.33

Table 2.9: Estimated inefficiency and the difference of revenue

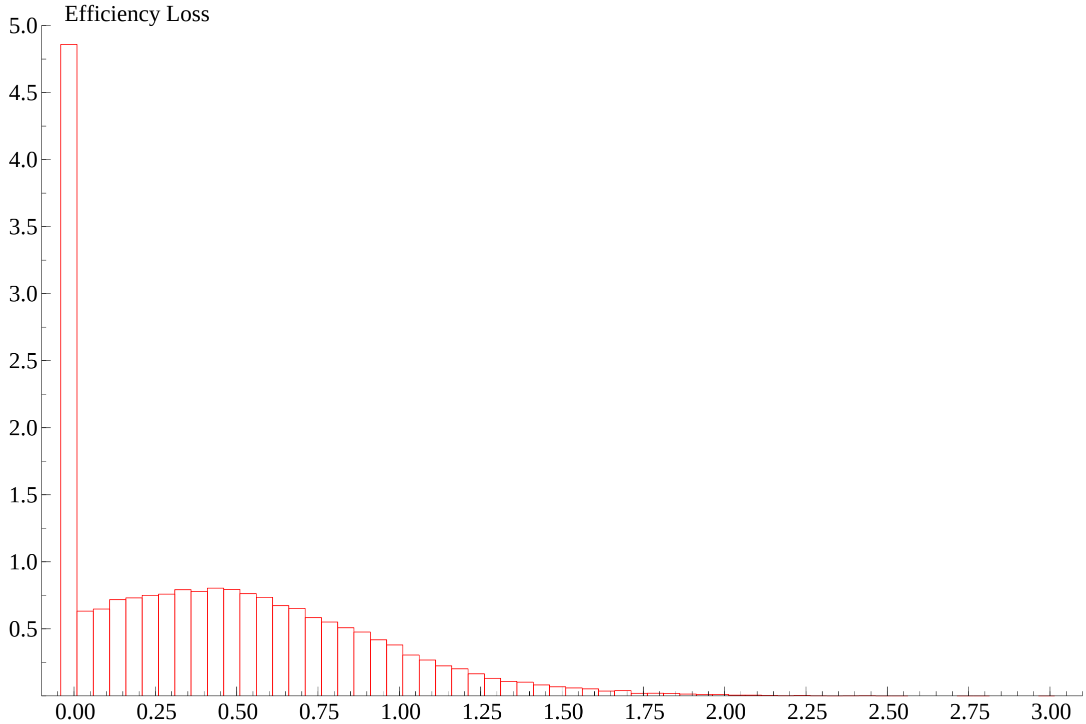

Figure 2.3: Histogram of Estimated Inefficiency from the eBay PlayStation 3 data

Note that since we divide the winning bids by 100 when we estimate the parameters, the estimated value of inefficiency is also divided by 100. From Table 2.9, the average inefficiency is about \$40. In other words, on average, the total surplus will increase by \$40 in efficient auctions. Since the market price of PlayStation 3 in 2009 was \$400, the value of estimated inefficiency is not small. The probability that inefficiency will occur can be computed numerically. The estimated probability that inefficiency will occur is equal to 0.762. That is, in more than 75% auctions, the objects (PlayStation 3) are not awarded to the bidders with highest willingness to pay.

We estimate the revenue difference between the counterfactual efficient auctions and the online auctions using the values of $\bar{\alpha}$, $\bar{\beta}$, $\bar{\boldsymbol{\gamma}}$, $\bar{\boldsymbol{\delta}}$ and $(\overline{\text{Pos.Rep.}}, \overline{\text{Neg.Rep.}})$. Table 2.9 shows the result of the estimated revenue difference. Figure 2.4 is a histogram of the estimated revenue difference.

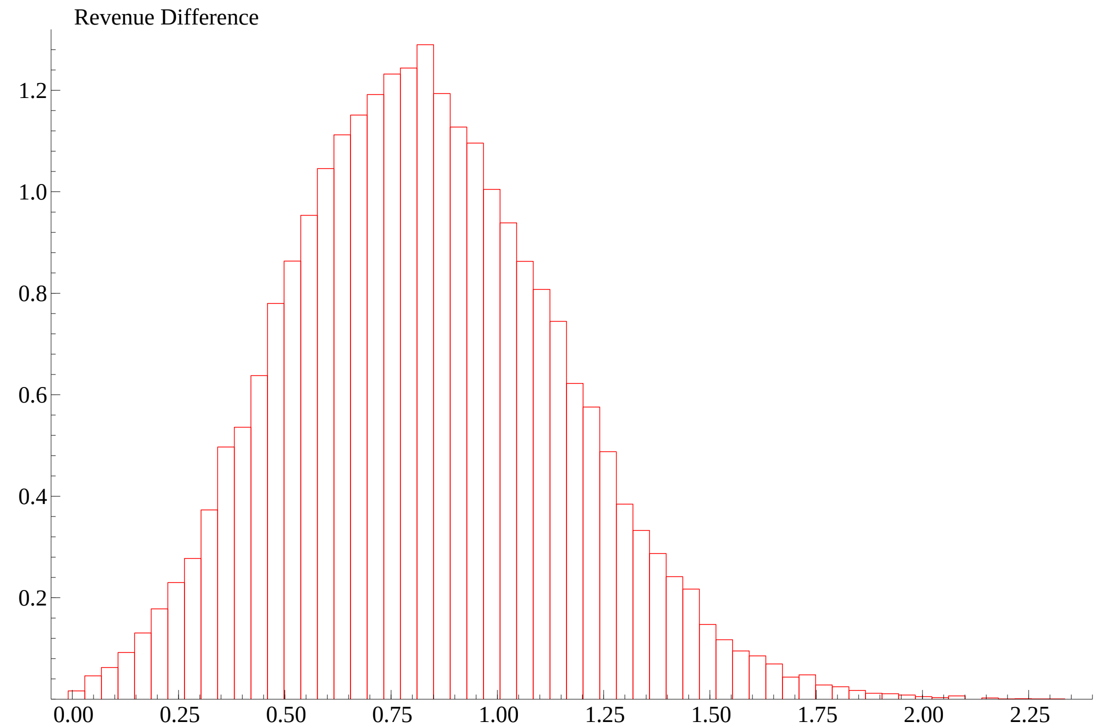

Figure 2.4: Histogram of the revenue difference from the eBay PlayStation 3 data

The mean of the revenue difference between efficient auctions and online auctions is \$83.0. Therefore, if there were efficient auctions, sellers could gain additional revenue of \$83. Since the market price of PlayStation 3 in 2009 was about \$400, the value of the additional gain is not small.

2.6 Conclusions

In the paper, we estimate the inefficiency in online auction markets. Online auctions may be inefficient due to the Internet fraud. A typical example of Internet fraud related to online auctions is when sellers do not send objects to winners even though they have received payment. Since bidders always bear a risk of fraud, online auctions can be inefficient.

We propose that the online auction is inefficient due to Internet fraud. A bidder who does not trust a seller's action can fail to obtain the object even if he or she has a high willingness to pay. As a result, the objects are awarded to the bidders with a low willingness to pay.

We discuss the identification and estimation strategies to estimate the structural parameters in the inefficient online auction model. We use the Bayesian MCMC method to estimate the structural parameters. In the Monte Carlo experiments, our estimation method works well.

Our empirical example is eBay PlayStation 3 auctions in 2009. We found that the mode of bidders' estimates of the risk not to be defrauded is increasing with respect to the number of positive reputations and decreasing with respect to the number of negative reputations. Using the values of estimated parameters, we compute the inefficiency and the revenue difference. The inefficiency which is the difference between the total surplus of the efficient auctions and the total surplus of the online auctions, is estimated $43.50. The probability of the inefficient online auctions is estimated at 0.762. Therefore, 76.2% of online auctions are inefficient auctions. The revenue difference between the efficient auctions and the online auctions is estimated at $83.0.

2.A Implementation of the Metropolis-Hastings Algorithm

We describe the MCMC implementation for the procedure in Section 2.3.

First, we set initial values. For example, in Section 2.4, we set initial values $\alpha^{(0)} = 25$, $\beta^{(0)} = 10$, $\boldsymbol{\gamma}^{(0)} = (\gamma_0^{(0)}, \gamma_1^{(0)}, \gamma_2^{(0)}) = (2.0, 0.1, -0.4)$, and $\boldsymbol{\delta}^{(0)} = (\delta_0^{(0)}, \delta_1^{(0)}, \delta_2^{(0)}) = (-1.0, 0.0, 0.0)$.

2.A.1 Sampling α and β

From equation (2.5), the posterior density of (α, β) is

$$\pi(\alpha, \beta | w_1, ..., w_L) \propto \prod_{l=1}^{L} [1 - F(w_l|\alpha, \beta)] f(w_l|\alpha, \beta) [F(w_l|\alpha, \beta)]^{N-2} \pi(\alpha, \beta),$$

where $\pi(\alpha, \beta)$ is the prior density of (α, β). In Section 2.4, the prior distributions of α and β are

$$\alpha \sim \mathrm{N}(0, 1000) \text{ and } \beta \sim \mathrm{N}(0, 1000).$$

The most commonly used algorithm for simulating from posterior distribution is the Metropolis-Hastings (MH) algorithm. At iteration t, we generate the proposal value, α^*, from

$$\alpha^* \sim \mathrm{N}(\alpha^{(t-1)}, \sigma_\alpha^2),$$

where $\alpha^{(t-1)}$ is the draw at iteration $t-1$ and σ_α is the standard deviation of the proposal density. The proposal draw, α^*, is accepted into the posterior sample with probability

$$\rho(\alpha^{(t-1)}, \alpha^*) = \min\left[\frac{\pi(\alpha^* | \beta^{(t-1)}, w_1, ..., w_L)}{\pi(\alpha^{(t-1)} | \beta^{(t-1)}, w_1, ..., w_L)}, 1\right],$$

where $\pi(\alpha | \beta^{(t-1)}, w_1, ..., w_L)$ is the posterior density conditional on $\beta^{(t-1)}$. If α^* is rejected, then $\alpha^{(t-1)}$ is included in the posterior sample.

Similarly, we draw the posterior sample $\beta^{(t)}$. We draw the proposal value β^* from

$$\beta^* \sim \mathrm{N}(\beta^{(t-1)}, \sigma_\beta^2),$$

where $\beta^{(t-1)}$ is the draw at iteration $t-1$ and σ_β is the standard deviation of the proposal density. The acceptance probability, $\rho(\beta^{(t-1)}, \beta^*)$, is

$$\rho(\beta^{(t-1)}, \beta^*) = \min\left[\frac{\pi(\beta^*|\alpha^{(t)}, w_1, ..., w_L)}{\pi(\beta^{(t-1)}|\alpha^{(t)}, w_1, ..., w_L)}, 1\right],$$

where $\pi(\beta|\alpha^{(t)}, w_1, ..., w_L)$ is the posterior density conditional on $\alpha^{(t)}$. Then we obtain the posterior sample $\beta^{(t)}$ by the following rule:

$$\beta^{(t)} = \begin{cases} \beta^* & \text{with probability } \rho(\beta^{(t-1)}, \beta^*) \text{ and} \\ \beta^{(t-1)} & \text{with probability } 1 - \rho(\beta^{(t-1)}, \beta^*). \end{cases}$$

2.A.2 Sampling $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$

From equation (2.6), the posterior density of $(\boldsymbol{\gamma}, \boldsymbol{\delta})$ conditional on $(\alpha^{(t)}, \beta^{(t)})$ is

$$\pi(\gamma, \delta|\alpha^{(t)}, \beta^{(t)}, \Omega) \propto \prod_{l=1}^{L}[1 - F(w_l|\alpha, \beta)]f(w_l|\alpha, \beta)[F(w_l|\alpha, \beta)]^{N-2}\pi(\alpha, \beta),$$

where $\Omega = (w_1, ..., w_L, X_1, ..., X_L, Y_1, ..., Y_L)$ and $\pi(\alpha, \beta)$ is the prior density of $(\boldsymbol{\gamma}, \boldsymbol{\delta})$. In Section 2.4, the prior distributions of $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ are

$$\boldsymbol{\gamma} \sim \mathrm{N}(0, 1000I_3) \text{ and } \boldsymbol{\delta} \sim \mathrm{N}(0, 1000I_3),$$

where I_3 is the identity matrix of size 3. We use the random walk-based MH algorithm to draw the posterior sample, $\boldsymbol{\gamma}^{(t)}$. For each $k \in \{1, 2, 3\}$, we draw the proposal value

γ_k^* from

$$\gamma_k^* \sim \mathrm{N}(\gamma_k^{(t-1)}, \sigma_{\gamma_k}^2),$$

where $\gamma_k^{(t-1)}$ is the draw at iteration $t-1$ and σ_{γ_k} is the standard deviation of the proposal density. The acceptance probability, $\rho(\gamma_k^{(t-1)}, \gamma_k^*)$, is

$$\rho(\gamma_k^{(t-1)}, \gamma_k^*) = \min\left[\frac{\pi(\gamma_k^*|\alpha^{(t)}, \beta^{(t)}, \boldsymbol{\gamma}_{-k}^{(t-1)}, \boldsymbol{\delta}^{(t-1)}, \Omega)}{\pi(\gamma_k^{(t-1)}|\alpha^{(t)}, \beta^{(t)}, \boldsymbol{\gamma}_{-k}^{(t-1)}, \boldsymbol{\delta}^{(t-1)}, \Omega)}, 1\right],$$

where $\pi(\gamma_k|\alpha^{(t)}, \beta^{(t)}, \boldsymbol{\gamma}_{-k}^{(t-1)}, \boldsymbol{\delta}^{(t-1)}, \Omega)$ is the posterior density conditional on $(\alpha^{(t)}, \beta^{(t)}, \boldsymbol{\gamma}_{-k}^{(t-1)}, \boldsymbol{\delta}^{(t-1)})$. Then we obtain the posterior sample $\gamma_k^{(t)}$ by the following rule:

$$\gamma_k^{(t)} = \begin{cases} \gamma_k^* & \text{with probability } \rho(\gamma_k^{(t-1)}, \gamma_k^*) \text{ and} \\ \gamma_k^{(t-1)} & \text{with probability } 1 - \rho(\gamma_k^{(t-1)}, \gamma_k^*). \end{cases}$$

Similarly, we draw the posterior sample $\delta_k^{(t)}$ for each $k \in \{1, 2, 3\}$. We draw the proposal value δ_k^* from

$$\delta_k^* \sim \mathrm{N}(\delta_k^{(t-1)}, \sigma_{\delta_k}^2),$$

where $\delta_k^{(t-1)}$ is the draw at iteration $t-1$ and σ_{δ_k} is the standard deviation of the proposal density. The acceptance probability, $\rho(\delta_k^{(t-1)}, \delta_k^*)$, is

$$\rho(\delta_k^{(t-1)}, \delta_k^*) = \min\left[\frac{\pi(\delta_k^*|\alpha^{(t)}, \beta^{(t)}, \boldsymbol{\gamma}^{(t)}, \boldsymbol{\delta}_{-k}^{(t-1)}, \Omega)}{\pi(\delta_k^{(t-1)}|\alpha^{(t)}, \beta^{(t)}, \boldsymbol{\gamma}^{(t)}, \boldsymbol{\delta}_{-k}^{(t-1)}, \Omega)}, 1\right],$$

where $\pi(\delta_k|\alpha^{(t)}, \beta^{(t)}, \boldsymbol{\gamma}^{(t)}, \boldsymbol{\delta}_{-k}^{(t-1)}, \Omega)$ is the posterior density conditional on $(\alpha^{(t)}, \beta^{(t)}, \boldsymbol{\gamma}^{(t)}, \boldsymbol{\delta}_{-k}^{(t-1)})$. Then we obtain the posterior sample $\gamma_k^{(t)}$ by the following rule:

$$\delta_k^{(t)} = \begin{cases} \delta_k^* & \text{with probability } \rho(\delta_k^{(t-1)}, \delta_k^*) \text{ and} \\ \delta_k^{(t-1)} & \text{with probability } 1 - \rho(\delta_k^{(t-1)}, \delta_k^*). \end{cases}$$

Chapter 3

An Empirical Model of Online Common Value Auctions with Buy-It-Now Prices

3.1 Introduction

Recently, auctions have become familiar for many people with the emergence of online auction sites. For example, eBay, the largest online auction marketplace, hosted 700 million listings with 14 billion dollars of goods traded and had 370 million registered users around the world in the third quarter of 2008.

However, there are some differences between online auctions and conventional auctions. One example of these difference is "buy-it-now price" option.[1] In an auction with a buy price, a seller sets a fixed price and a bidder can get the item if he or she accepts it. In other words, in auctions with buy prices, buyers can purchase goods without the auctions.

[1]For example, in eBay, the largest online auction site in the world, the buy price option is called "Buy-It-Now" option.

In standard second-price auctions (i.e., auctions without buy prices), each bidder submits a bid. At the end of the auction, the bidder with the highest bid wins the object, and he or she pays the winning bid, which equals the second-highest bid. On the other hand, in buy price trading, a buyer can purchase the object for the fixed price set by the seller. In online auction markets, identical objects are often sold in both auction and buy-it-now formats. Thus, bidders in online auctions with buy-it-now options must participate in auctions observing fixed buy prices.

There have been many studies regarding structural estimation, which focuses on online auction markets. However, to the best of our knowledge, few studies have focused on the buy price options. Most empirical studies in online auction literature ignore the buy price options and estimate the revenue difference between online auctions and (counterfactual) real-world auctions. Houser and Wooders (2006) assume a log-linear model and examine the effect of reputation on a winning bid. They reported that the seller's reputation has a statistically significant effect on the winning bid but that the bidder's reputation does not. Melnik and Alm (2002) applied the Tobit model and estimated the impact of the seller's reputation on the willingness of bidders to bid on items using data concerning coin sales. They found that the seller's reputation has a positive but small impact on the price paid. Livingston (2005) examined the effect of the seller's reputation on the bidders' decision to participate and the willingness of bidders to bid on item. Empirical results using data for golf clubs sold show that the seller's reputation has a positive impact on both the bidders' decision to participate and the willingness of bidders to bid on item. Resnick et al. (2006) conducted a controlled field experiment. In the experiment, the same honest seller sold to many bidders under his regular identity, which has a strong reputation, and under a new seller's identity. Their results show that the established identity fared better; the difference in bidder's willingness to pay was 8.1 % of the selling price.

However, since these papers do not focus on the buy price option, these models may cause incorrect estimates. In this paper, we construct a structural econometric model of online common value auction model with a buy-it-now option. Shahriar (2008) constructed a common value auction model with a buy-it-now option. In experimental economics literature, Shahriar and Wooders (2011) conducted controlled experiments for both private and common value auctions with buy prices. While nether study empirical using real data, we focused on empirical study and the estimation strategy. Ackerberg et al. (2011) constructed an econometric model of online auctions that focused on buy price option. They identified risk preference parameters and time impatience parameters using buy price auction data. While they focused on the independent private values model, we have focused on the pure common value model.

While the common value auction model is an important theoretical auction model, few papers have studied the structural estimation of common value auctions. Most studies on the structural estimation of auction models focus on the private values model. Few empirical studies focus on the common value model due to the negative result of non-parametric identification of common value auctions (Athey and Haile (2002) and Athey and Haile (2007)). [2] Therefore, we specify the parametric forms of the distribution of structural parameters to avoid the identification problem. A few empirical researches which study the common value online auctions. Bajari and Hortaçsu (2003) proposed the Bayesian estimation method for online common value auction models, specifying normality for the common value. Wegmann and Villani (2011) also proposed the Bayesian estimation method for online common value auction models, specifying the gamma distribution for the common value.

Our empirical example involves eBay mint coin auctions in 2013. We found that when we ignore the buy prices, we underestimate the mean of bidders' signals corresponding to

[2] Recently, some papers studied the identification condition of the common value auction model. Li et al. (2000) showed the identification under the additive separability of common value component. Février (2008) restricted the shape of density function of common value and showed the identification of the common value auction model. d'Haultfoeuille and Février (2008) proposed the identification condition of common value auction model assuming the support of private signal is finite and variates depending on the common value.

the value of the good as well as the effect on the signals of positive rating for sellers. We found that the percentage of positive reputations has a positive effect on the mean of the signal. We computed the optimal buy price that maximizes the sellers' expected revenue using the estimated parameters. The estimated optimal buy price is \$52.20, which is almost equal to the average buy prices. We also conducted a revenue comparison. We compared the revenue between auctions with buy prices and auctions without buy prices. We found that the mean of the revenue difference between auctions with buy prices and those without is \$0.05.

The rest of this paper is organized as follows. In Section 3.2, following Shahriar (2008), we describe the theoretical model of online common value auctions with buy-it-now options. Section 3.3 describes the estimation strategy for the model described in Section 3.2. Monte Carlo experiments are conducted in Section 3.4. In Section 3.5, we explain the eBay mint coin auction data used in our empirical example. We also present the estimation results from the eBay mint coin auction data. In Section 3.6, we show counterfactual simulations. We compute the optimal buy price that maximizes sellers' expected revenues. In addition, we compare revenue between auctions with and without buy-it-now prices. Section 3.7 makes some concluding remarks.

3.2 The Model

Since our inference is based on the model of Shahriar (2008), it is worthwhile to review the theoretical result of Shahriar (2008). Following the precedent of Shahriar (2008), and Shahriar and Wooders (2011), we describe a common value model of auctions with a buy-it-now option.

Consider a seller who sells an individual object through an auction. In the auction, there are $n \geq 2$ potential bidders. Each bidder, i, receives a private signal, S_i, which is identically and independently distributed random variable from the distribution function,

F (the density function f). In this model, we consider the pure common value model, i.e., the *ex post* value of the item is the same for each bidder. Furthermore, we consider a specific functional form for the common values. We assume that the *ex post* valuation is equal to the average of all signals. That is, the valuation takes the form of

$$v = \frac{1}{n} \sum_{i=1}^{n} s_i.$$

Each bidder, i, knows the value of its own signal, s_i, but does not know the realization of others' signals, s. Therefore, each bidder does not know the realization of the common value, v. However, the distribution of signal, $F(\cdot)$, is common knowledge among bidders.

We assume that each bidder has a risk neutral utility. Therefore, if bidder i buys the item and pays a price, p, her utility is $v - p$.

3.2.1 Auctions with Buy Prices

We regard auctions with buy prices as two-stage games. In the first-stage, the seller sets a buy price, p, and all bidders decide to accept or reject buy price p simultaneously. If at least one bidder accepts buy price p, the auction game ends and the bidder who accepts wins the item and pays price p.[3] If no bidder accepts buy price p, the first-stage of the game ends and the auction proceeds to the second stage. In the second stage, the winner and winning price are determined via a second-price sealed-bid auction. That is, the winning bidder is the bidder who makes the highest bid, and the winning price is equal to the second highest bid.

In the first-stage, after bidder i observes the realization of her signal, s_i, she decides whether to accept or reject the buy price, p. Following Shahriar (2008), we considered symmetric cutoff strategies. A cutoff strategy for bidder i is a constant, c, such that the

[3]If $n \leq 2$ bidder accept the buy price p, we assume that the bidder who accepts the buy price win the item with probability $1/n$.

bidder will

$$\begin{cases} \text{accept} & \text{if } s_i > c \\ \text{reject} & \text{otherwise.} \end{cases}$$

Suppose that all bidders except i follow the same cutoff strategy. Then, the expected payoff to bidder i with signal s_i from accepting buy price p is given by

$$U^A(s_i, c) = \sum_{l=0}^{n-1} \left[\binom{n-1}{l} F(c)^{n-1-l} (1 - F(c))^l \left(\frac{1}{l+1} \right) u_l(s_i) \right], \tag{3.1}$$

where l is the number of bidders except bidder i who accepts buy price p, and

$$u_l(s) = \int_{-\infty}^{c} \cdots \int_{-\infty}^{c} \int_{c}^{\infty} \cdots \int_{c}^{\infty} \left[\frac{1}{n} \left(s + \sum_{j \neq i} s_j \right) - p \right]$$
$$\times \frac{f(s_1)}{1 - F(c)} \cdots \frac{f(s_l)}{1 - F(c)} \frac{f(s_{l+2})}{F(c)} \cdots \frac{f(s_n)}{F(c)} d\boldsymbol{s}_{-i}.$$

Next, consider the case in which bidder i rejects buy-it-now price p. Then, she can get the item only if all of her opponents also reject buy price p and the realization of bidder i's signal s_i is the highest among all bidders' signals. Let z be the highest signal among rivals (i.e., $z = \max_{j \neq i} s_j$) and $b(\cdot)$ be the equilibrium bidding function of the second-stage auction.[4] Then, if bidder i wins, her payment is $b(z)$. Note that since we assume the pure common value paradigm, the equilibrium bidding function is

$$b(s) = \mathrm{E}(V | S_i = s, z \equiv \max_{j \neq i} S_j = s).$$

Thus, the expected payoff of bidder i with signal s_i from rejecting buy price p is given

[4] In the second-stage, each bidder knows that no bidder accepts the buy price at the first-stage game. Therefore, in the second-stage auction, each bidder knows that all bidders' signals are sufficiently low to reject the buy price.

by

$$
\begin{aligned}
U^R(s_i, c) = &\int_{-\infty}^{\min\{s_i,c\}} \left\{ \int_{-\infty}^{s_2} \cdots \int_{-\infty}^{s_2} \left[\frac{1}{n} \left(s_i + \sum_{j \neq i} s_j \right) - b(s_2) \right] \frac{f(s_n)}{F(s_2)} ds_n \cdots \frac{f(s_3)}{F(s_2)} ds_3 \right\} \\
&\quad \times (n-1) F(s_2)^{n-2} f(s_2) ds_2 \\
= &(n-1) \int_{-\infty}^{\min\{s,c\}} \int_{-\infty}^{s_2} \cdots \int_{-\infty}^{s_2} \left[\frac{1}{n} \left(s_i + \sum_{j \neq i} s_j \right) - b(s_2) \right] dF(s_n) \cdots dF(s_2),
\end{aligned}
\tag{3.2}
$$

where $x_2 = z$.

A cutoff, c^*, is a symmetric Bayesian Nash equilibrium if a bidder gets a higher-than-expected payoff by accepting buy price p if $x > c^*$ and she gets a higher-than-expected payoff by rejecting buy price p if $x < c^*$. Therefore, we obtain the following proposition.

Proposition 3 (Shahriar (2008)). *A symmetric equilibrium cutoff, c^*, satisfies*

$$U^A(c^*, c^*) = U^R(c^*, c^*). \tag{3.3}$$

3.3 Estimation Procedures

Before discussing how to estimate the auction model with a buy price, we briefly describe the estimation strategy for the simple auction model (i.e., an auction without a buy price).

We observe T auctions, indexed by $t = 1, ..., T$. We observe the number of bidders, n_t, and the winning bid, w_t, in auction t. Furthermore, we observe the auction-specific covariate, X_t. The same item is sold in each auction, $t \in \{1, ..., T\}$. While we focus on the pure common value auction model, few researchers have studied the econometric model of the pure common value auction model. One reason that few empirical studies focus on the the pure common value auction model is that the common value auction model is not nonparametrically identified from observed bids.[5] Therefore, we specify the parametric form of the distribution of signal, $F(\cdot)$. We assume that bidders' signals are normally distributed with mean μ_t and variance σ_t^2. That is,

$$S_i \sim N(\mu_t, \sigma_t^2),$$

where

$$\mu_t = \boldsymbol{\alpha}' X_t$$

and

$$\sigma_t = \exp(\boldsymbol{\beta}' X_t),$$

where $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is the unknown coefficient parameter vector to estimate.

Recall that the equilibrium bidding function of the second-stage auction, $b(\cdot)$, is given by

$$\begin{aligned} b(s) &= \mathrm{E}(V|S_i = s, Z = \max_{j \neq i} S_j = s) \\ &= \mathrm{E}\bigg(\frac{1}{n}\sum_{i=1}^{n} S_i | S_i = s, Z = s\bigg). \end{aligned}$$

Since $b(\cdot)$ is a strictly increasing function, there exists an inverse function, $\phi(\cdot)$. Therefore, if we observe only second-stage auctions, the likelihood function of winning bids, w_t, is given by

$$\begin{aligned} L(w_1, ..., w_T | \boldsymbol{\mu}, \boldsymbol{\sigma}) = \prod_{t=1}^{T} \prod_{i=1}^{n_t} &\begin{pmatrix} & n_t & \\ 1 & 1 & n_t - 2 \end{pmatrix} [F(\phi(w_t)|\mu_t, \sigma_t)]^{n_t - 2} \\ &\times f(\phi(w_t)|\mu_t, \sigma_t) \frac{1}{b'(\phi(w_t))} [1 - F(\phi(w_t)|\mu_t, \sigma_t)], \qquad (3.4) \end{aligned}$$

where $\boldsymbol{\mu} = (\mu_1, ..., \mu_T)$, and $\boldsymbol{\sigma} = (\sigma_1, ..., \sigma_T)$.

[5] See Athey and Haile (2002) for detail.

Next, we discuss how to estimate the structural parameters of auctions with buy prices. As described in the previous section, each bidder must decide whether to accept or reject buy-it-now price p in the first-stage. We assume that the econometricians can observe the buy prices, p_t, in auction t. If at least one bidder accepts buy-it-now price p_t, the likelihood is given by

$$1 - F(c^*|\mu_t, \sigma_t)^{n_t}.$$

If each bidder rejects buy-it-now price p_t, cutoff value c^* exceeds signal s_i for all $i \in \{1, ..., n_t\}$. Then, the likelihood of winning bid w_t is given by

$$\prod_{i=1}^{n_t} \begin{pmatrix} & n_t & \\ 1 & 1 & n_t - 2 \end{pmatrix} [F(\phi(w_t)|\mu_t, \sigma_t)]^{n_t - 2} f(\phi(w_t)|\mu_t, \sigma_t) \frac{1}{b'(\phi(w_t))} [F(c^*|\mu_t, \sigma_t) - F(\phi(w_t)|\mu_t, \sigma_t)].$$

Let $\mathcal{R} = \{1, ..., R\}$ where $R < T$ is the set of buy-it-now trading. Therefore, $\{1, ..., T\} \backslash \mathcal{R}$ is the set of auctions without buy-it-now prices. Then, the likelihood function of $\boldsymbol{w} = (w_1, ..., w_T)$ and $\boldsymbol{p} = (p_1, ..., p_T)$ is given by

$$\begin{aligned} L(\boldsymbol{w}, \boldsymbol{p}|\boldsymbol{\mu}, \boldsymbol{\sigma}) = & \prod_{t \in \mathcal{R}} \prod_{i=1}^{n_t} [1 - F(c_t^*|\mu_t, \sigma_t)^{n_t}] \\ & \times \prod_{t \in \mathcal{R}^c} \prod_{i=1}^{n_t} \begin{pmatrix} & n_t & \\ 1 & 1 & n_t - 2 \end{pmatrix} [F(\phi(w_t)|\mu_t, \sigma_t)]^{n_t - 2} \\ & \times f(\phi(w_t)|\mu_t, \sigma_t) \frac{1}{b'(\phi(w_t))} [F(c_t^*|\mu_t, \sigma_t) - F(\phi(w_t)|\mu_t, \sigma_t)]. \end{aligned} \tag{3.5}$$

Note that from equation (3.4) and equation (3.5), when we estimate the structural parameters, ignoring the buy prices, the estimator may be incorrect.

We estimate the structural parameters using the Bayesian method. We compute the posterior distribution by the standard Markov Chain Monte Carlo (MCMC) simulation method. Let $\boldsymbol{\alpha}^{(0)}$ and $\boldsymbol{\beta}^{(0)}$ be the initial value of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, respectively. Then we repeat the following algorithm for a sufficiently large number, $j \in \{1, ..., J\}$:

1. Generate $\boldsymbol{\alpha}^{(j)}|\boldsymbol{\beta}^{(j-1)}, \boldsymbol{w}, \boldsymbol{p}$
2. Generate $\boldsymbol{\beta}^{(j)}|\boldsymbol{\alpha}^{(j),}, \boldsymbol{w}, \boldsymbol{p}$.

We generate random samples from posterior distributions via random walk-based Metropolis-Hastings algorithm. Note that from equations (3.1) - (3.3), observing the buy price, p_t, and the random draws, $\boldsymbol{\alpha}^{(j)}$ and $\boldsymbol{\beta}^{(j)}$, the value of equilibrium cutoff strategy c_t^* can be computed numerically. Since the inverse bidding function, $\phi(\cdot) = b^{-1}(\cdot)$, cannot be obtained analytically, we compute the inverse bidding function, $\phi(\cdot)$, numerically. Analogous to cutoff strategy c_t^*, observing buy price p_t and random draws $\boldsymbol{\alpha}^{(j)}$ and $\boldsymbol{\beta}^{(j)}$, the signal corresponding to winning bid w_t can be computed by the Newton-Raphson method.

3.4 Simulation Experiments

In this section, we estimate the structural parameters in our model using simulation data. The number of observed auctions is T. We examine the performance of our estimator with T = 150, 300 and 700. In our simulation experiments, the number of potential bidders for all auction is $N = 5$. We set the constant buy price, $p = 45.0$, for all $t \in \{1, ..., T\}$. We generate the auction-specific covariate, X_t, from a standard normal distribution. We draw the signals, S, from the normal distribution. That is,

$$S_i \sim i.i.d. \, \mathrm{N}(\mu_t, \sigma_t^2),$$

where $\mu_t = \alpha_0 + \alpha_1 X_t$ and $\sigma_t = \exp(\beta_0 + \beta_1 X_t)$. The true values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are

$$\alpha_0 = 40.0, \alpha_1 = 0.4, \beta_0 = 7.0, \text{ and } \beta_1 = 0.7.$$

The prior distribution of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are

$$\boldsymbol{\alpha} \sim \mathrm{N}(0, 100\boldsymbol{I})$$

and

$$\boldsymbol{\beta} \sim \mathrm{N}(0, 100\boldsymbol{I}),$$

where $\boldsymbol{I}$ is the identity matrix of size 2.

We use the random walk-based Metropolis-Hastings algorithm to generate the random draws from posterior distributions. The number of iteration is 20000, and the burn-in period is 2000 in each case. We estimate the parameters using both the *true* likelihood (3.5) and the *wrong* likelihood (3.4) to compare the performance of the estimation results.

3.4.1 The Case of $T = 150$

Tables 3.1 and 3.2 show results using the true likelihood (3.5) and the wrong likelihood (3.4), respectively. From Table 3.1, the 95% credible intervals contain the true values using the likelihood (3.5).

		(True) Likelihood (3.5)		
	True	Mean	Stdev.	95% interval
α_0	40.00	39.56	0.46	(38.62, 40.40)
α_1	0.40	-0.05	0.66	(-1.32, 1.27)
β_0	7.00	7.12	0.12	(6.89, 7.37)
β_1	0.70	0.77	0.14	(0.50, 1.04)

Table 3.1: Estimation result with true likelihood (3.5) ($T = 150$)

	True	(Wrong) Likelihood (3.4) Mean	Stdev.	95% interval
α_0	40.00	36.01	1.22	(33.66, 38.48)
α_1	0.40	-2.63	1.13	(-4.94, -0.42)
β_0	7.00	6.94	0.13	(6.69, 7.20)
β_1	0.70	0.64	0.15	(0.35, 0.92)

Table 3.2: Estimation result with wrong likelihood (3.4) ($T = 150$)

From Table 3.2, however, the 95% credible intervals do not contain the true values for α_0 and α_1 using likelihood (3.4). Therefore, we find that the estimation using likelihood (3.5) performs well. On the other hand, we find that when we ignore the buy prices, p, we fail to estimate the structural parameters correctly.

Figures 3.1 and 3.2 show the posterior densities using likelihood (3.5) and likelihood (3.4), respectively.

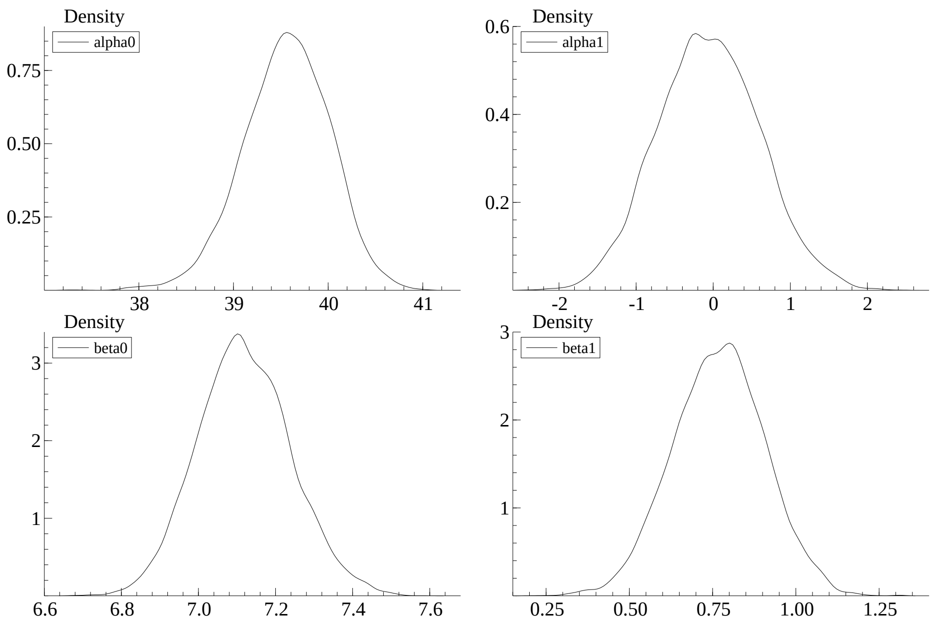

Figure 3.1: Posterior densities using True Likelihood (3.5) with $T = 150$

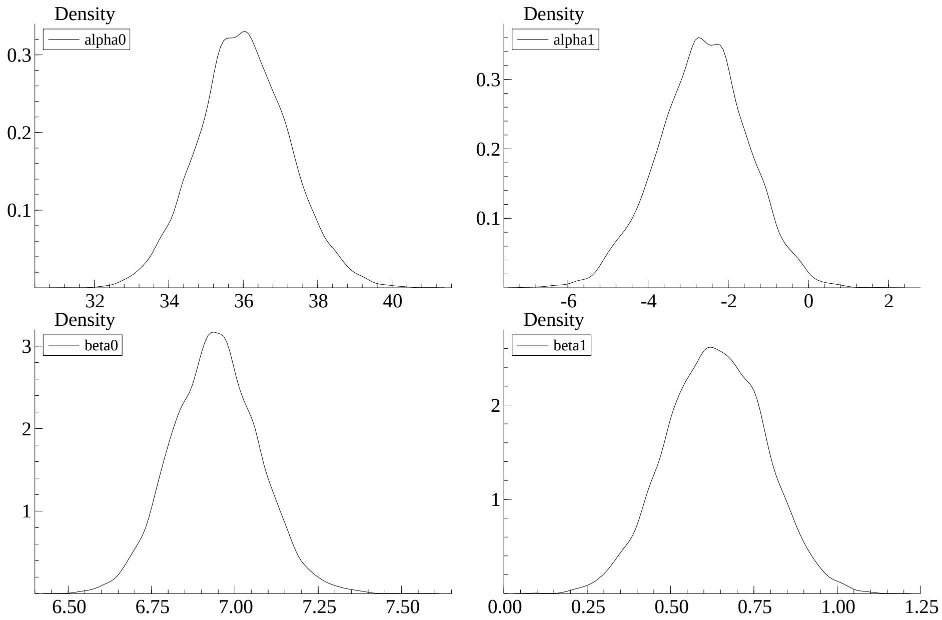

Figure 3.2: Posterior densities using Wrong Likelihood (3.4) with $T = 150$

	(True) Likelihood (3.5)		(Wrong) Likelihood (3.4)	
	CD	IF	CD	IF
α_0	0.23	22.71	0.86	10.98
α_0	0.97	8.70	0.38	11.15
β_0	0.26	12.92	0.40	5.22
β_0	0.38	8.28	0.43	5.24

Table 3.3: The Convergence Diagnostics for the MCMC (CD) and the Inefficiency Factors (IF) ($T = 150$)

Table 3.3 reports p-values of the convergence diagnostics for the MCMC (CD) and the inefficiency factors (IF).[6] In Table 3.3, from column 2 to column 3, we use *true* likelihood (3.5) (i.e., likelihood with buy prices). From column 4 to column 5, we use *wrong* likelihood (3.4) (i.e., likelihood ignoring buy prices). All p-values of the convergence diagnostics are more than 0.2. Furthermore, the inefficiency factor values are sufficiently

[6]The CD test statistic tests the equality of the means of the first and last parts of the sample path. The definition of inefficiency factor is $1 + 2\sum_{k=1}^{\infty} \rho(k)$, where $\rho(k)$ is the sample autocorrelation at lag k.

low. The inefficiency factors are 5.22 to 22.71, which imply that we would gain the same variance of the posterior means from 880 uncorrelated draws, even in the worst case. Figures 3.3 and 3.4 show the sample paths of the estimated parameters using likelihood (3.5) and likelihood (3.4), respectively. From Figures 3.3 and 3.4, it can be seen that the sample paths of these parameters converge to posterior distributions. Thus, we conclude that the sample paths of the estimated parameters converge to posterior distributions.

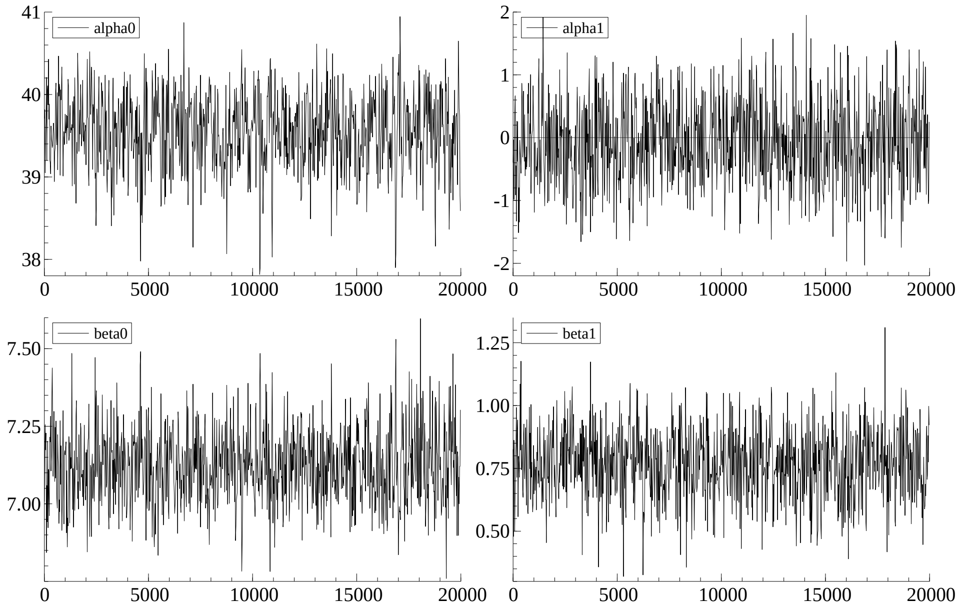

Figure 3.3: Sample paths of parameters with $T = 150$ (true Likelihood (3.5))

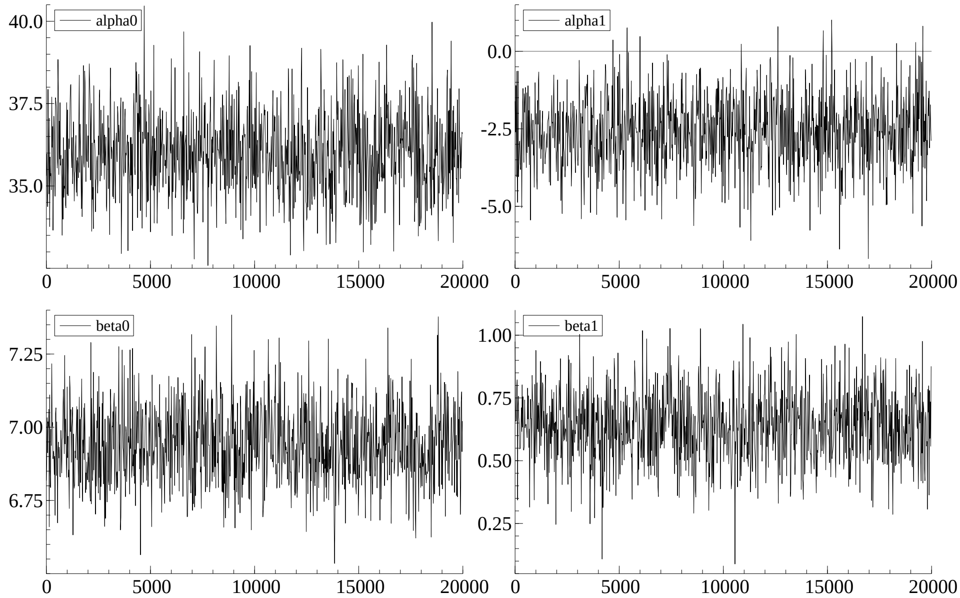

Figure 3.4: Sample paths of parameters with $T = 150$ (Wrong Likelihood (3.4))

3.4.2 The Case of $T = 300$

Results are shown in Tables 3.4 and 3.5. In Table 3.4, we use the *true* likelihood (3.5) (i.e., likelihood with buy prices) to compute the posterior distributions. In Table 3.5, in contrast, we use the *wrong* likelihood (3.4) (i.e., likelihood ignoring buy prices) to compute the posterior distributions.

		(True) Likelihood (3.5)		
	True	Mean	Stdev.	95% interval
α_0	40.00	39.80	0.28	(39.24, 40.32)
α_1	0.40	-0.26	0.33	(-0.88, 0.43)
β_0	7.00	7.01	0.08	(6.85, 7.17)
β_1	0.70	0.68	0.08	(0.52, 0.82)

Table 3.4: Estimation result with true likelihood (3.5) ($T = 300$)

	True	(Wrong) Likelihood (3.4) Mean	Stdev.	95% interval
α_0	40.00	37.38	0.82	(35.76, 39.01)
α_1	0.40	-1.65	0.70	(-3.04, -0.27)
β_0	7.00	6.88	0.09	(6.71, 7.06)
β_1	0.70	0.63	0.09	(0.45, 0.79)

Table 3.5: Estimation result with wrong likelihood (3.4) ($T = 300$)

From Table 3.4, the 95% credible intervals contain the true values using likelihood (3.5). However, from Table 3.5, the 95% credible intervals do not contain the true values for α_0 and α_1 using likelihood (3.4). Therefore, we find that the estimation using the likelihood (3.5) performs well. On the other hand, we find that when we ignore the buy prices, p, we fail to estimate the structural parameters correctly.

Figures 3.5 and 3.6 show the posterior densities using likelihood (3.5) and likelihood (3.4), respectively.

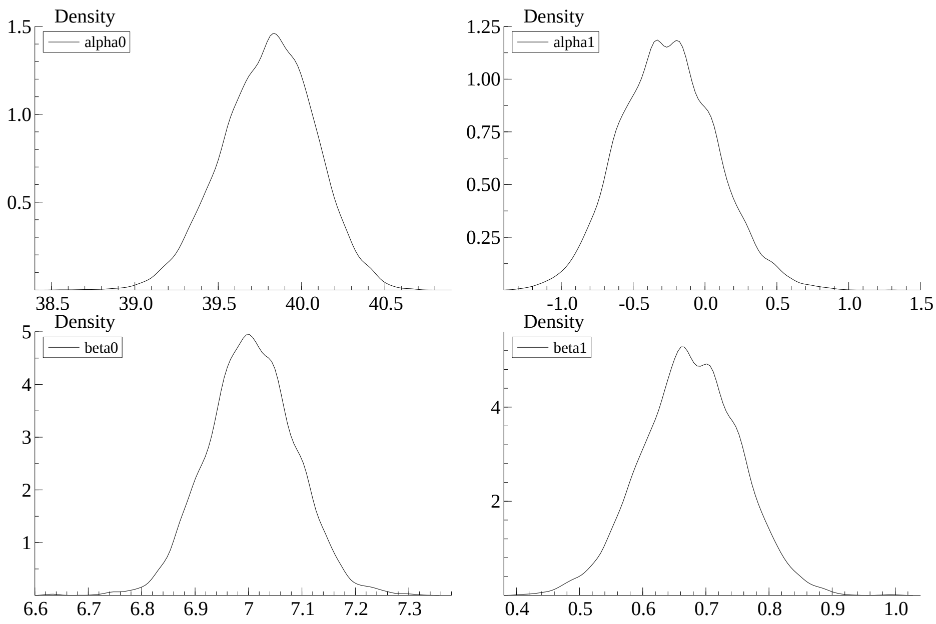

Figure 3.5: Posterior densities using *true* likelihood (3.5) with $T = 300$

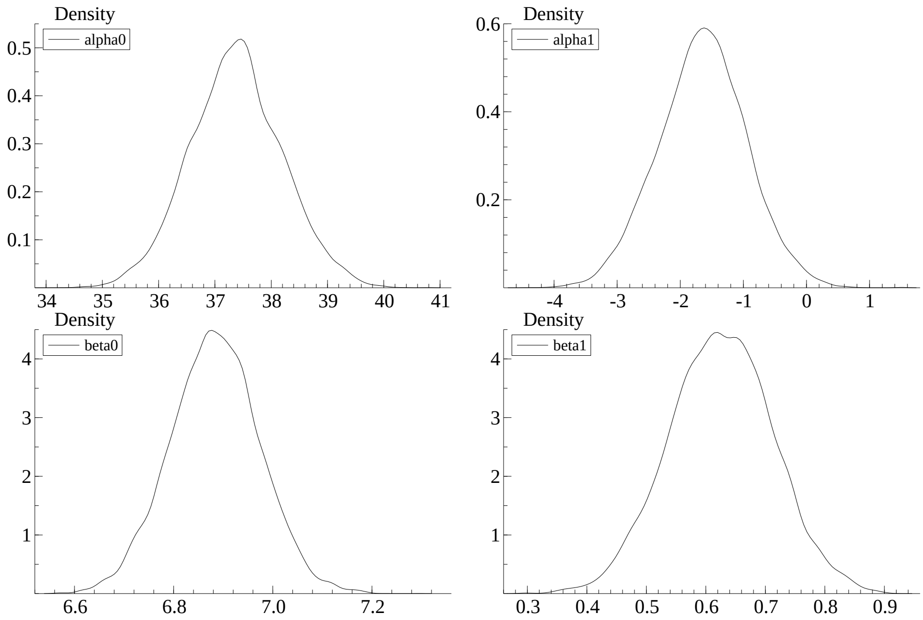

Figure 3.6: Posterior densities using *wrong* likelihood (3.4) with $T = 300$

	(True) Likelihood (3.5)		(Wrong) Likelihood (3.4)	
	CD	IF	CD	IF
α_0	0.46	21.36	0.70	10.34
α_0	0.74	13.05	0.72	11.40
β_0	0.68	17.57	0.77	4.57
β_0	0.99	6.88	0.42	5.77

Table 3.6: The Convergence Diagnostics for the MCMC (CD) and the Inefficiency Factors (IF) ($T = 300$)

Table 3.6 reports p-values of the convergence diagnostics for the MCMC (CD) and the inefficiency factors (IF). In Table 3.6, from column 2 to column 3, we use *true* likelihood (3.5) (i.e., likelihood with buy prices). From column 4 to column 5, we use *wrong* likelihood (3.4) (i.e., likelihood ignoring buy prices). All p-values of the convergence diagnostics are more than 0.4. Furthermore, the inefficiency factor values are sufficiently low. The inefficiency factors are 4.57 to 21.36, which imply that we would gain the same variance of the posterior means from 930 uncorrelated draws, even in the worst case.

Figure 3.7 and 3.8 show the sample paths of the estimated parameters using likelihood (3.5) and likelihood (3.4), respectively. From Figures 3.7 and 3.8, it can be seen that the sample paths of these parameters converge to posterior distributions. Thus, we conclude that the sample paths of the estimated parameters converge to posterior distributions.

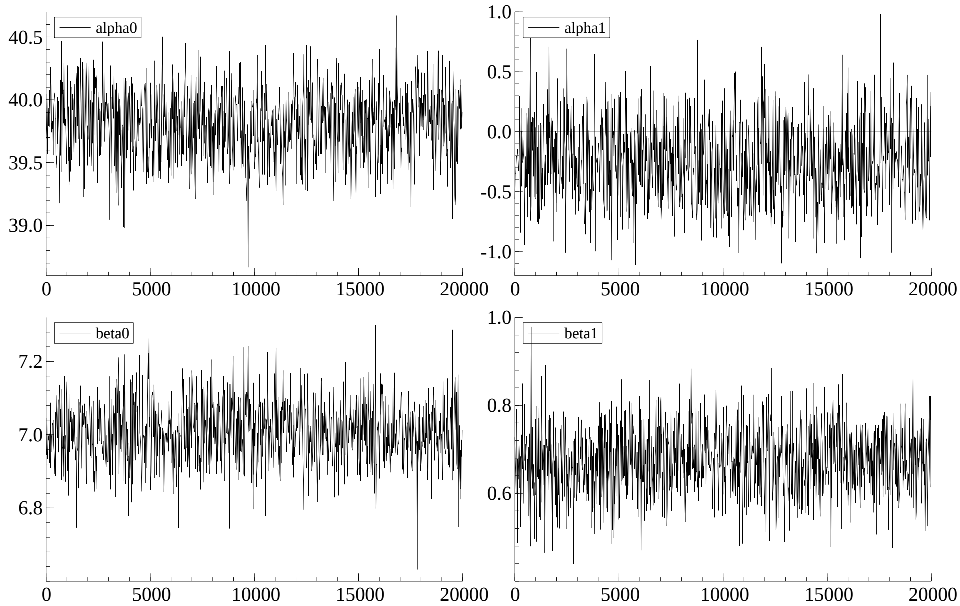

Figure 3.7: Sample paths of parameters with $T = 300$ (true likelihood (3.5))

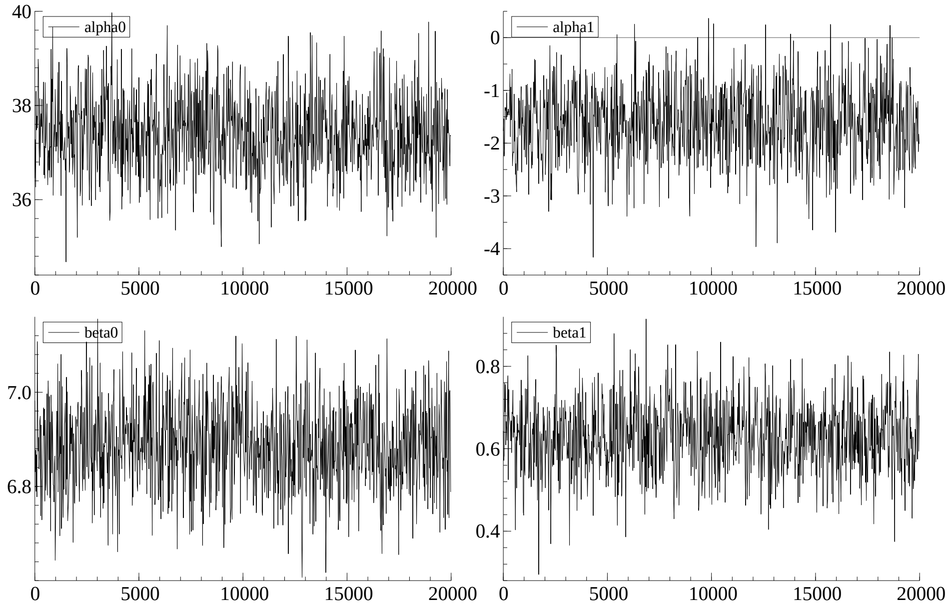

Figure 3.8: Sample paths of parameters with $T = 300$ (wrong likelihood (3.4))

3.4.3 The Case of $T = 700$

Results are shown in Tables 3.7 and 3.8. In Table 3.7, we use the *true* likelihood (3.5) (i.e., likelihood with buy prices) to compute the posterior distributions. In Table 3.8, in contrast, we use the *wrong* likelihood (3.4) (i.e., likelihood ignoring buy prices) to compute the posterior distributions.

		(True) Likelihood (3.5)		
	True	Mean	Stdev.	95% interval
α_0	40.00	40.12	0.17	(39.77, 40.44)
α_1	0.40	0.58	0.22	(0.15, 1.01)
β_0	7.00	6.96	0.05	(6.86, 7.07)
β_1	0.70	0.66	0.06	(0.54, 0.77)

Table 3.7: Estimation result with true likelihood (3.5) ($T = 700$)

	True	(Wrong) Likelihood (3.4) Mean	Stdev.	95% interval
α_0	40.00	37.44	0.52	(36.41, 38.46)
α_1	0.40	-1.54	0.49	(-2.51, -0.63)
β_0	7.00	6.80	0.06	(6.68, 6.92)
β_1	0.70	0.51	0.07	(0.38, 0.64)

Table 3.8: Estimation result with wrong likelihood (3.4) ($T = 700$)

From Table 3.7, the 95% credible intervals contain the true values using likelihood (3.5). However, from Table 3.8, the 95% credible intervals do not contain the true values for all parameters using likelihood (3.4). Therefore, we find that the estimation using the likelihood (3.5) performs well. On the other hand, we find that when we ignore the buy prices, p, we fail to estimate the structural parameters correctly.

Figures 3.9 and 3.10 show the posterior densities using likelihood (3.5) and likelihood (3.4), respectively.

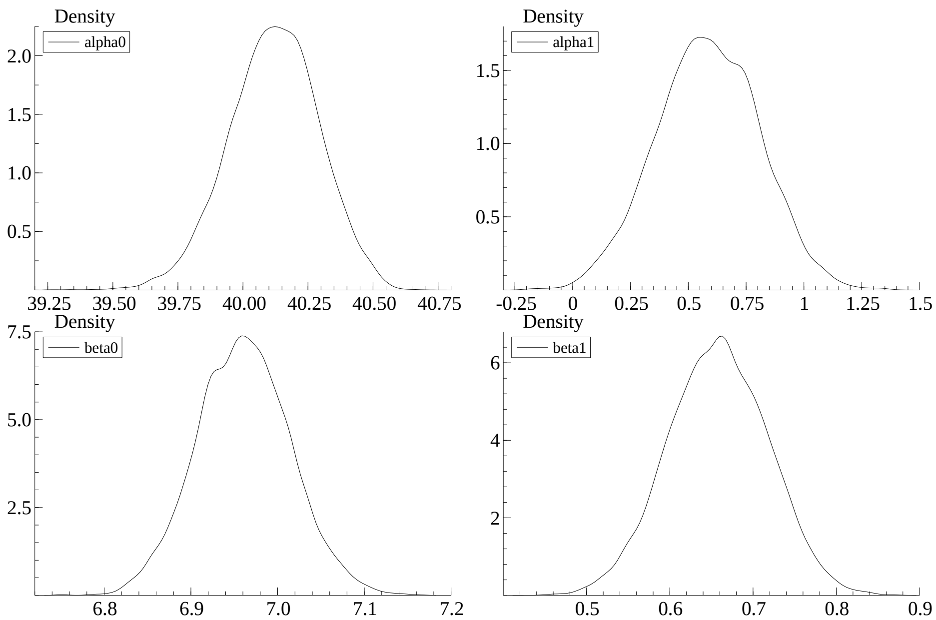

Figure 3.9: Posterior densities using true likelihood (3.5) with $T = 700$

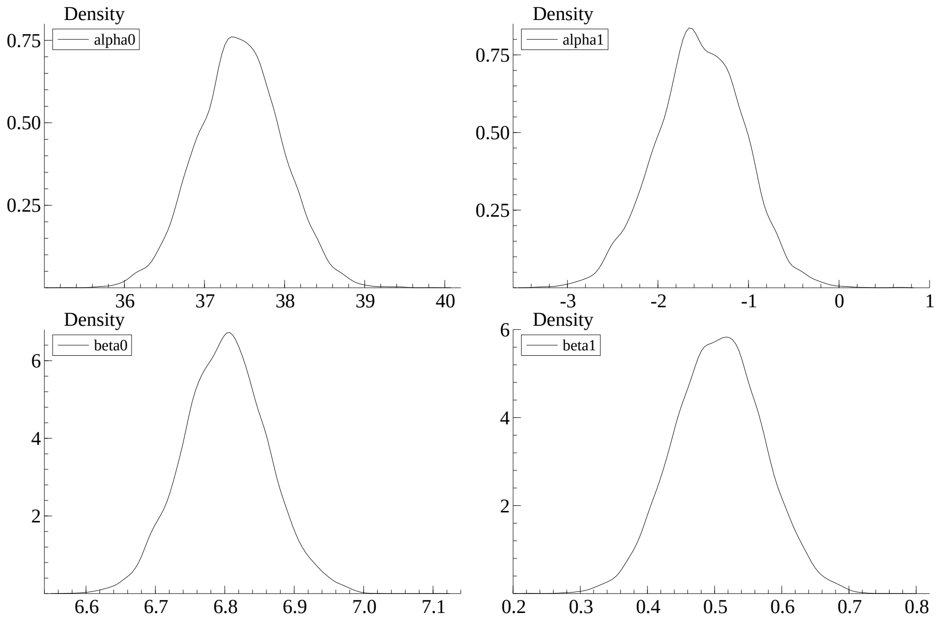

Figure 3.10: Posterior densities using wrong likelihood (3.4) with $T = 700$

	(True) Likelihood (3.5)		(Wrong) Likelihood (3.4)	
	CD	IF	CD	IF
α_0	0.22	11.89	0.42	10.22
α_0	0.84	5.05	0.20	10.26
β_0	0.98	8.32	0.64	4.02
β_0	0.34	11.06	0.44	5.41

Table 3.9: The Convergence Diagnostics for the MCMC (CD) and the Inefficiency Factors (IF) ($T = 700$)

Table 3.9 reports p-values of the convergence diagnostics for the MCMC (CD) and the inefficiency factors (IF). In Table 3.9, from column 2 to column 3, we use *true* likelihood (3.5) (i.e., likelihood with buy prices). From column 4 to column 5, we use *wrong* likelihood (3.4) (i.e., likelihood ignoring buy prices). All p-values of the convergence diagnostics are more than 0.2. Furthermore, the inefficiency factor values are sufficiently low. The inefficiency factors are 5.05 to 11.89 with true likelihood. This implies that we would gain the same variance of the posterior means from 1680 uncorrelated draws, even in the worst case. Similarly, the inefficiency factors are 4.02 to 10.26 when we ignore the buy prices. Therefore, we would gain the same variance of the posterior sample means from 1949 uncorrelated draws even in the worst case. Figures 3.11 and 3.12 show the sample paths of the estimated parameters using likelihood (3.5) and likelihood (3.4), respectively. From Figures 3.11 and 3.12, it can be seen that the sample paths of these parameters converge to posterior distributions. Thus, we conclude that the sample paths of the estimated parameters converge to posterior distributions.

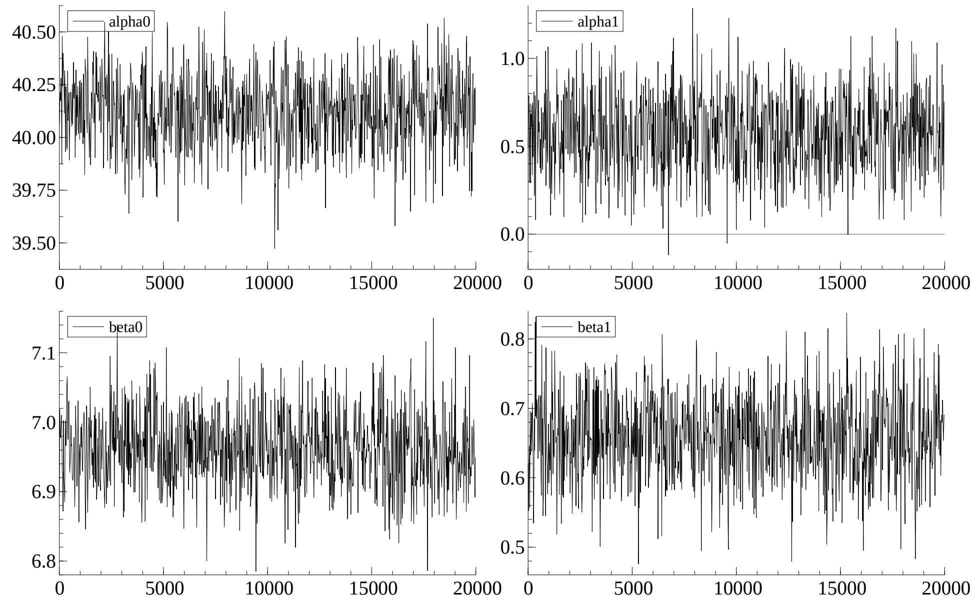

Figure 3.11: Sample paths of parameters with $T = 700$ (true likelihood (3.5))

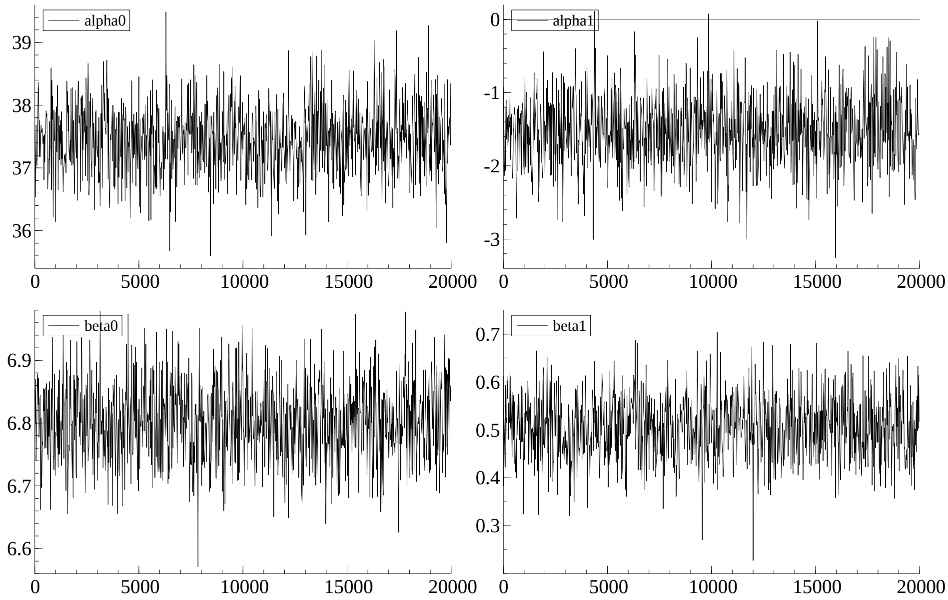

Figure 3.12: Sample paths of parameters with $T = 700$ (wrong likelihood (3.4))

3.5 Empirical Illustrations

3.5.1 Data Description

Our empirical example examines auctions of 2005 U.S. mint silver proof coin sets held on eBay in 2013. Data were collected from 152 completed eBay auctions from June through July of 2013. Auctions with fewer than two actual bidders were dropped since there is no competition with no bidder or one bidder.

As we can see from the studies of Bajari and Hortaçsu (2003) and Wegmann and Villani (2011), who studied coin auctions in their empirical illustrations, coin auctions are excellent examples in the empirical study of common value auction models. While both Bajari and Hortaçsu (2003) and Wegmann and Villani (2011) collected various kinds of coins in their empirical illustrations, we collected only 2005 U.S. mint silver proof coin sets. Therefore, we can estimate the distribution of signals, $F(\cdot)$, with fewer covariates.

	Mean	Median	Std	Max	Min
Winning bid	38.74	39.07	3.22	46.43	29.53
Rating ratio	0.91	0.98	0.15	1.00	0.22
Number of actual bidders	4.32	4.00	1.12	8.00	2.00
Buy-it-now price	45.74	45.96	3.19	54.15	37.91

Table 3.10: Summary statistics (2005 U.S. mint silver proof coin set, # of obs. = 152)

Table 3.10 summarizes the statistics. The first column describes the variables. The "winning bid" is the transaction price and the second-highest bid in the eBay auction. Note that the "winning bid" does not contain the transaction price via the "buy-It-Now" option. From Table 3.10, on average, one could get the mint silver proof coin for $39 via auction. The "rating ratio" is the percentage of "positive ratings" in the total ratings (i.e., sum of "positive ratings" and "negative ratings"). "Positive ratings" denotes the number of positive ratings a seller receives. "Negative ratings" is the sum of the number of negative ratings and the number of neutral ratings a seller receives. Since the number of neutral ratings and the number of negative ratings are usually small relative to the number of positive ratings, we regard neutral ratings as negative ratings. "Number of actual bidders" is the number of participants who actually bid at auction t. "Buy price" denotes the transaction price via the buy-it-now option. From Table 3.10, the average "buy-it-now price" is about $46. Therefore, the transaction price via the "buy-it-now" option is higher than the transaction price via auction by an average of $7.

3.5.2 Estimation Results

We estimate the structural parameters using the U.S. mint silver proof coin data described above. We assume that the signal, S_i, follows the normal distribution. That is,

$$S_i \sim i.i.d.\, \mathrm{N}(\mu_t, \sigma_t^2),$$

where $\mu_t = \alpha_0 + \alpha_1 X_t$ and $\sigma_t = \exp(\beta_0 + \beta_1 X_t)$. The parameters $\boldsymbol{\alpha} = (\alpha_0, \alpha_1)$ and $\boldsymbol{\beta} = (\beta_0, \beta_1)$ are unknown to econometricians. In this empirical illustration, the auction-specific covariate, X_t, is the "rating ratio." That is, X_t is the percentage of "positive ratings" in the total ratings.

The prior distributions of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are

$$\boldsymbol{\alpha} \sim \mathrm{N}(0, 100\boldsymbol{I})$$

and

$$\boldsymbol{\beta} \sim \mathrm{N}(0, 100\boldsymbol{I}),$$

where $\boldsymbol{I}$ is the identity matrix of order 2.

Similar to the simulation experiments in Section 3.4, we estimate the parameters using likelihood (3.5) (i.e., true likelihood) and likelihood (3.4) (i.e., wrong likelihood). We use the random walk-based Metropolis-Hastings algorithm to generate the random draws from posterior distributions. The number of iterations is 100000, and the burn-in period is 10000 when we use the likelihood (3.5). Similarly, The number of iterations is 70000, and the burn-in period is 5000 for the estimation using likelihood (3.4).

	(True) Likelihood (3.5)		(Wrong) Likelihood (3.4)	
	CD	IF	CD	IF
α_0	0.15	335.12	0.06	238.44
α_0	0.14	333.25	0.05	236.95
β_0	0.43	331.09	0.97	256.15
β_0	0.45	327.86	0.98	255.10

Table 3.11: The Convergence Diagnostics for the MCMC (CD) and the Inefficiency Factors (IF)

Table 3.11 reports p-values of the convergence diagnostics for the MCMC (CD) and inefficiency factors (IF). All p-values of the convergence diagnostics are more than 0.05. Furthermore, the values of the inefficiency factor values are sufficiently low. For likelihood (3.5), the inefficiency factors are 327.86 to 335.12, which imply that we would gain the same variance of the posterior means from 298 uncorrelated draws, even in the worst case. Similarly, for likelihood (3.4), the inefficiency factors are 236.95 to 256.15, which imply that we would gain the same variance of the posterior means from 273 uncorrelated draws, even in the worst case.

Figures 3.13 and 3.14 shows the sample paths of the estimated parameters using true likelihood (3.5) and wrong likelihood (3.4), respectively. From Figures 3.13 and 3.14, it can be seen that the sample paths of these parameters converge to posterior distributions. Thus, we conclude that the sample paths of the estimated parameters converge to posterior distributions.

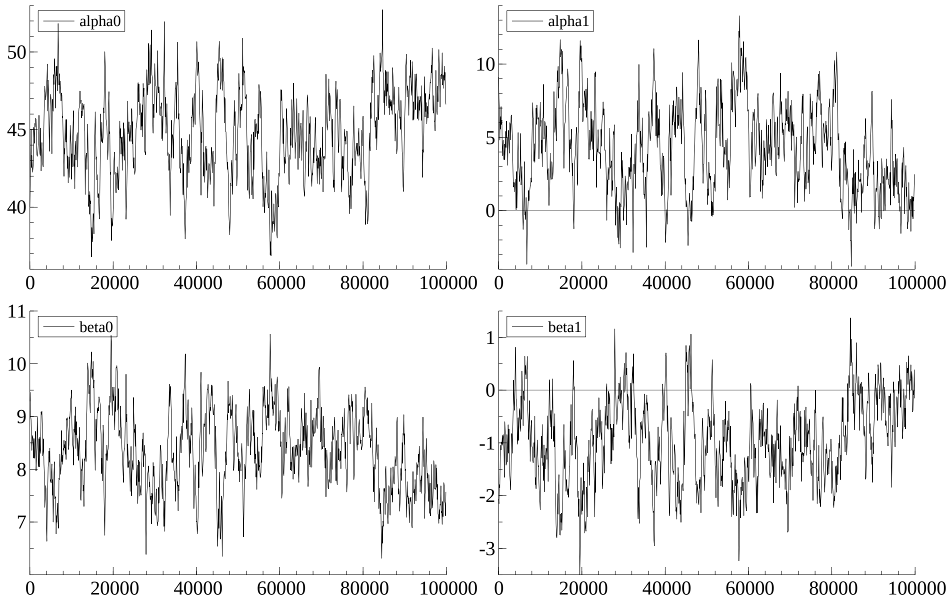

Figure 3.13: Sample paths of parameters using true likelihood (3.5)

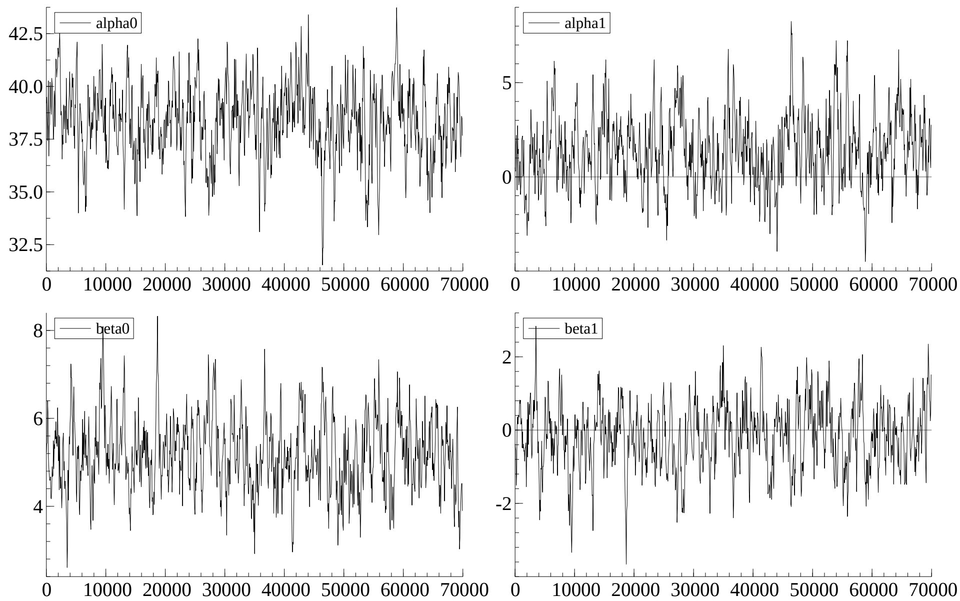

Figure 3.14: Sample paths of parameters using wrong likelihood (3.4)

Tables 3.12 and 3.13 provide the posterior inferences using likelihood (3.5) and (3.4), respectively. In Tables 3.12 and 3.13, the second column is the posterior mean, the third column is the posterior standard deviation, and the fourth column provides the 95% credible intervals of the posterior distributions.

	(True) Likelihood (3.5)		
	Mean	Stdev.	95% interval
α_0	44.78	2.80	(39.10, 49.71)
α_1	4.23	2.97	(-1.06, 10.24)
β_0	8.33	0.74	(6.94, 9.77)
β_1	-0.95	0.80	(-2.49, 0.55)

Table 3.12: Estimation result with true likelihood (3.4)

	(Wrong) Likelihood (3.4)		
	Mean	Stdev.	95% interval
α_0	38.37	1.78	(34.48, 41.65)
α_1	1.39	1.92	(-2.18, 5.51)
β_0	5.18	0.84	(3.57, 6.95)
β_1	-0.17	0.91	(-2.08, 1.59)

Table 3.13: Estimation result with wrong likelihood (3.4)

Figures 3.15 and 3.16 show the posterior densities.

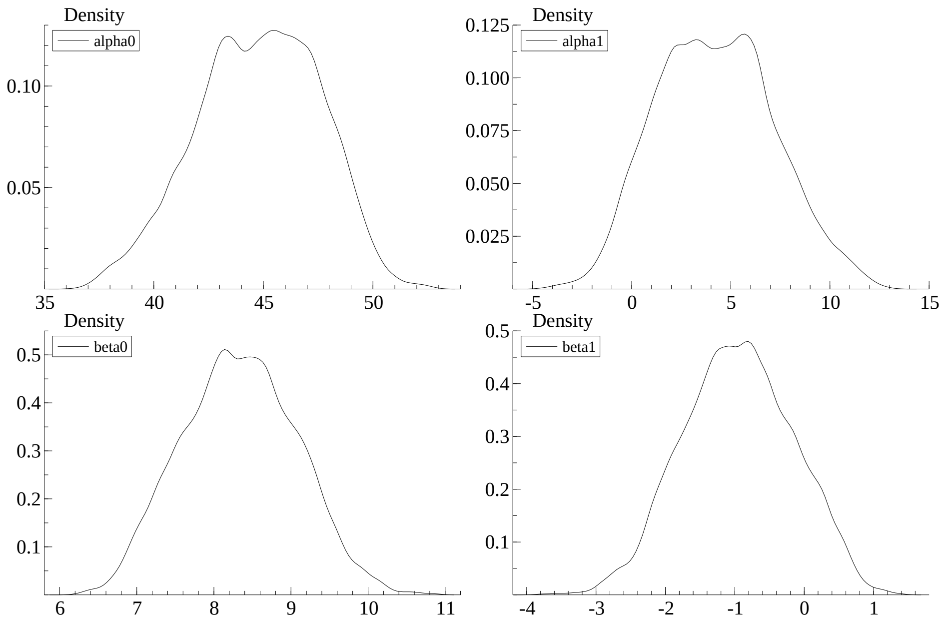

Figure 3.15: Posterior densities using true likelihood (3.5)

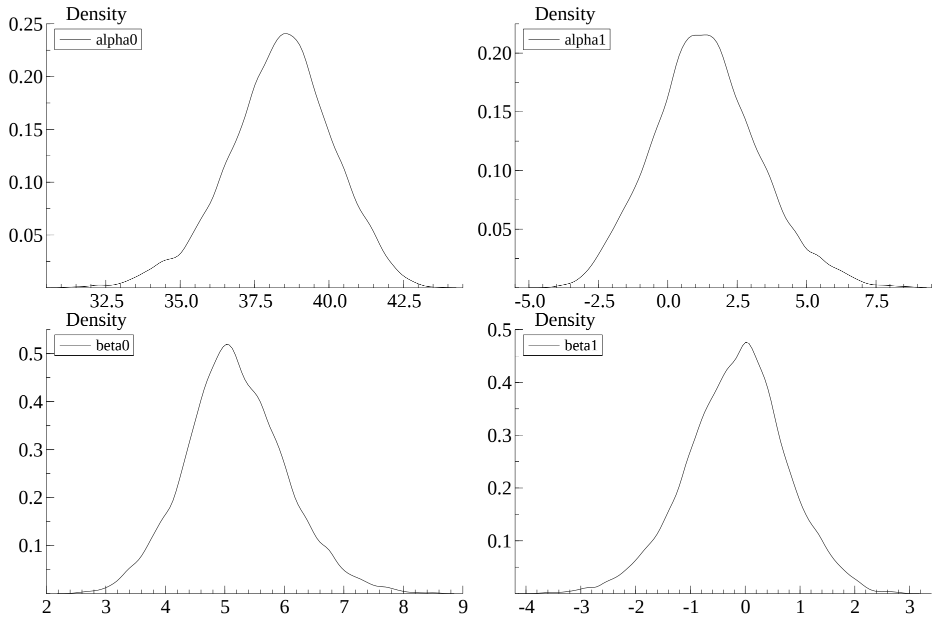

Figure 3.16: Posterior densities using wrong likelihood (3.4)

Comparing the estimation results, both the values of α_0 and α_1 in Table 3.12 are higher than those of α_0 and α_1 in Table 3.13. Since $\mu_t = \alpha_0 + \alpha_1 X_t$ is the mean of each bidder's signal, S_{it}, estimation without buy prices (i.e., estimation result using likelihood (3.4)) may underestimate the true distribution of bidders' signals. The posterior mean of α_1 with buy prices is higher than that of α_1 without buy prices. Since the covariate is the percentage of "positive ratings," estimates without buy prices may underestimate the effect of "positive ratings." Both the 95% credible intervals of α_1 in Table 3.12 and 3.13 contain zero. However, Figure 3.15 shows that most of α_1 takes positive values using likelihood (3.5). The probability that α_1 takes negative values is 0.07 when we use likelihood (3.5). In contrast, Figure 3.16 shows that α_1 takes negative values at a rate that cannot be ignored when we estimate without buy prices. The probability that α_1 takes negative values is 0.23 when we use likelihood (3.4). Similarly, the 95% credible intervals of β_1 in Table 3.12 and 3.13 contain zero. However, Figure 3.15 shows that most of β_1 takes negative values using likelihood (3.5). The probability that β_1 takes negative values is 0.87. In contrast, Figure 3.16 shows that α_1 takes positive values at a significant rate using likelihood (3.4). The probability that β_1 takes negative values is 0.56.

From the estimation results, we find that the posterior mean of α_1 is positive. This implies that the percentage of positive reputations has a positive effect on the mean of bidders' signals, S. On the other hand, the posterior mean of β_1 is negative. This result implies that if the ratio of the positive reputations increases, the variance of signal becomes smaller. These results seem plausible.

3.6 Counterfactual Simulations

In this section, we present several counterfactual simulations using the estimated parameters. First, we compute the optimal buy price which maximizes the sellers' expected revenues.

3.6.1 Optimal Buy Price

In our theoretical model described in Section 3.2, each bidder decides whether to accept or reject buy prices after watching the buy-it-now prices. Therefore, the expected revenue (i.e., the expected transaction price) of the seller depends on the buy prices. In this subsection, we compute the optimal buy price that maximizes the expected revenue of sellers using the estimated parameters, $\boldsymbol{\alpha} = (\alpha_0, \alpha_1)$ and $\boldsymbol{\beta} = (\beta_0, \beta_1)$.

Let $\bar{\boldsymbol{\alpha}} = (\bar{\alpha}_0, \bar{\alpha}_1)$ and $\bar{\boldsymbol{\beta}} = (\bar{\beta}_0, \bar{\beta}_1)$ be the posterior means of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. In addition, let "$\mathrm{P\bar{R}N}$" be the sample average of the "rating ratio," i.e.,

$$\bar{\boldsymbol{\alpha}} = (44.78, 4.23),\ \bar{\boldsymbol{\beta}} = (8.33, -0.95),\ \text{and } \mathrm{P\bar{R}N} = 0.91.$$

We compute the optimal buy price using the parameters $\bar{\boldsymbol{\alpha}}$, $\bar{\boldsymbol{\beta}}$, and $\mathrm{P\bar{R}N}$. We employ the grid search method to compute the optimal buy price. From the buy prices of

$p = \$40.00$ to $\$60.00$ with increment $\$0.01$, we execute the following procedure.

First, we generate random draws, $s_1^{(t)}, ..., s_N^{(t)}$ from $N(\mu, \sigma^2)$, where $\mu = \bar{\alpha}_0 + \bar{\alpha}_1 \mathrm{P\bar{R}N}$ and $\sigma^2 = \exp(\bar{\beta}_0 + \bar{\beta}_1 \mathrm{P\bar{R}N})$. Then we compute the transaction prices of auctions with buy price, p, and the winning bids of auctions without buy prices. The equilibrium transaction prices of auctions with buy prices can be computed using the procedure described in Section 3.2 for each buy price of $p = 40.00, 40.01, ..., 60.00$. Let $w_*^{(t)}$ be the transaction price of auctions with buy prices. Similarly, since bidder i with signal s_i in auctions without buy prices bids $b_i = \mathrm{E}(\frac{1}{N}\sum_{i=1}^n S_i | S_i = s, \max_{j \neq i} S_j = s)$, we can compute the bids of auctions without buy prices from $s_1^{(t)}, ..., s_N^{(t)}$ and the estimated parameters. Iterate this procedure until t becomes a large number, T. In our example, $T = 5000$. Since the expected revenue is $\mathrm{E}(W_*)$, the sample average of $w_*^{(t)}$ estimates the expected revenue.

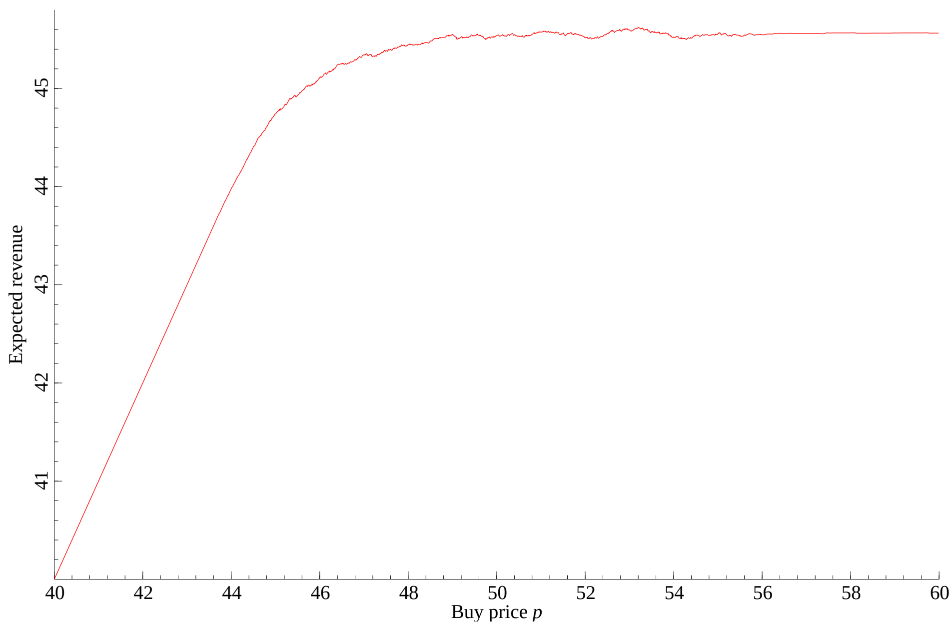

Figure 3.17: Expected revenue and buy prices from the eBay mint coin data

Figure 3.17 shows the expected revenue of auctions with buy prices and corresponding buy prices from \$40.0 to \$60.0. The horizontal axis is the buy price, and the vertical axis represents the expected revenue of auctions with buy prices.

When the seller sets the buy price too low, each bidder accepts the buy prices. Therefore, the expected revenue of sellers is equal to the buy price that the seller sets when the buy price is sufficiently low. In Figure 3.17, the expected revenues are equal to buy prices from \$40.0 to \$43.0. That is, we find that bidders will accept the buy prices when $p \in [40.0, 43.0]$. On the other hand, when the seller sets the price too high, every bidder rejects the buy price. Thus, the expected revenue of a buy price auction is equal to that of a standard auction (i.e., auction without buy-it-now prices) when the buy price is sufficiently high. In our empirical example, the estimated expected revenue of standard auctions is about \$45.56. In Figure 3.17, we find that the expected revenues of auctions with a buy-it-now option are equal to those of standard auctions, \$45.56, when buy prices are higher than \$56.24. Besides, for $p > 49.00$, the expected revenue does not substantially change in our empirical example.

According to the results of our computation, the optimal buy price that maximizes expected revenue is \$53.20. Note that from Table 3.10, the average buy prices is about \$45.74. Therefore, we find that in eBay mint coin auctions, the optimal buy price is higher than the average buy price that sellers set.

3.6.2 Revenue Comparison

As discussed in Section 3.2, the transaction prices in auctions with buy prices may not be equal to the transaction prices in auctions without buy prices since buyers may accept the buy prices. Therefore, the expected revenue of auctions with buy prices may not equal that of auctions without buy prices. In this subsection, we compare the revenues of auctions with buy prices and those without.

We estimate the expected revenue difference (i.e., the difference between the expected revenue with buy prices and those without) in the eBay mint coin auction market using the estimated parameters. Similar to the previous subsection, let $\bar{\boldsymbol{\alpha}} = (\bar{\alpha}_0, \bar{\alpha}_1)$ and $\bar{\boldsymbol{\beta}} = (\bar{\beta}_0, \bar{\beta}_1)$ be the posterior means of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. In addition, let "P$\bar{\text{R}}$N" be the sample average of the "rating ratio," i.e.,

$$\bar{\boldsymbol{\alpha}} = (44.78, 4.23),\ \bar{\boldsymbol{\beta}} = (8.33, -0.95),\ \text{and } \mathrm{P\bar{R}N} = 0.91.$$

We estimate the revenue difference using $\bar{\boldsymbol{\alpha}}$, $\bar{\boldsymbol{\beta}}$, and P$\bar{\text{R}}$N. The procedure is similar to the computation of the expected revenues of price auctions. First, we generate random draws $s_1^{(t)}, ..., s_N^{(t)}$ from $N(\mu, \sigma^2)$, where $\mu = \bar{\alpha}_0 + \bar{\alpha}_1 \mathrm{P\bar{R}N}$ and $\sigma^2 = \exp(\bar{\beta}_0 + \bar{\beta}_1 \mathrm{P\bar{R}N})$. Then, we compute the transaction prices of auctions with buy prices and the winning bids of auctions without buy prices. Since bidder i with signal s_i in auctions without buy prices bids $b_i = \mathrm{E}(\frac{1}{N}\sum_{i=1}^{n} S_i | S_i = s, \max_{j \neq i} S_j = s)$, we can compute the bids of auctions without buy prices from $s_1^{(t)}, ..., s_N^{(t)}$ and the estimated parameters. Let $w^{(t)}$ be the winning bid of an auction without a buy price. Similarly, the equilibrium transaction prices of auctions with buy prices can be computed following the procedure described in Section 3.2. Let $w_*^{(t)}$ be the transaction price of an auction with a buy price. Then, the revenue difference between auctions with buy prices and auctions without them can be obtained by $w_*^{(t)} - w^{(t)}$. Iterate this procedure until t becomes a large number, T. In our example, $T = 5000$. We execute the procedure for buy prices $p = \$40.0$ to $\$60.0$ with the increment $\$0.01$.

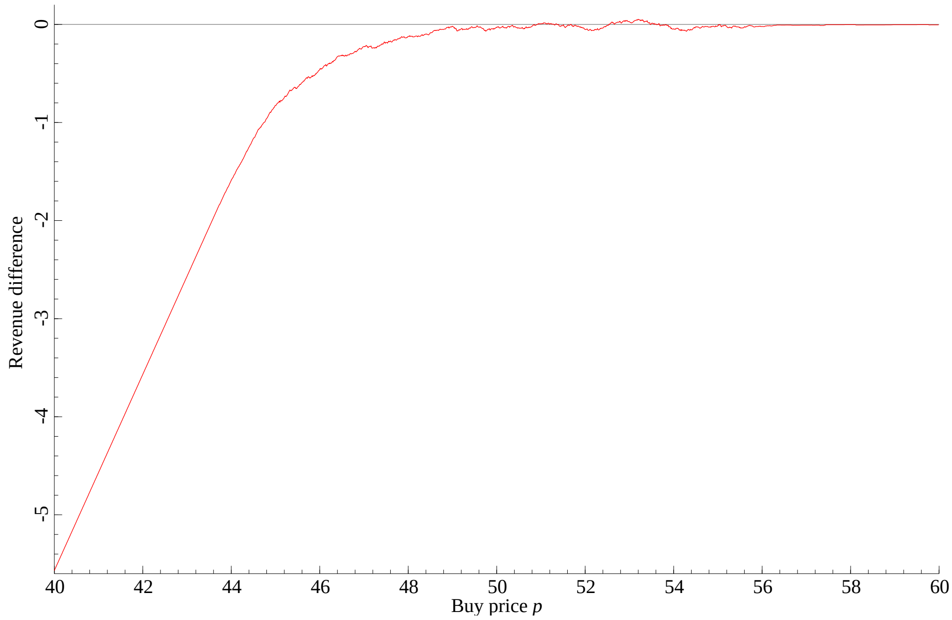

Figure 3.18: Expected revenue difference from the eBay mint coin data

Figure 3.18 shows the expected revenue difference, $\mathrm{E}(W_* - W)$, and corresponding buy prices from \$40.0 to \$60.0. The horizontal axis is the buy price, and the vertical axis represents the expected revenue difference.

Since the expected revenue of standard auctions does not depend on the buy price and equals \$45.56 in eBay mint coin auctions, the graph of the expected revenue difference in Figure 3.18 is same shape as that of the expected revenue of buy price auctions in Figure 3.17 except the scale of vertical axis. Therefore, the buy price that maximizes the revenue difference is \$53.20, which is the optimal buy price that maximizes the expected revenue of auctions with a buy-it-now option. The maximized revenue difference is \$0.05. This result implies that, on average, sellers can realize an additional profit of \$0.05 when they set the buy price at \$53.20.

When the seller sets price too low, every bidder accepts the buy price. Therefore, the graph for the buy prices $p \in [40.0, 43.0]$, has a linear shape. On the other hand, when the seller sets prices too high, every bidder rejects the buy price. Thus, the expected revenue of an auction with a buy-it-now option is equal to that of a standard auction when the buy price is sufficiently high. Then, the revenue difference equals zero. In Figure 3.18, we find that the expected revenues of auctions with buy-it-now prices are zero when the prices are higher than \$56.24. However, the revenue difference is nearly zero and does not substantially change for $p > 49.00$. Furthermore, for $p \leq 49.00$, the revenue of auctions without buy price is greater than that of auctions with buy price for $p \leq 49.00$. As a result, the revenue difference is negative or nearly zero for any buy price p.

Table 3.14 presents the summarized statistics of the revenue differences and buy prices. In Table 3.14, the "acceptance rate" represents the estimated probability that at least one bidder will accept the buy price in first-stage game.

Buy price	AR	Mean	Stdev.	25% quantile	Median	75% quantile	IQR
40.00	1.00	-5.57	11.26	-13.52	-6.20	1.81	15.33
43.00	1.00	-2.57	11.26	-10.52	-3.20	4.80	15.33
45.00	0.99	-0.83	10.92	-8.52	-1.20	6.48	15.00
50.00	0.65	-0.03	7.90	-3.52	0.00	1.75	5.27
52.00	0.41	-0.05	6.44	0.00	0.00	0.00	0.00
53.20	0.27	0.05	5.47	0.00	0.00	0.00	0.00
56.00	0.01	-0.02	0.82	0.00	0.00	0.00	0.00
60.00	0.00	0.00	0.31	0.00	0.00	0.00	0.00

Table 3.14: Summary statistics of revenue differences and buy prices (AR: Acceptance rate, IQR: Interquartile range)

As described above, when the buy price is \$53.20, the expected revenue difference is maximized and equal to \$0.05. Since the acceptance rates are 1 for $p \in [40.00, 43.00]$, the transaction prices are equal to the buy prices. In addition, the winning bids without buy prices do not depend on those prices. Therefore, for $p \in [40.00, 43.00]$, the standard deviation and interquartile range are invariant. We find that the acceptance rate decreases quickly for buy prices $p > 47$. The standard deviation decreases as the buy price increases. Similarly, the interquartile range of revenue difference (i.e., difference between the 75% quantile and the 25% quantile of revenue difference) decreases. In particular, for $p \in [52, 60]$, both the 25% quantile and the 75% quantile are 0, and the standard deviation is quite small. These results imply that the distribution of revenue differences converges to the degenerate distribution that only takes 0 as p increases in probability. This result is reasonable, since the transaction prices with buy prices become the winning bids without buy prices when the buy prices are too high. When the buy price, p, is too high, no bidder accepts the price in the first stage. Then, the transaction prices are determined in the second-stage auction games. Since the second-stage auction games coincide with auctions without buy prices, the transaction prices with buy prices equal the winning bids without buy prices when the buy prices are sufficiently high. Decreases in the standard deviation and the interquartile range of revenue difference represent this phenomenon.

3.7 Conclusions

In this paper, we provide a method for estimating online common value auction models with buy price options. In online auction markets, buyers must submit their bids while observing the fixed buy prices. Therefore, when we estimate the structural parameters of online auction models, we must take into account the buy price. When we ignore the buy-it-now prices, estimates may be incorrect.

Our empirical example is eBay mint coin auctions in 2013. We found that when we ignore the buy prices, we underestimate the mean of bidders' signals corresponding to the value of the good as well as the effect on the signals of positive ratings for sellers. Furthermore, we found that the percentage of positive reviews has a positive effect on the mean of the signal. We computed the optimal buy price that maximizes the sellers' revenue using the estimated parameters. We found that the optimal buy price was \$53.20, which is higher than the average buy prices that we observed. We also conducted a revenue comparison. We compared the revenue between auctions with and without buy prices. We found that the mean of the revenue difference between auctions with and without buy prices was \$0.05. Therefore, sellers can realize additional gain from auctions with buy-it-now price options when they set the optimal price, $p = \$53.20$. Furthermore, we found that at least one bidder accepts the optimal buy price with probability 0.27.

Our findings open the door to areas of future research. In this paper, we assume that the bidder is risk neutral. However, Ackerberg et al. (2011) cover risk aversion and time impatience in their model. Therefore, risk aversion and time impatience can be considered in this model.

Chapter 4

An Empirical Analysis of Bundling Sales in Online Auction Markets

4.1 Introduction

Today, many people use consumer-to-consumer electronic commerce sites to buy (or sell) goods. In particular, with the emergence of online auction sites (e.g., eBay and Yahoo!), many people have become familiar with auctions.

Many studies have focused on online auctions. Examples include Melnik and Alm (2002), Livingston (2005), Houser and Wooders (2006), and Resnick et al. (2006) regarding the effect of reputation on sellers' revenue; Bajari and Hortaçsu (2003) regarding common value auctions and the winners' curse; Bapna et al. (2008) and Giray et al. (2009) regarding consumer surplus; Adams (2007) regarding demand on eBay; Hossain and Morgan (2005) regarding the revenue equivalence theorem; and Roth and Ockenfels (2002) regarding snipe bidding.

In such tradings, sellers often sell two or more items as bundling auctions. However, other sellers sell the same items separately. In this paper, we focus on bundling auctions of online auction markets. We propose an empirical model of online common value auctions for both bundling auctions and separate auctions.

Some papers focus on the bundling auction model in theoretical literature. Palfrey (1983) studied bundling auctions with two bidders. He found that bundling auctions generate more expected revenues with two bidders within the private values paradigm. Chakraborty (1999) extended Palfrey (1983) to a general number of bidders. He found that if the number of bidders grows large, the expected revenue of separate sales becomes greater than that of bundling auctions. While Chakraborty (1999) studied the private values model, Chakraborty (2002) studied the common value auction model and found the effect that they call *the winner's curse reduction effect* in bundling auctions. He also compared the expected revenues between bundling auctions and separate auctions.

Our empirical example involves eBay mint coin auctions in 2014. In our data set, there are two kinds of coin sets: 11-coin sets and 22-coin sets. We regard the 11-coin sets as separate items and the 22-coin sets as bundled items. We also conduct some counterfactual simulations using the estimated parameters. We evaluated the winner's curse reduction effect in the sense of Chakraborty (2002) and compared revenue between bundling auctions and separate auctions. Chakraborty (2002) showed that bidders will bid more aggressively in separate auctions than in bundling auctions; he named this effect *the winner's curse reduction effect*. We measured the magnitude of the winner's curse reduction effect. We found that bidders in separate auctions will bid \$2.5 higher than in bundling auctions. For revenue comparison, we found that the expected revenue in a bundling auction is higher than that in separate auctions by \$0.37. Since the average transaction price of bundled items (22-coin sets) is \$8.98, the value of additional gains are not negligible.

The rest of this paper is organized as follows. In Section 4.2, we describe the model of online auctions within the pure common value paradigm. Additionally, following

Chakraborty (2002), we review the theoretical results for the bundling auctions. Section 4.3 describes the estimation strategy for the model described in Section 4.2. We utilize Bayesian estimation to estimate the structural parameters. In Section 4.4, we conduct some simulation experiments. In Section 4.5, we estimate the structural parameters using real auction data. Our empirical example is eBay mint coin set auctions in 2014. In Section 4.6, we compute the winner's curse reduction effect in the sense of Chakraborty (2002) and compare the revenue between separate auctions and bundling auctions using the estimated parameters. Section 4.7 features some concluding remarks.

4.2 The Model

There are N risk neutral potential bidders and a seller. The number of potential bidders, N, is a random variable and an exogenous variable. The seller sells two different objects $k = 1$ and 2. In this model, we consider the pure common value model in which the *ex post* valuation of the item is the same for each bidder. Let V_1 and V_2 denote the values for items 1 and 2, respectively. The realizations of values are unknown to the bidders. Instead, each bidder, i, receives her private signals corresponding to V_1 and V_2, which are denoted by S_{1i} and S_{2i}, respectively. Each bidder knows the realization of her own private signal but does not know the others' before auctions. However, both the distribution of S_{1i} and the distribution of S_{2i} are common knowledge among bidders. In this paper, we consider a specific functional form for V_1 and V_2. We assume that the value of each item to bidders is the average of their signals. That is, the valuations take the form of

$$V_1 = \frac{1}{N}\sum_{i=1}^{N} S_{1i} \text{ and}$$

$$V_2 = \frac{1}{N}\sum_{i=1}^{N} S_{2i},$$

respectively. [1] For the bundled item, we impose the additive separability on bidder i's signal for the bundled items, S_i. Namely, we assume that $S_i = S_{1i} + S_{2i}$. Then, from our specific functional form for the value, the valuation of the bundled item, V, is

$$\begin{aligned} V &= \frac{1}{N}\sum_{i=1}^{N} S_i \\ &= \frac{1}{N}\sum_{i=1}^{N}(S_{1i} + S_{2i}) \\ &= V_1 + V_2. \end{aligned}$$

We assume that $S_{k1}, ..., S_{kN}$ are independently and identically distributed. Namely,

$$S_{ki} \sim i.i.d.\, F_k(x)$$

for $k \in \{1, 2\}$. We assume that S_{1i} and S_{2i} are independently distributed. Furthermore, we assume that for each $k \in \{1, 2\}$, S_{ki} is affiliated with $S_{1i} + S_{2i}$ in the sense of Milgrom and Weber (1982).

4.2.1 Equilibrium

In this paper, we regard online eBay auctions as second-price auctions. That is, each bidder submits her bid and the bidder with the highest bid among bidders wins the object and pays the second highest bid. Then, the equilibrium bidding strategies in separate auctions for items $k = 1$ and 2 are straightforward arguments from Milgrom and Weber (1982).

Let Y_{ki} be the highest signal except bidder i's signal, S_{ki}. That is, $Y_{ki} = \max_{j \neq i} S_{kj}$. Then, the equilibrium bidding functions for bidder i with private signals $S_{1i} = s_1$ and

[1] This specification is the special case of Chakraborty (2002) and has been used in several papers. Example are Goeree and Offerman (2002) for auctions within common value and private values paradigm and Shahriar (2008) and Shahriar and Wooders (2011) for auctions with buy prices model.

$S_{2i} = s_2$ in separate auctions are given by

$$b_1(s_1) = \mathrm{E}[V_1 | S_{1i} = s_1, Y_{1i} = s_1] \quad (4.1)$$

and

$$b_2(s_2) = \mathrm{E}[V_2 | S_{2i} = s_2, Y_{2i} = s_2] \quad (4.2)$$

for items $k = 1$ and 2, respectively.

Analogously, the equilibrium bidding function for bundling auctions can be derived in the same manner. Let $S_i = S_{1i} + S_{2i}$ be the sum of bidder i's signals, S_{1i} and S_{2i}. Furthermore, let $G(\cdot)$ denote the cumulative distribution function of $S_i = S_{1i} + S_{2i}$. In other words, $G(\cdot)$ is the convolution of $F_1(\cdot)$ and $F_2(\cdot)$. Then, $S_1, ..., S_N$ are also independently and identically distributed with the CDF $G(\cdot)$. Namely,

$$S_i \sim i.i.d.\, G(s).$$

Let Y_i be the highest signal except bidder i's signal, S_i. That is, $Y_i = \max_{j \neq i} S_j$. Then, using an argument similar to that of a separate auction, we gain the equilibrium bidding function for bidder i with signal $S_i = s$ in the bundling auctions

$$b(s) = \mathrm{E}[V_1 + V_2 | S_i = s, Y_i = s]. \quad (4.3)$$

4.2.2 Bundling Auctions versus Separate Auction

Since we computed the various effects of bundling auctions in our empirical example, it is worthwhile to review the theoretical result of bundling auctions within the common value paradigm. Chakraborty (2002) discussed the bundling auctions model and the separate auctions model within the pure common value paradigm. Furthermore, he discussed the effect of bundling auctions and separate auctions with some useful examples. In this subsection, we review the results of Chakraborty (2002).

Chakraborty (2002) discussed that bundling auctions have a winner's curse reducing effect. The intuitive explanation of the winner's curse reducing effect is as follows. In separate auctions of $k = 1$ and 2, winning the items $k = 1, 2$ implies that each winner has the highest signal on each item. On the other hand, in a bundling auction, winning the bundled item implies that the winner has the highest signal for the bundled item but not for individual items, $k = 1$ and 2. Therefore, winning the bundling auction is not as bad as winning two separate auctions. The following theorem is the Theorem 1 in Chakraborty (2002). They call the result of Theorem 1 *the winner's curse reducing effect.*

Theorem 1 (Chakraborty (2002))**.** *A bidder bids more aggressively when the objects are bundled. That is,*

$$b(s) \geq b_1(s_1) + b_2(s_2),$$

where $s = s_1 + s_2$.

4.3 Estimation

The results of equilibrium bidding strategies (4.1), (4.2), and (4.3) are familiar to economists. However, few empirical studies focus on the structural estimation of common value auction models. The main reason is the negative result of nonparametric identification on the common value auction model. Athey and Haile (2002) and Athey and Haile (2007) showed the conditional distribution of S_{ki}, given V_k is not identified from the observed bids in the common value auction model without additional identification conditions.

Therefore, most studies of structural estimation of the auction model focus on the private values model. Recently, some papers have studied the identification condition of the common value auction model. For example, Li et al. (2000) showed the identification under the additive separability of the common value component. Février (2008) restricted the shape of the density function of the common value and showed the identification of the common value auction model. d'Haultfoeuille and Février (2008) proposed the identification condition of the common value auction model, assuming the support of a private signal is finite and varies depending on the common value. In this paper, we impose parametric specification to avoid the identification problem.

4.3.1 Estimation Procedure

We observe T_k auctions indexed by $t = 1, ..., T_k$ for item $k \in \{1, 2\}$. The same items are each sold in separate auctions. Analogously, we observe T auctions indexed by $t = 1, ..., T$ for bundling auctions. We can observe each bidder's bid, B_{kit}, and the number of actual bidders, n_t, for bidder i, for item $k\{1, 2\}$, and for auction $t\{1, ..., T_k\}$. We cannot observe each bidder's signals, S_{kit} and S_{it}, the common value, V_{kt} and V_t, and the number of potential bidders, N_t.

An unknown number of potential bidders, N_t, can be a problem for identification, in general. Within the private values paradigm, several papers proposed a novel method for identifying the structural parameters when econometricians cannot observe the number of potential bidders (Paarsch (1997); Song (2004); An et al. (2010); and Shneyerov and Wong (2011)). However, for common value auctions, to the best of our knowledge, no paper has focused on this issue. Therefore, we assume that the number of potential bidders is constant among auctions as is the maximum number of actual number of bidders observable by econometricians such as Guerre et al. (2000).

Following the example of Bajari and Hortaçsu (2003), we assume that bidders' signals, S_{kit}, are normally distributed with mean, μ_{kt}, and variance, σ_{kt}^2. That is, for $k \in 1, 2$,

$$S_{kit} \sim N(\mu_{kt}, \sigma_{kt}^2),$$

where

$$\mu_{kt} = \boldsymbol{\alpha}_k' X_{kt}$$

and

$$\sigma_{kt_k}^2 = (\exp(\beta_{k1}), ..., \exp(\beta_{kd}))X_{kt},$$

where d represents the dimensionality of the vector of the coefficient parameter, and X_{kt} is the vector of the auction-specific covariate. The values of $\boldsymbol{\alpha_k} = (\alpha_{k1}, ..., \alpha_{kd})$ and $\boldsymbol{\beta}_k = (\beta_{k1}, ..., \beta_{kd})$ are unknown to econometricians; therefore, we estimate these parameters.

Recall that the equilibrium bidding function $b_k(\cdot)$ is given by

$$b_k(s_k) = \mathrm{E}(V_{kt}|S_{kit} = s_k, Y_{kit} = s_k).$$

Since $b_k(\cdot)$ is a strictly increasing function, there exists an inverse function $\phi_k(\cdot)$. Note that since we considered second-price auctions, the winning bid of item k, w_{kt}, in auction t is the second-highest bid in auction t. Therefore, observing the winning bids, the likelihood function for separate auctions is given by

$$\begin{aligned} L(w_1, ..., w_{kt}|\boldsymbol{\alpha}_k, \boldsymbol{\beta}_k, (X_{k1}, ..., X_{kt})) = \prod_{t=1}^{T_k} \begin{pmatrix} & N & \\ 1 & 1 & N-2 \end{pmatrix} & [F_k(\phi_k(w_{kt})|\mu_{kt}, \sigma_{kt}^2)]^{N-2} \\ & \times f_k(\phi_k(w_{kt})|\mu_{kt}, \sigma_{kt}^2) \frac{1}{b_k'(\phi_k(w_{kt}))} \\ & \times [1 - F_k(\phi_k(w_{kt})|\mu_{kt}, \sigma_{kt}^2)], \end{aligned} \tag{4.4}$$

where $f_k(\cdot)$ is the probability density function of S_{kit}. In this case, $f_k(\cdot)$ is the normal density function.

The likelihood function for bundling auctions can be derived in the same manner.

We assume that S_{it} is the normal random draw with mean μ_t and variance σ_t^2. That is,

$$S_{it} \sim N(\mu_t, \sigma_t^2)$$

where

$$\mu_t = \boldsymbol{\alpha}' X_t$$

and

$$\sigma_t^2 = (\exp(\beta_1), ..., \exp(\beta_d))X_t,$$

where $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_d)$ and $\boldsymbol{\beta} = (\beta_1, ..., \beta_d)$ are the unknown coefficient parameter vector to be estimated.

Since the equilibrium bidding function, $b(\cdot)$, is a strictly increasing function, there exists an inverse function, $\phi(\cdot)$. Similar to the separate auctions, since we considered second-price auctions, the winning bid, w_t, in auction t is the second-highest bid in auction t. Therefore, in observing the winning bids, the likelihood function for the bundling auction is given by

$$\begin{aligned} L(w_1, ..., w_t|\boldsymbol{\alpha}, \boldsymbol{\beta}, (X_1, ..., X_t)) = \prod_{t=1}^{T} \begin{pmatrix} & N & \\ 1 & 1 & N-2 \end{pmatrix} & [G(\phi(w_t)|\mu_t, \sigma_t^2)]^{N-2} \\ & \times g(\phi(w_t)|\mu_t, \sigma_t^2) \frac{1}{b'(\phi(w_t))} \\ & \times [1 - G(\phi(w_t)|\mu_{kt}, \sigma_{kt}^2)], \end{aligned} \tag{4.5}$$

where $g(\cdot)$ is the probability density function of S_{it}. In this case, $g(\cdot)$ is the normal density function.

4.4 Simulation Experiments

In this section, we estimate the structural parameters in our model using simulation data. The numbers of observed auction markets are $T = 200, 500$ and 1000 for item $k \in \{1, 2\}$ and the bundling auctions. In our simulation experiments, the number of potential bidders is $N = 5$ for all auctions.

Throughout this section, we assume that S_{1it} and S_{2it} are random variables drawn independently from identical distributions. We draw the signals for item $k \in \{1, 2\}$ from the normal distribution. That is,

$$S_{kit} \sim i.i.d.\, \mathrm{N}(\mu_{kt}, \sigma_{kt}^2),$$

where $\mu_{kt} = \alpha_{k0} + \alpha_{k1} X_{kt}$, and $\sigma_{kt}^2 = \exp(\beta_{k0}) + \exp(\beta_{k1}) X_{kt}$. We draw covariate X_{kt} from gamma distribution $\mathrm{Ga}(7, 2)$. The true values of $\boldsymbol{\alpha}_k$ and $\boldsymbol{\beta}_k$ are

$$\alpha_{k0} = 36.5, \alpha_{k1} = 0.2, \beta_{k0} = 2.5, \text{ and } \beta_{k1} = -0.7.$$

The bundling auction features the bundling of items 1 and 2. From the reproductive property of normal distributions,

$$S_{it} \sim i.i.d.\, \mathrm{N}(\mu_t, \sigma_t^2),$$

where $\mu_t = \alpha_0 + \alpha_1 X_t$, and $\sigma_t^2 = \exp(\beta_0) + \exp(\beta_1) X_t$. The true values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are

$$\alpha_0 = 73.0, \alpha_1 = 0.4, \beta_0 = \log 2 + 2.5, \text{ and } \beta_1 = \log 2 - 0.7.$$

We estimate the structural parameters using the Bayesian method. We utilize the Markov Chain Monte Carlo (MCMC) method to compute the posterior distribution of parameters. We use the random walk-based Metropolis-Hastings (MH) algorithm to compute the posterior distribution of the parameters.

For each case, the prior distributions of $\boldsymbol{\alpha}_k$, $\boldsymbol{\beta}_k$ are

$$\boldsymbol{\alpha}_k \sim \mathrm{N}(\mathbf{0}, 100\boldsymbol{I}), \text{ and } \boldsymbol{\beta}_k \sim \mathrm{N}(\mathbf{0}, 100\boldsymbol{I}),$$

where $\boldsymbol{I}$ is the identity matrix of order 2. Similarly, the prior distribution of $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ are

$$\boldsymbol{\alpha} \sim \mathrm{N}(\mathbf{0}, 100\boldsymbol{I}), \text{ and } \boldsymbol{\beta} \sim \mathrm{N}(\mathbf{0}, 100\boldsymbol{I}).$$

4.4.1 The Case of $T = 200$

Hereafter, we omit subscript $k \in \{1, 2\}$ for the separate auctions since signals for the two separate items follow the identical distributions in our setting. That is, $(\alpha_0, \alpha_1) \equiv (\alpha_{k0}, \alpha_{k1})$ and $(\beta_0, \beta_1) \equiv (\beta_{k0}, \beta_{k2})$. In this case, we draw 20000 random samples from the random walk-based MH algorithm, and the burn-in period is 2000 for both the separate item and bundled item.

	True value	Mean	Stdev.	95% interval	CD	IF
α_0	36.5	36.99	1.37	(34.34, 39.98)	0.18	186.15
α_1	0.2	0.13	0.10	(-0.10, 0.32)	0.07	181.58
β_0	2.5	2.29	0.27	(1.68, 2.70)	0.24	152.40
β_1	-0.7	-0.37	0.28	(-0.95, 0.11)	0.12	179.13

Table 4.1: Estimation results for separate auctions (Sample size: $T = 200$)

Table 4.1 shows the estimated posterior distributions of the parameters for separate auctions. Our estimator contains the true values in 95% credible intervals. All p-values of the convergence diagnostics (CD) are more than 0.07.[2] Furthermore, the inefficiency factor (IF) values are sufficiently low.[3] The values of inefficiency factors are, at most, 187, which implies that we would obtain the same variance of the posterior sample means

[2]The CD test statistic tests the equality of the means of the first and last parts of the sample path.

[3]The definition of inefficiency factor is $1 + 2\sum_{k=1}^{\infty} \rho(k)$, where $\rho(k)$ is the sample autocorrelation at lag k.

from 105 uncorrelated draws, even in the worst case. Figure 4.1 represents the sample paths from estimated posterior distributions for the separate auctions. Figure 4.2 shows the estimated posterior densities. As a result, our MCMC simulation performs well.

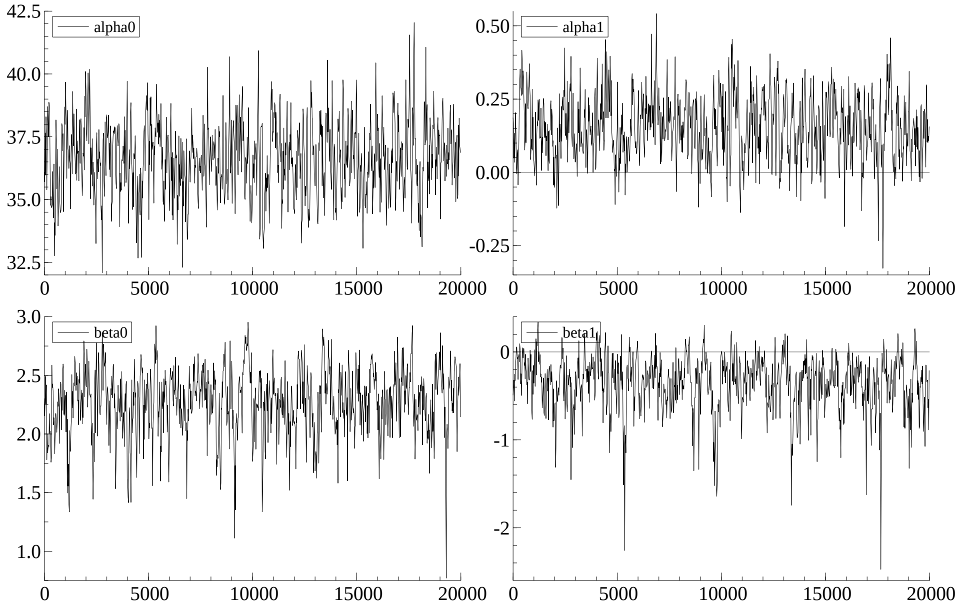

Figure 4.1: Sample paths for separate auctions (Sample size: $T = 200$)

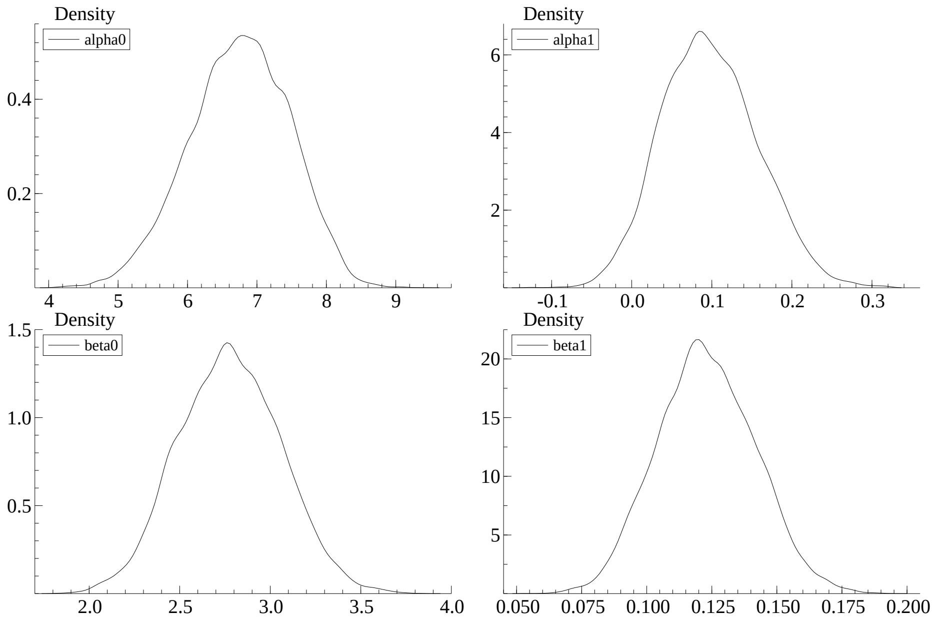

Figure 4.2: Posterior densities for separate auctions (Sample size: $T = 200$)

	True value	Mean	Stdev.	95% interval	CD	IF
α_0	73.0	73.06	0.48	(72.17, 74.00)	0.16	67.72
α_1	0.40	0.40	0.03	(0.33, 0.46)	0.19	67.56
β_0	3.19	3.59	0.21	(3.02, 3.81)	0.31	99.22
β_1	-0.01	-5.01	4.23	(-15.10, 0.31)	0.71	31.91

Table 4.2: Estimation results for bundling auctions (Sample size: $T = 200$)

Table 4.2 shows the estimated posterior distributions of the parameters for bundling auctions. Our estimator contains the true values in 95% credible intervals. All p-values of the convergence diagnostics (CD) are more than 0.16. Furthermore, the inefficiency factor values are sufficiently low. The values of inefficiency factors are, at most 99, which implies that we would obtain the same variance of the posterior sample means from 202 uncorrelated draws, even in the worst case. Figure 4.3 represents the sample paths from estimated posterior distributions for the bundling auctions. Figure 4.4 shows the estimated posterior densities. As a result, our MCMC simulation performs well.

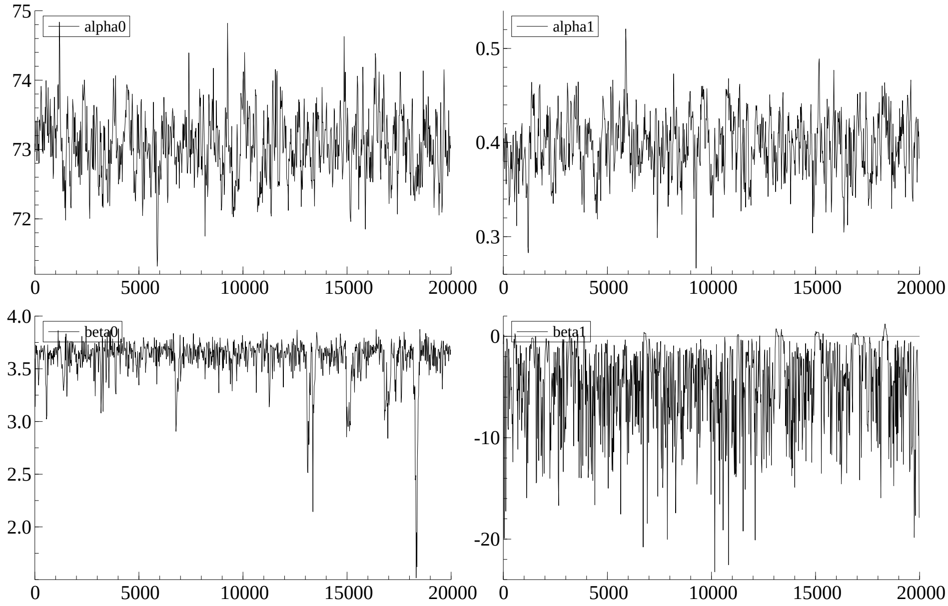

Figure 4.3: Sample paths for bundling auctions (Sample size: $T = 200$)

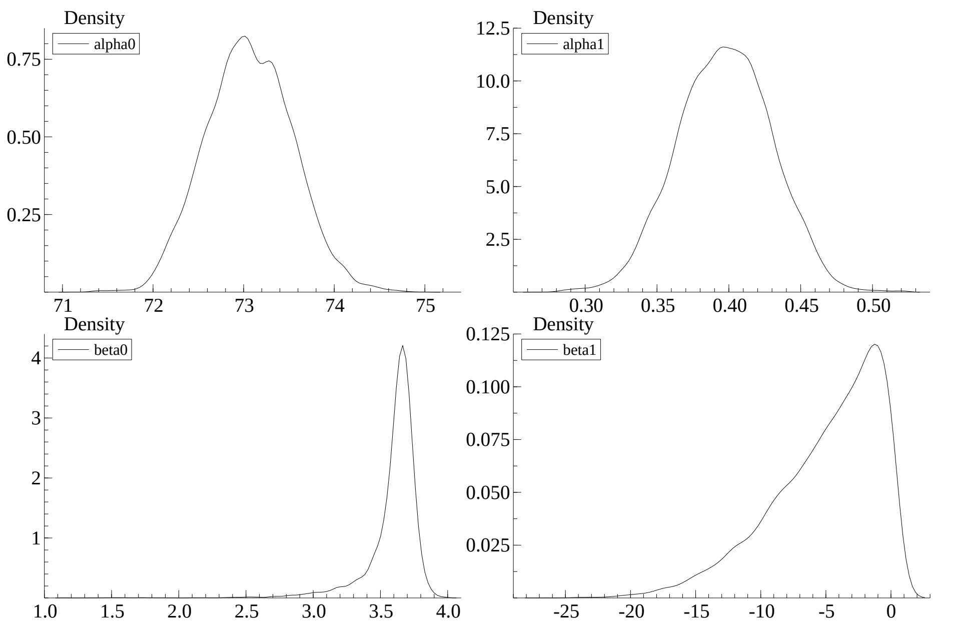

Figure 4.4: Posterior densities for bundling auctions (Sample size: $T = 200$)

4.4.2 The Case of $T = 500$

In this case, we draw 20000 random samples from the random walk-based MH algorithm, and the burn-in period is 2000 for the separate items. For the bundled item, the number of iteration is 50000 and burn-in period is 2000.

	True value	Mean	Stdev.	95% interval	CD	IF
α_0	36.5	37.17	0.92	(35.34, 38.93)	0.25	68.61
α_1	0.20	0.17	0.07	(0.04, 0.30)	0.38	68.05
β_0	2.5	2.46	0.12	(2.21, 2.69)	0.05	54.46
β_1	-0.7	-0.68	0.21	(-1.14, -0.31)	0.09	55.27

Table 4.3: Estimation results for separate auctions (Sample size: $T = 500$)

Table 4.3 shows the estimated posterior distributions of the parameters for separate auctions. Our estimator contains the true values in 95% credible intervals. All p-values of the convergence diagnostics (CD) are more than 0.05. Furthermore, the inefficiency factor values are sufficiently low. The values of inefficiency factors are, at most 69, which implies that we would obtain the same variance of the posterior sample means from 289 uncorrelated draws, even in the worst case. Figure 4.5 represents the sample paths from estimated posterior distributions for the separate auctions. Figure 4.6 shows the estimated posterior densities. As a result, our MCMC simulation performs well.

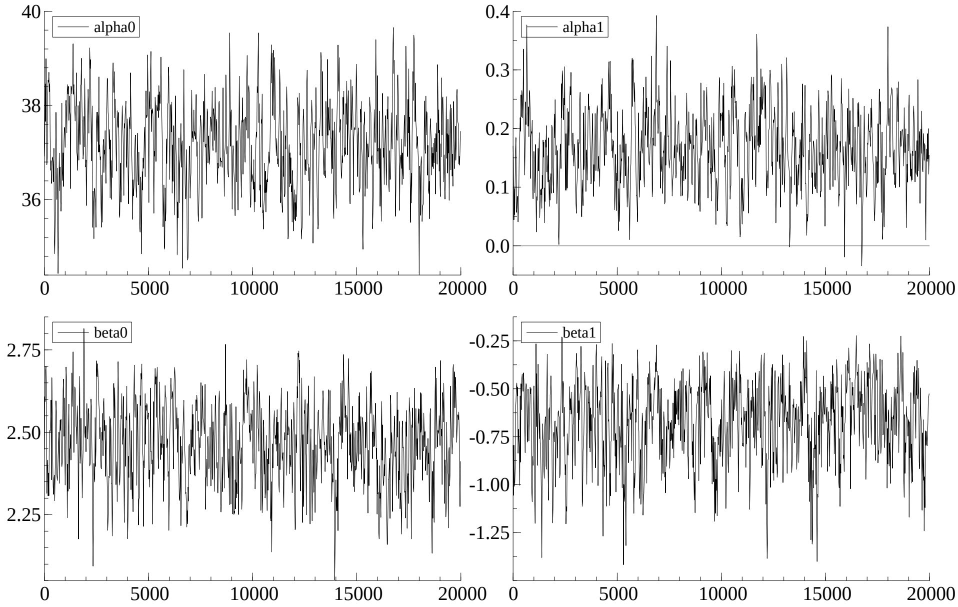

Figure 4.5: Sample paths for separate auctions (Sample size: $T = 500$)

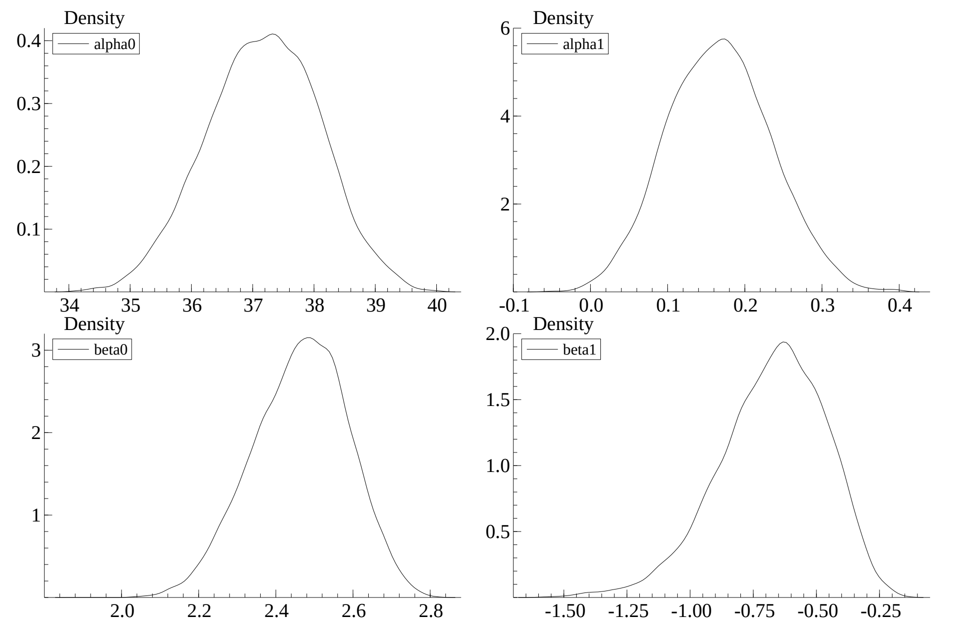

Figure 4.6: Posterior densities for separate auctions (Sample size: $T = 500$)

	True value	Mean	Stdev.	95% interval	CD	IF
α_0	73.0	72.79	0.32	(72.18, 73.43)	0.95	88.75
α_1	0.40	0.42	0.02	(0.38, 0.47)	0.78	92.15
β_0	3.19	3.31	0.32	(2.63, 3.83)	0.02	197.81
β_1	-0.01	-0.54	2.31	(-8.41, 0.83)	0.10	96.28

Table 4.4: Estimation results for bundling auctions (Sample size: $T = 500$)

Table 4.4 shows the estimated posterior distributions of the parameters for the bundling auctions. Our estimator contains the true values in 95% credible intervals. All p-values of the convergence diagnostics (CD) are more than 0.02. Furthermore, the inefficiency factor values are sufficiently low. The values of inefficiency factors are, at most 198, which implies that we would obtain the same variance of the posterior sample means from 252 uncorrelated draws, even in the worst case. Figure 4.7 represents the sample paths from estimated posterior distributions for the separate auctions. Figure 4.8 shows the estimated posterior densities. As a result, our MCMC simulation performs well.

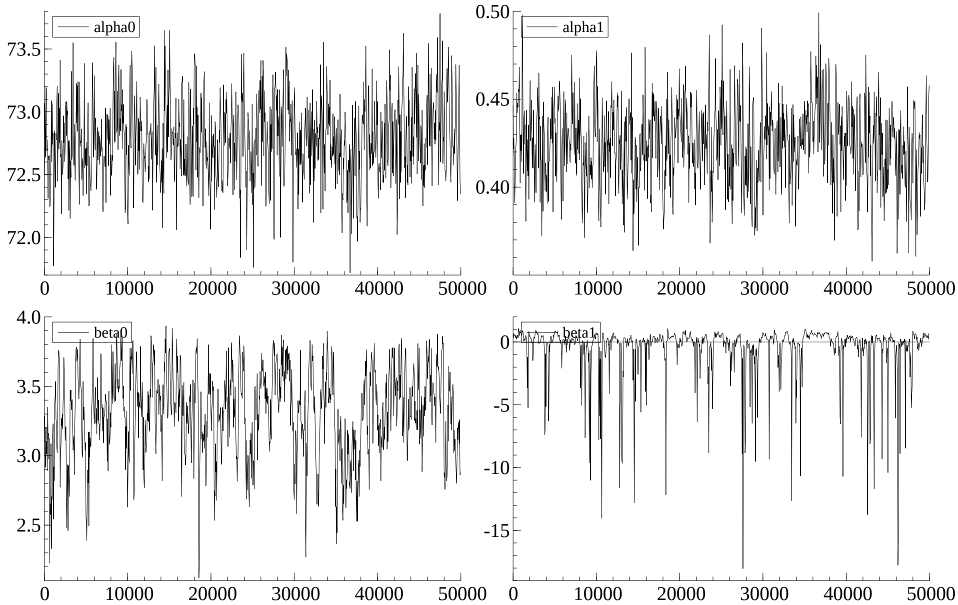

Figure 4.7: Sample paths for bundling auctions (Sample size: $T = 500$)

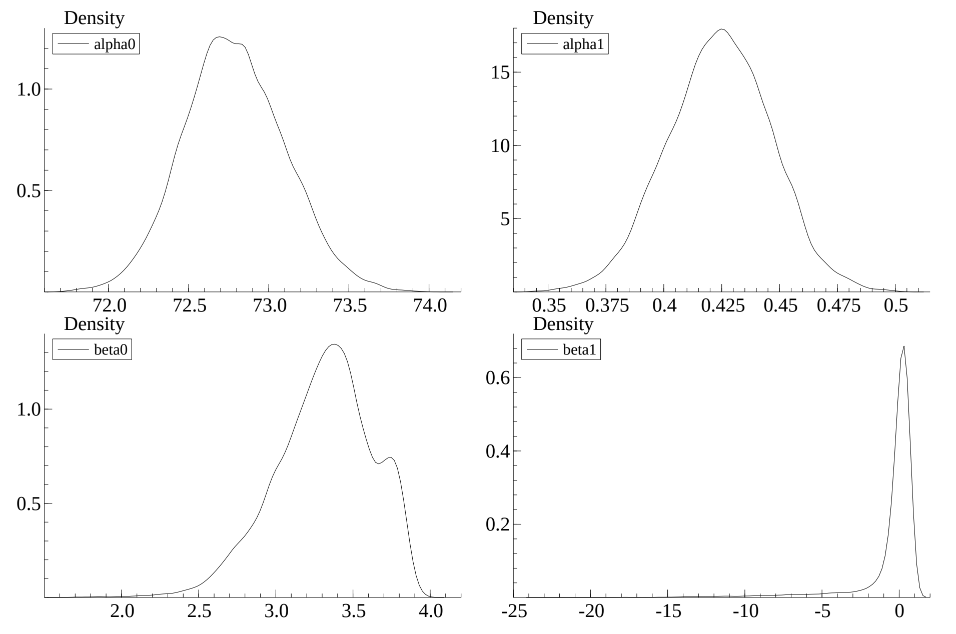

Figure 4.8: Posterior densities for bundling auctions (Sample size: $T = 500$)

4.4.3 The Case of $T = 1000$

In this case, we draw 20000 random samples from the random walk-based MH algorithm, and burn-in period is 2000 for both the separate items and bundled item.

	True value	Mean	Stdev.	95% interval	CD	IF
α_0	36.5	3.66	0.64	(35.31, 37.79)	0.01	67.46
α_1	0.2	0.27	0.05	(0.17, 0.36)	0.01	66.38
β_0	2.5	2.52	0.09	(2.34, 2.68)	0.01	63.49
β_1	-0.7	-0.72	0.16	(-1.07, -0.43)	0.02	63.94

Table 4.5: Estimation results for separate auctions (Sample size: $T = 1000$)

Table 4.5 shows the estimated posterior distributions of the parameters for separate auctions. Our estimator contains the true values in 95% credible intervals. All p-values of the convergence diagnostics (CD) are more than 0.01. Furthermore, the inefficiency factor values are sufficiently low. The values of inefficiency factors are, at most 68, which implies that we would obtain the same variance of the posterior sample means from 294 uncorrelated draws, even in the worst case. Figure 4.9 represents the sample paths from estimated posterior distributions for the separate auctions. Figure 4.10 shows the estimated posterior densities. As a result, our MCMC simulation performs well.

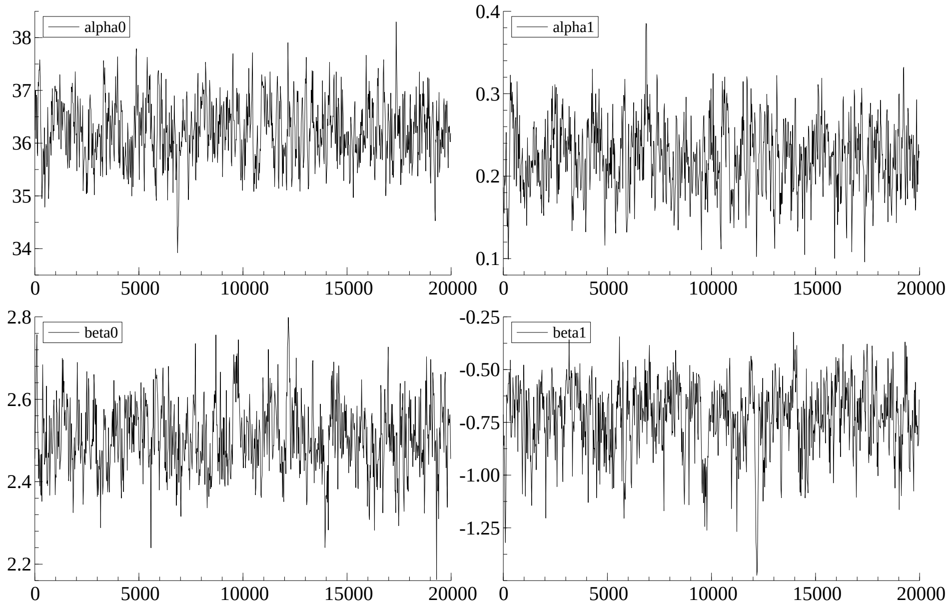

Figure 4.9: Sample paths for separate auctions (Sample size: $T = 1000$)

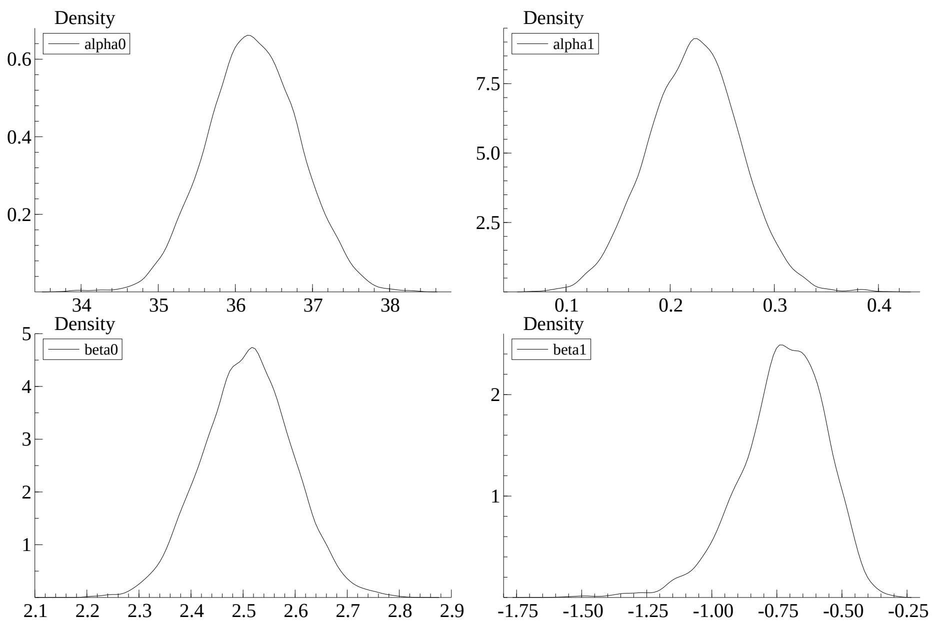

Figure 4.10: Posterior densities for separate auctions (Sample size: $T = 1000$)

	True value	Mean	Stdev.	95% interval	CD	IF
α_0	73.0	72.92	0.20	(72.52, 73.31)	0.08	71.70
α_1	0.40	0.41	0.01	(0.38, 0.44)	0.15	72.41
β_0	3.19	3.07	0.25	(2.42, 3.44)	0.59	162.33
β_1	-0.01	0.30	0.27	(-0.30, 0.81)	0.61	166.61

Table 4.6: Estimation results for bundling auctions (Sample size: $T = 1000$)

Table 4.6 shows the estimated posterior distributions of the parameters for the bundling auctions. Our estimator contains the true values in 95% credible interval. All p-values of the convergence diagnostics (CD) are more than 0.08. Furthermore, the inefficiency factor values are sufficiently low. The values of inefficiency factors are, at most 167, which implies that we would obtain the same variance of the posterior sample means from 119 uncorrelated draws, even in the worst case. Figure 4.11 represents the sample paths from estimated posterior distributions for the separate auctions. Figure 4.12 shows the estimated posterior densities. As a result, our MCMC simulation performs well.

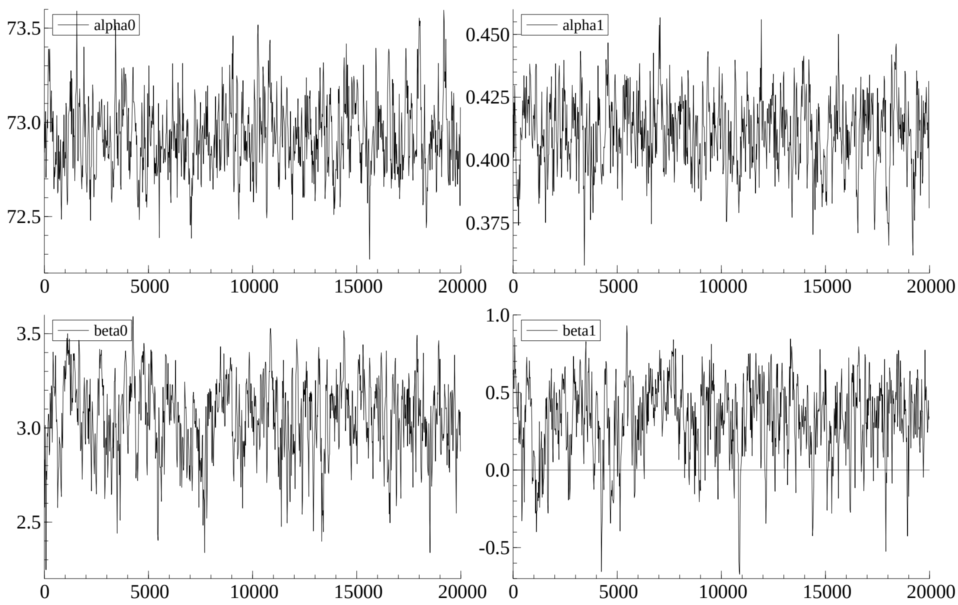

Figure 4.11: Sample paths for bundling auctions (Sample size: $T = 1000$)

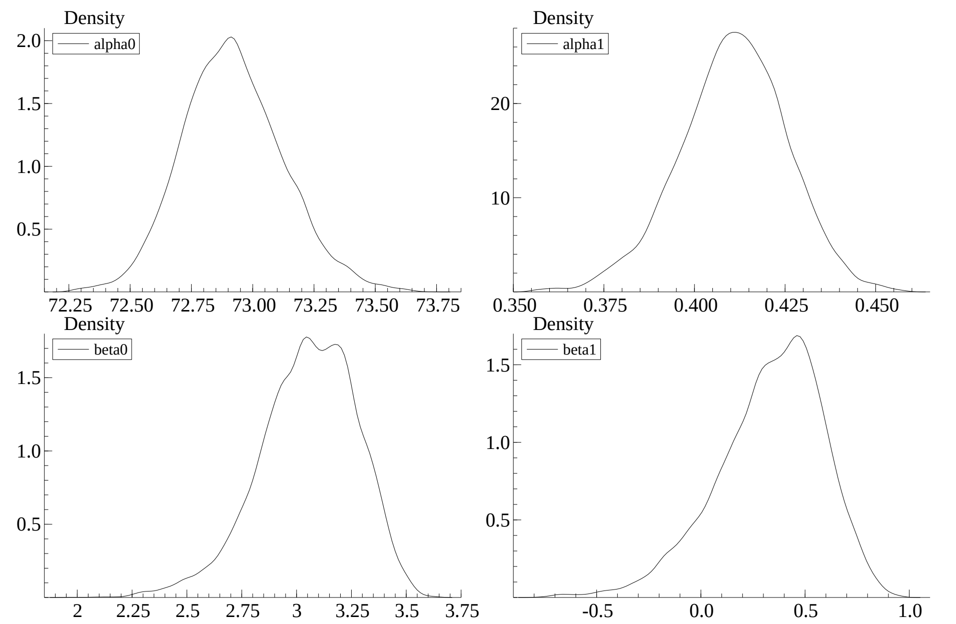

Figure 4.12: Posterior densities for bundling auctions (Sample size: $T = 1000$)

4.5 Empirical Examples

4.5.1 Data Description

Our empirical example consists of auctions of 2005 U.S. mint coin set held on eBay in 2014. Data were collected from 208 eBay auctions completed in October, 2014. There are two types of goods in our data set. One is the 11-coin mint set and the other is the 22-coin mint set. The 22-coin mint set includes two packages of the 11-coin mint set. The sample sizes are 107 and 101, respectively.

As Bajari and Hortaçsu (2003) and Wegmann and Villani (2011) studied coin auctions in their empirical illustrations, coin auctions are excellent examples in the empirical study of the common values auction model. While Bajari and Hortaçsu (2003) and Wegmann and Villani (2011) both collected various kinds of coins in their empirical illustrations, we only collected 2005 U.S. mint coin sets (11-coin sets and 22-coin sets). Therefore, we estimated the distribution of signals with fewer covariates.

	Mean	Std	Median	Max	Min
Winning bid	6.55	2.64	5.99	15.5	2.25
Positive reputation	6515.74	17105.87	388.00	73913	5
Negative reputation	9.52	28.49	0.00	194	0.00
Number of actual bidders	2.87	1.61	3.00	6.00	1.00
Days	5.26	2.28	7.00	10.00	0.00

Table 4.7: Summary statistics (2005 U.S. mint coin sets, (11-coin set) # of obs. = 107)

	Mean	Std	Median	Max	Min
Winning bid	8.98	3.35	8.25	17.0	3.3
Positive reputation	22553.29	33006.16	1303.00	73892.00	0.00
Negative reputation	13.39	17.17	3.00	57.00	0.00
Number of actual bidders	3.47	2.04	3.00	7.00	1.00
Days	5.98	2.06	7.00	10.00	1.00

Table 4.8: Summary statistics (2005 U.S. mint coin sets, (22-coin set) # of obs. = 101)

Tables 4.7 and 4.8 provide the summary of statistics for the 11-coin set and 22-coin set, respectively. The first column describes the variables. "Winning bid" is the second highest bid in the eBay auction. As seen in Tables 4.7 and 4.8, on average, one could purchase a mint coin set for \$7.3 or \$9.2 for the 11-coin set or 22-coin set, respectively. "Positive reputation" denotes the number of positive ratings a seller has received. Similarly, "negative reputation" is the sum of the number of negative ratings and the number of neutral ratings a seller receives. Since the number of neutral ratings and the number of negative ratings are usually small relative to the number of positive ratings, we regard neutral ratings as negative ratings. "Number of actual bidders" is the number of participants who actually bid at auction t. "Days" denotes the duration of the auctions held.

4.5.2 Estimation Results

We estimate the structural parameters using the U.S. mint coin data described above.

The 11-Coin Set

For the 11-coin set, we assume that the signal, S_i, follows the normal distribution. That is,

$$S_{it} \sim i.i.d.\,\mathrm{N}(\mu_{1t}, \sigma_{1t}^2),$$

where $\mu_{1t} = \alpha_0 + \alpha_1 X_{t1} + \alpha_2 X_{t2}$, and $\sigma_{1t}^2 = \exp(\beta_0) + \exp(\beta_1)X_{t1} + \exp(\beta_2)X_{t2}$. The parameters $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_2)$ and $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2)$ are unknown to econometricians.[4] In this empirical illustration, the auction-specific covariates, $X_t = (X_{t1}, X_{t2})$ are the logarithm of "Positive reputation + 1" and "Negative reputation + 1"; that is,

$$X_{t1} = \log(\text{Positive reputation} + 1) \text{ and } X_{t2} = \log(\text{Negative reputation} + 1)$$

[4] Analogous to the simulation experiments (Section 4.4), we omit subscript $k = 1$ for the coefficient parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

for observed auction t.

The prior distribution of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are

$$\boldsymbol{\alpha} \sim \mathrm{N}(0, 10\boldsymbol{I})$$

and

$$\boldsymbol{\beta} \sim \mathrm{N}(0, 10\boldsymbol{I}),$$

where $\boldsymbol{I}$ is the identity matrix of order 3.

Similar to the simulation experiments described in Section 4.4, we used the random walk-based Metropolis-Hastings algorithm to generate random draws from the posterior distributions. The number of iteration is 20000, and the burn-in period is 1000.

Table 4.9 reports the probabilities parameters take positive (PP), the p-values of convergence diagnostics for the MCMC (CD) and Inefficiency Factors (IF). All p-values of the convergence diagnostics are more than 0.06. Furthermore, the inefficiency factor values are sufficiently low. In particular, the inefficiency factors are 39.88 to 95.65, which implies that we would obtain the same variance of the posterior sample means from 209 uncorrelated draws, even in the worst case. Figure 4.13 shows the sample paths of estimated parameters. From Figure 4.13 it can be seen that the sample paths of these parameters converge to posterior distributions. Thus, we conclude that the sample paths of estimated parameters converge to posterior distributions.

Parameter	Covariate (Coefficient Parameter)	PP	CD	IF
μ_1	Const. (α_0)	1.00	0.77	87.41
	log(Pos.Rep. + 1) (α_1)	1.00	0.85	95.65
	log(Neg.Rep. + 1) (α_2)	0.46	0.96	66.27
σ_1^2	Const. (β_0)	0.51	0.06	54.85
	log(Pos.Rep. + 1) (β_1)	1.00	0.32	47.15
	log(Neg.Rep. + 1) (β_2)	0.44	0.77	39.88

Table 4.9: The convergence diagnostics for the MCMC (CD) and the inefficiency factors (IF) for the 11-coin set

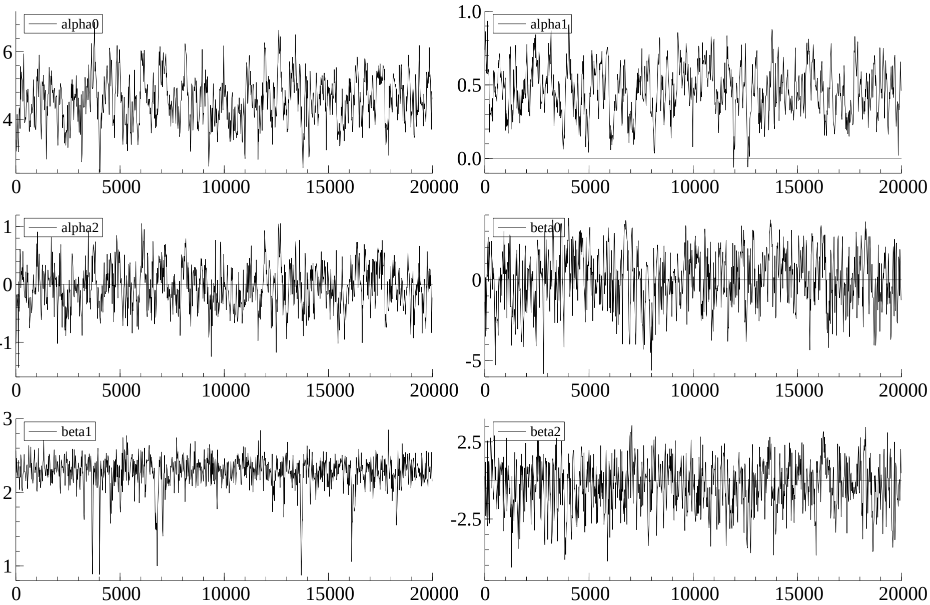

Figure 4.13: Sample paths of parameters (11-coin set)

Figure 4.14 shows the posterior densities of parameters for the 11-coin set. Table 4.10 and Figure 4.14 provide some posterior inferences. In Table 4.10, "Mean," "Stdev," and "95% interval" represent the posterior mean, the posterior standard deviation, the 95% credible interval, respectively.

Parameter	Covariate (Coefficient Parameter)	Mean	Stdev.	95% credible interval
μ_1	Const. (α_0)	4.59	0.71	(3.23, 6.00)
	log(Pos.Rep. + 1) (α_1)	0.46	0.17	(0.12, 0.79)
	log(Neg.Rep. + 1) (α_2)	-0.03	0.39	(-0.78, 0.73)
σ_1^2	Const. (β_0)	0.01	1.76	(-3.56, 3.12)
	log(Pos.Rep. + 1) (β_1)	2.27	0.22	(1.71, 2.60)
	log(Neg.Rep. + 1) (β_2)	-0.35	1.60	(-3.70, 2.44)

Table 4.10: Posterior inferences for the 11-coin set

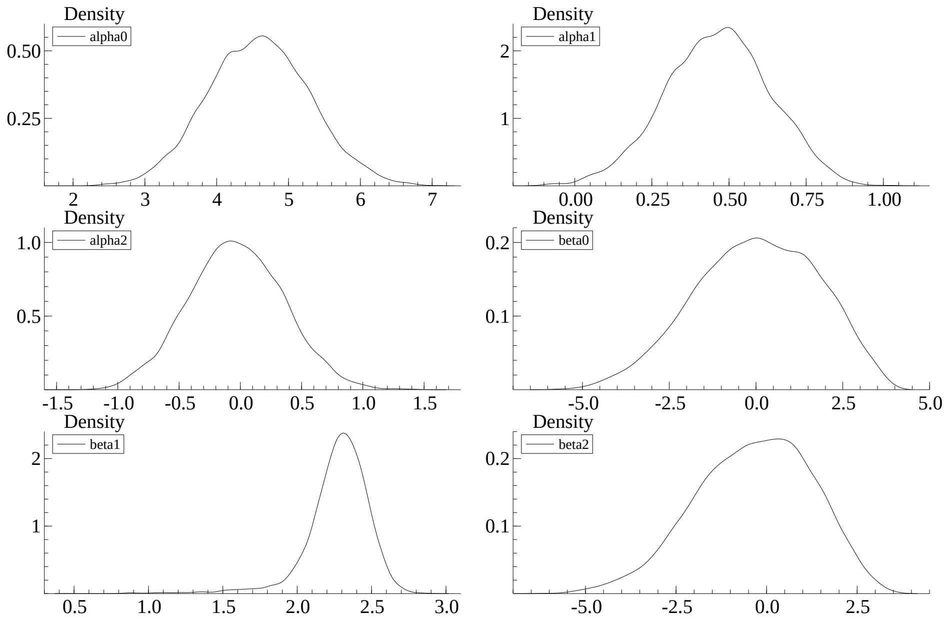

Figure 4.14: Posterior densities (11-coin set)

As seen in Table 4.10, the posterior mean of α_0 is 4.59. Since α_0 is the constant term corresponding to the mean parameter μ_1, when a seller has no reputation (i.e., a new entrant), the mean of the bidders' signal is \$4.59. As seen in Table 4.10, the posterior mean of α_1 is 0.46, which is the coefficient parameter of the covariate $\log(\text{Positive reputation} + 1)$ corresponding to the mean parameter μ_1. Therefore, if a seller earns a more positive reputation, the mean of the bidders' signals will increase. This result seems intuitively plausible. The posterior mean of α_2 is -0.03, and α_2 takes a positive value with probability 0.46. Since α_2 is the coefficient parameter of the covariate $\log(\text{Negative reputation} + 1)$ corresponding to the mean parameter μ_1, the number of negative ratings does not have much effect on the mean of the bidder's signal. This result is not intuitively plausible. One possible reason for the tiny effect of negative reputations on the mean of bidders' signals is the positive correlation between positive reputations and negative reputations. The correlation coefficient between positive reputations and negative reputations is 0.86, which represents a high positive correlation.

The scatter plot is given in Figure 4.15.

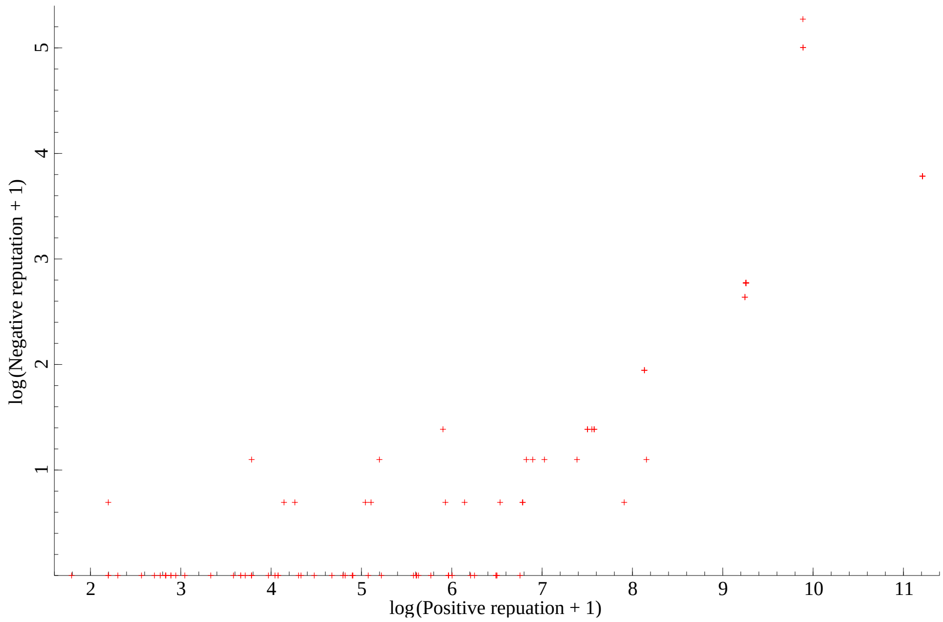

Figure 4.15: Logarithms of “Positive reputation + 1” and “Negative reputation + 1” (11-coin set)

From Table 4.7 , the number of negative ratings is small relative to the number of positive ratings. There are very few auctions in which sellers receive negative ratings. In most cases, sellers receive positive ratings. From Figure 4.15, many sellers with $\log(\text{Positive reputation} + 1) < 7$ (i.e., sellers with positive reputations, less than 1100 total) had no negative ratings. All sellers with $\log(\text{Positive reputation} + 1) \geq 7$ had some negative ratings. Therefore, sellers with more trades receive more (both positive and negative) ratings. From these facts, we conclude that the number of negative ratings does not represent the insincerity of seller but, rather, the abundance of the seller's experience, in our empirical example.

The 22-Coin Set

Similar to the case of 11-coin set, we assume that the signal S_i follows the normal distribution. That is,

$$S_{it} \sim i.i.d.\,\mathrm{N}(\mu_t, \sigma_t^2),$$

where $\mu_t = \alpha_0 + \alpha_1 X_{t1} + \alpha_2 X_{t2}$, and $\sigma_t^2 = \exp(\beta_0) + \exp(\beta_1)X_{t1} + \exp(\beta_2)X_{t2}$. The parameters $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_2)$ and $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2)$ are unknown to econometricians. In this empirical illustration, the auction-specific covariates, $X_t = (X_{t1}, X_{t2})$ are the logarithm of "Positive reputation + 1" and "Negative reputation + 1".

The prior distribution of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are

$$\boldsymbol{\alpha} \sim \mathrm{N}(0, 100\boldsymbol{I})$$

and

$$\boldsymbol{\beta} \sim \mathrm{N}(0, 100\boldsymbol{I}),$$

where $\boldsymbol{I}$ is the identity matrix of order 3.

Similar to the case of 11-coin set, we use the random walk-based MH algorithm to generate random draws from the posterior distributions. We draw 30000 random samples from the posterior distribution via MH algorithm for each parameter. The burn-in period is 3000.

Table 4.11 provides the summary of statistics of posterior distributions and the p-values of convergence diagnostics for the MCMC (CD) and Inefficiency Factors (IF). All p-values of the convergence diagnostics are more than 0.06. Furthermore, the inefficiency factors are less than 188. Therefore, we would obtain the same variance of the posterior sample means from 159 uncorrelated draws, even in the worst case. Figure 4.16 shows the sample paths of estimated parameters. We conclude that the sample paths of estimated parameters converge to posterior distributions.

Parameter	Covariate (Coefficient Parameter)	PP	CD	IF
μ_1	Const. (α_0)	1.00	0.19	166.34
	log(Pos.Rep. + 1) (α_1)	0.99	0.10	187.68
	log(Neg.Rep. + 1) (α_2)	0.30	0.06	160.56
σ_1^2	Const. (β_0)	1.00	0.85	53.65
	log(Pos.Rep. + 1) (β_1)	0.18	0.81	21.81
	log(Neg.Rep. + 1) (β_2)	0.24	0.41	16.99

Table 4.11: The convergence diagnostics for the MCMC (CD) and the inefficiency factors (IF) for the 22-coin set

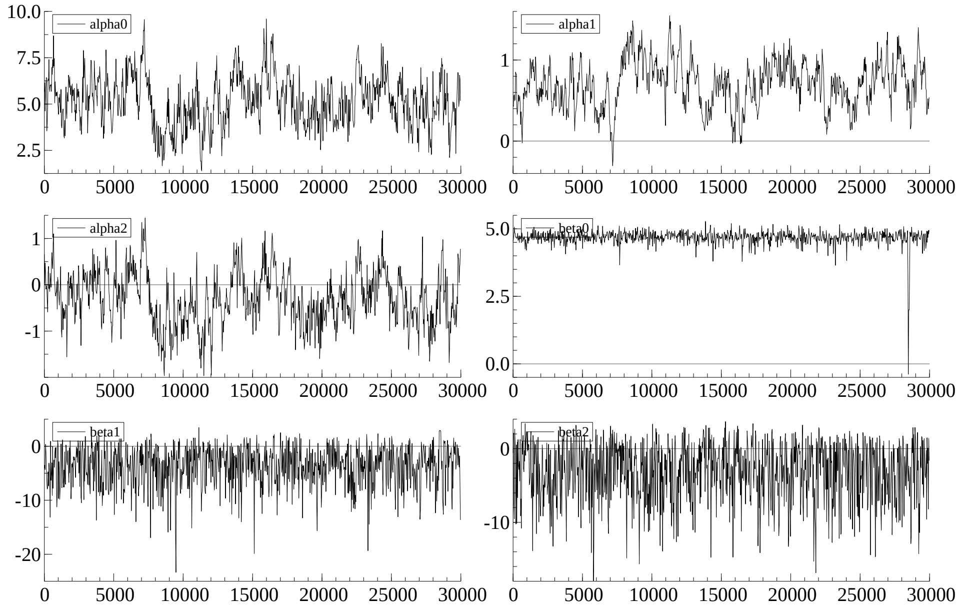

Figure 4.16: Sample paths of parameters (22-coin set)

Parameter	Covariate (Coefficient Parameter)	Mean	Stdev.	95% credible interval
μ	Const. (α_0)	5.15	1.36	(2.53, 7.90)
	log(Pos.Rep. + 1) (α_1)	0.70	0.29	(0.12, 1.25)
	log(Neg.Rep. + 1) (α_2)	-0.28	0.58	(-1.42, 0.89)
σ^2	Const. (β_0)	4.68	0.24	(4.27, 5.02)
	log(Pos.Rep. + 1) (β_1)	-3.56	3.79	(-12.37, 1.71)
	log(Neg.Rep. + 1) (β_2)	-3.18	3.94	(-12.21, 2.48)

Table 4.12: Posterior inferences for the 22-coin set

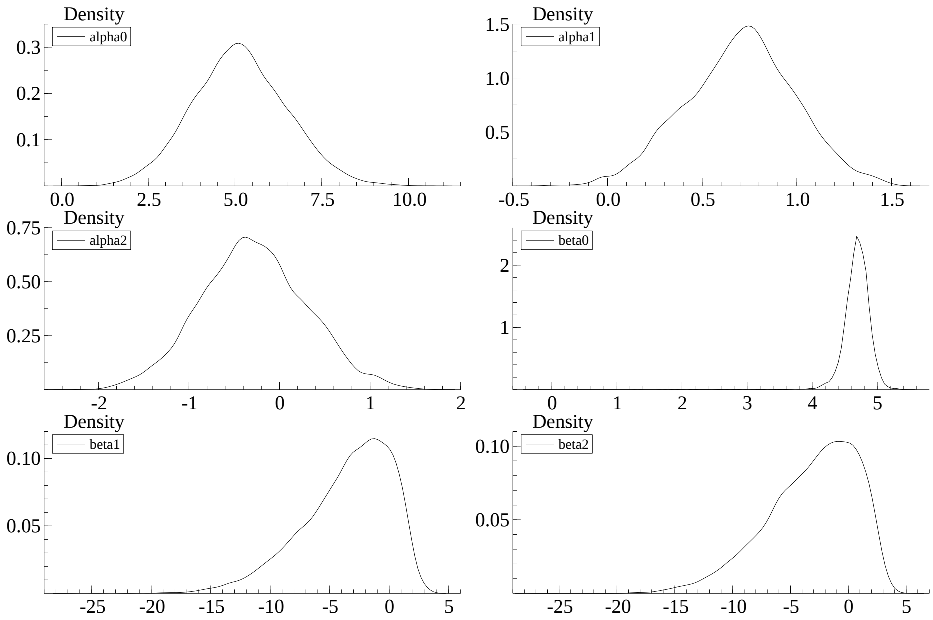

Figure 4.17: Posterior densities (22-coin set)

Figure 4.17 shows the posterior densities of parameters for the 22-coin set. Table 4.12 and Figure 4.17 provide some posterior inferences. As seen in Table 4.12, the posterior mean of α_0 is 5.15. Since α_0 is the constant term corresponding to the mean parameter μ. Therefore, when a seller has no reputation (i.e., a new entrant), the mean of the bidders' signal will be \$5.15. The posterior mean of α_1 is 0.70. Since α_1 is the coefficient parameter of the covariate $\log(\text{Positive reputation} + 1)$ corresponding to the mean parameter μ, we find that positive reputation has positive effect on the mean of bidders' signals. The posterior mean of α_2 is -0.28 and α_2 takes a positive value with probability 0.30. Recall that α_2 is the coefficient parameter of the covariate $\log(\text{Negative reputation} + 1)$ corresponding to the mean parameter μ. According to our results, the number of negative ratings does not have much effect on the mean of bidders' signals. This result is not plausible to our intuition. A possible reason is the same as in the case of 11-coin set. That is, a high positive correlation between positive reputations and negative reputations. The correlation coefficient between positive reputations and negative reputations is 0.92, which represents a high positive correlation between positive reputations and negative reputations. The scatter plot is shown in Figure 4.18.

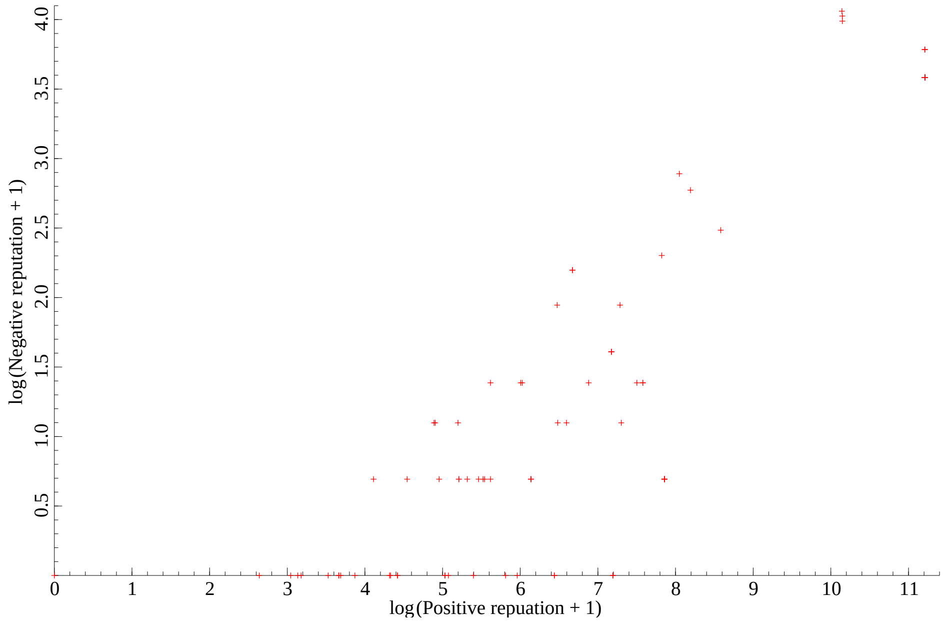

Figure 4.18: Logarithms of “Positive reputation + 1” and “Negative reputation + 1” (22-coin set)

Table 4.8 shows the number of negative ratings is small relative to the number of positive ratings. There are very few auctions in which sellers receive negative ratings. In most cases, sellers receive positive ratings. From Figure 4.18, many sellers with $\log(\text{Positive reputation} + 1) < 7.2$ (i.e., sellers with positive reputations, less than 1330 total) had no negative ratings. All sellers with $\log(\text{Positive reputation} + 1) \geq 7$ had some negative ratings. Analogous to the case of 11-coin set, we conclude that the number of negative ratings does not represent the insincerity of seller but, rather, the abundance of the seller’s experience. As a result, negative ratings do not have much impact on the mean of bidders’ signals.

4.6 Counterfactual Simulations

In this section, we compute the winner's curse reduction effect in the sense of Chakraborty (2002) and compare the revenue of separate auctions and bundling auctions using the estimated parameters from Section 4.5.[5]

In our empirical model, the distribution of bidders' signals depends on auction-specific covariates. We compute the winner's curse reduction effect and the expected revenue for a "representative" auction using the sample means of covariates, log(Positive reputation+ 1) and log(Negative reputation + 1), in Tables 4.7 and 4.8 and the posterior mean of the estimated parameters in Tables 4.10 and 4.12. The sample means of log(Positive reputation+ 1) and log(Negative reputation + 1) are

$$\overline{\log(\text{Positive reputation} + 1)} = 6.77 \text{ and } \overline{\log(\text{Negative reputation} + 1)} = 1.34,$$

respectively. The number of participants for a representative auction is $N = 7$. Subsequently, the bidding functions can be computed using equations (4.1) and (4.3).

In our empirical example, since the separate items, $k = 1$ and $k = 2$, are the same item, we cannot estimate the parameters for item $k = 2$ directly. In other words, we cannot obtain the estimates for coefficient parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ for item 2 from observed bids. However, for an arbitrary fixed covariates (and hence for the representative auction), the distribution of bidders' signals for item 2 can be identified. Since bidder i's private signal for item $k = 1$, S_{1i}, and bidder i's private signal for item $k = 2$, S_{2i}, are independent, the distribution of bidders' signals for item 2 can be recovered from the identified distributions of bidders' signals for item 1, S_{1i}, and bidders' signals for

[5]Chakraborty (2002) also discussed the expected revenues of both bundling auctions and separate auctions. Under the regularity conditions that are satisfied in our parametric specifications (i.e., normally distributed signals), He found that revenue ranking between the revenue of bundling auctions and separate auctions depends on the number of potential bidders, N. He found that bundling auctions generate more expected revenue than do separate auctions for all $N < N^*$, where N^* is a sufficiently small number.

bundled item, S_i. Note that while we assume the independence, we do not assume that S_{1i} and S_{2i} have identical distributions.[6]

Since our parametric specification imposes that S_{1i}, S_{2i}, and S_i are normal random variables, from the reproductive property of normal distributions, we have

$$S_{2i} \sim \mathrm{N}(\mu - \mu_1, \sigma^2 - \sigma_1^2),$$

where (μ, σ^2) and (μ_1, σ_1^2) are the parameters for distributions of S_i and S_{1i}, respectively. Let (μ_2, σ_2^2) be the parameter vector for distributions of S_{2i}. By the estimated parameters and the sample mean of covariates, we gain $\mu_2 = 1.85$ and $\sigma_2^2 = 40.53$. Note that, since the mean of the signals for item 1 is $\mu_1 = 7.66$, $\mathrm{E}(S_{1i}) > \mathrm{E}(S_{2i})$ holds. This inequality seems intuitively plausible because the willingness to pay for the second item is usually less than that for the first item.

The statement of Theorem 1 is the winner's curse reduction effect, as proposed by Chakraborty (2002). Namely, $b_i(s) \geq b_{1i}(s_1) + b_{2i}(s_2)$. For each fixed signal $s = 5, 10, 15$, varying the value of the signal for item 1, s_1, from 3.0 to s, we compute $b_i(s) - (b_{1i}(s_1) + b_{2i}(s_2))$.

[6]Our simulation experiments in Section 4.4 dealt with the special case of independent signals. In our simulation experiments, we assume the identical distribution of S_{1i} and S_{2i}.

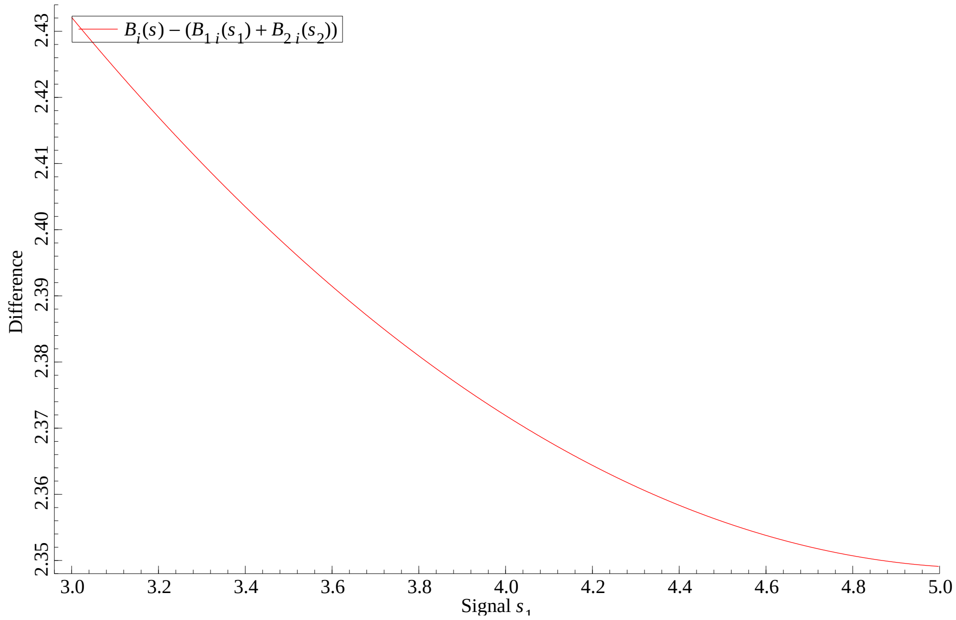

Figure 4.19: Difference of the bidding function with $s = 5$ (Independent signals case)

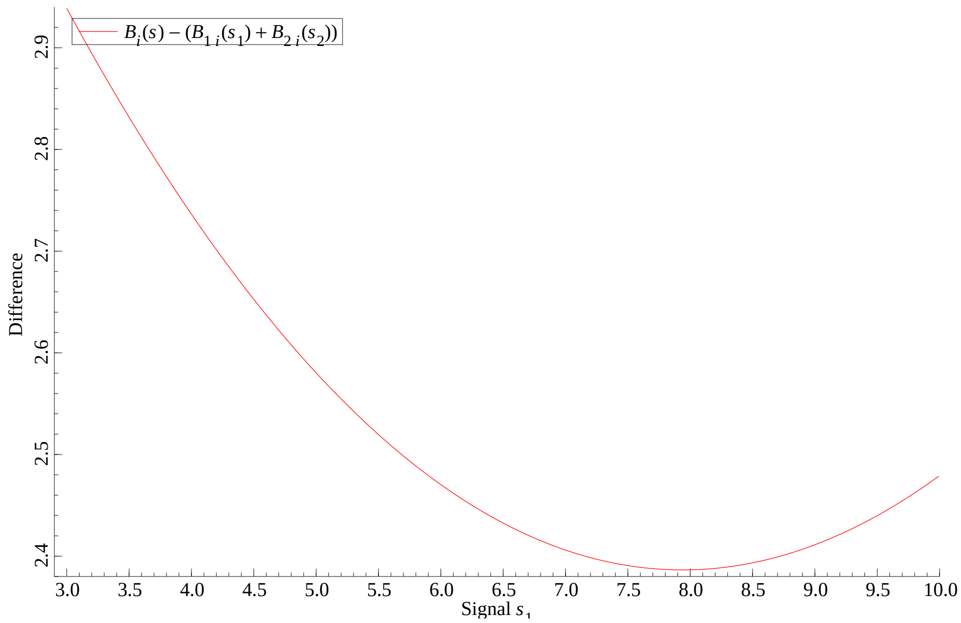

Figure 4.20: Difference of the bidding function with $s = 10$ (Independent signals case)

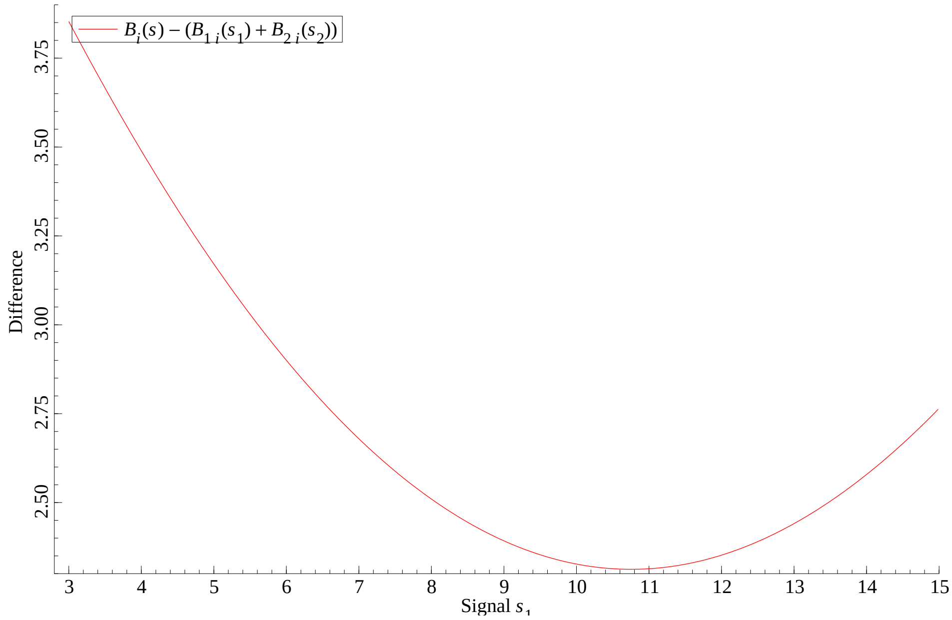

Figure 4.21: Difference of the bidding function with $s = 15$ (Independent signals case)

Figures 4.19, 4.20, and 4.21 report the difference of the bidding function $b_i(s) - (b_{1i}(s_1) + b_{2i}(s_2))$ for fixed signals $s = 5, 10, 15$, respectively. The shape of the graph with $s = 5$ is not similar to that of the graphs with $s = 10, 15$. When $s = 5$, the difference decreases as the signal for item 1, s_1, increases. On the other hand, when $s = 10$ and 15, the difference decreases for $s_1 \in (3.0, 8.0)$ and $s_1 \in (3.0, 11.0)$ and it increases for $s_1 > 8.0$ and $s_1 > 11.0$, respectively. The values of $b_i(s) - (b_{1i}(s_1) + b_{2i}(s_2))$ for $s = 5, 10, 15$ are similar, around \$2.50.

One may mistakenly conclude that Theorem 1 implies the revenue of bundling auctions is higher than that of separate auctions. Actually, Theorem 1 does not imply revenue ranking. In Theorem 1, for any signal of bundling auctions, $S_i = s$, the equation $s = s_{1i} + s_{2i}$ must hold. When we compare the revenues, the equation $s = s_{1i} + s_{2i}$ need not hold. The realizations of S_{1i} and S_{2i} are determined independently.

The expected revenues are computed by the Monte Carlo simulation method. Using the estimated parameters and the sample means of covariates, we draw the signals of bundling and separate auctions from the estimated distributions. We assume that the number of potential bidders is $N = 7$. Then, the equilibrium bids for signals are computed via equation (4.1), (4.2), and (4.3). The winning bids are the second-highest bids for both bundling and separate auctions. The revenue difference is computed by the difference between the bundling auction's winning bid and that of the separate auction. We iterate this procedure 5000 times.

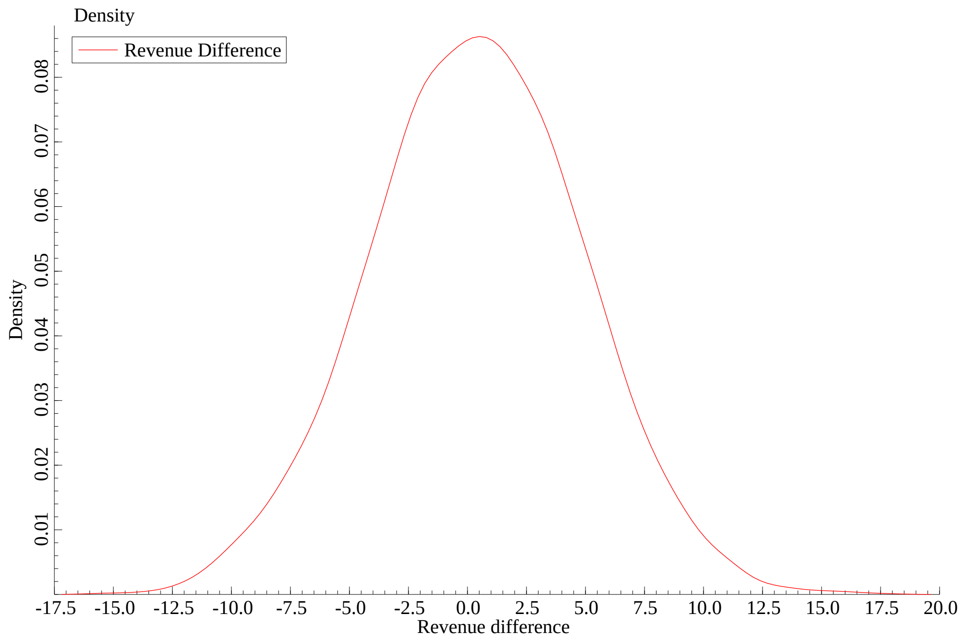

Figure 4.22: Density of revenue difference between bundling auctions and separate auctions (Independent signals case)

	Mean	Stdev.	.25 quantile	Median	.75 quantile	PP
Revenue (bundle)	8.67	3.12	6.61	8.85	10.88	-
Revenue (item 1)	6.99	2.48	5.39	7.10	8.73	-
Revenue (item 2)	1.32	1.90	0.09	1.41	2.67	-
Revenue difference	0.37	4.47	-2.60	0.38	3.40	0.53

Table 4.13: Summary statistics of revenue and revenue differences (Independent signals case)

The density of revenue differences between bundling and separate auctions is shown in Figure 4.22. The shape of the density is symmetrical at point 0. Table 4.13 reports the summary statistics of revenues and revenue differences. In Table 4.13, "Mean" and "Stdev." are the mean and the standard deviation of revenue differences, respectively. Similarly, ".25 quantile," "Median," and ".75 quantile" represent the first quartile, the second quartile, and the third quartile. The probability that the revenue of bundling auctions is higher than that of separate auctions is denoted by "PP."

According to Figure 4.22 and Table 4.13, the revenue of bundling auctions is higher than that of separate auctions with probability 0.53. The expected revenue difference is \$0.37. Therefore, sellers can gain an additional profit of \$0.37 by using a bundle auction rather than two separate auctions. Since the average transaction price of bundled items (22-coin sets) is \$8.98, we find that the value of additional gains are not negligible. In the theoretical literature, Chakraborty (2002) discussed the revenue ranking between the revenue of bundling auctions and separate auctions. He found that bundling auctions generate more expected revenue than do separate auctions when the number of bidders is sufficiently small. According to Tables 4.7 and 4.8, the number of participants at most 7. Therefore, our empirical example does not contradicts the result of Chakraborty (2002).

4.7 Conclusions

In this paper, we focused on bundling auctions in online auction markets. In online auction markets (e.g., eBay and Yahoo!), sellers often sell two or more items in bundling auctions. Conversely, other sellers sell the same items separately. We propose an estimation procedure for bundling auction models within the pure common value paradigm.

Our empirical example is eBay mint coin set auctions in 2014. In our data set, there are two kinds of coin sets: 11-coin sets and 22-coin sets. We regard the 11-coin sets as the separate item and the 22-coin set as the bundled item. We also conducted some counterfactual simulations using the estimated parameters. We computed the winner's curse reduction effect following Chakraborty (2002) precedent and compared the revenue of bundling auctions and separate auctions. We found that the value of the winner's curse reduction effect is about \$2.5. For revenue comparison, we found that the expected revenue in the bundling auctions is higher than that in the separate auctions by \$0.37. Since the average transaction price of bundled items (22-coin sets) is \$8.98, the value of additional gains are not negligible.

There are some avenues for future research in this paper. For one, we ignored the endogenous entry of bidders. In general, bidders will decide endogenously to participate, whether in bundling auctions or separate auctions. Analogously, we also ignored the seller's incentive to decide which item (the bundled item or separate items) to sell. The seller's decision as to which item to sell will depend on the revenue ranking between bundling auctions and separate auctions.

Chapter 5

Structural Estimation of the Scoring Auction Model

5.1 Introduction

Public sectors purchase a variety of goods and services from the private sector, from snow removal services to weapons systems. OECD (2007) reported that the amount of expenditure incurred for public procurement accounts for 10 to 15 % of GDP in OECD countries. For public funds to be spent efficiently and effectively, value for money (relevant prices and qualities of proposals in the whole procurement cycle are assessed) is the key principle in public procurement. Although low-bid auctions are a common awarding mechanism, more and more procurement buyers introduce competitive bidding processes in which the highest value-for-money offer is selected. The scoring auction, or equivalent multi-parameter bidding, is one of the most prevalent mechanisms that meets the objective.

In the scoring auction, bidders are asked to submit a set of multi-dimensional bids that include price and non-price attributes (quality), such as service life, delivery date, and the extent of environmental burden of the production processes. An *ex ante* publicly announced scoring rule maps the multiple-dimensional bid into a variable, a so-called *score*, and the awarder is the bidder whose score is the highest or lowest. Scoring auctions allow a procurement buyer to obtain more valuable (or greater *value-for-money*) contracts without reducing the bidders' profits than do price-only auctions (Milgrom (2004)).

A variety of forms of scoring rules are used in real-world public procurement. In US states' departments of transportation, for instance, Delaware, Idaho, Oregon, Massachusetts, Utah, and Virginia, use quasilinear (QL) rules in the first-score (FS) auction,[1] whereas Alaska, Colorado, Florida, Michigan, North Carolina, and South Dakota use price-over-quality ratio (PQR) rules, in which the score is equal to the price bid divided by a quality measure that aggregates all nonmonetary bids. The PQR scoring rule is also used in most public procurement scoring auctions in Japan and some in Australia.[2] In addition, some governments in EU countries use the scoring auction in which the score is the sum of the price and quality measurements but the score is nonlinear in the price bid. Note that any monotonic function cannot transform these nonquasilinear scoring rules into a QL form, because a necessary condition for quasi-linearity requires price to be linear in score.

A growing number of empirical works on FS auctions have been developed (*e.g.,* Bajari et al. (2007) and Lewis and Bajari (2009)). Nevertheless, they are confined to either nonstructural approaches or FS auctions with awarding rules and no reservation price. Theoretical literature, such as Asker and Cantillon (2008), has shown that, unless the scoring auction is quasilinear and the reserve price is nonbinding, the bidder's optimal choice in non-price attributes hinges on the bidder's score. This implies that the bidder's *pseudo-type* in Asker and Cantillon (2008) may not be monotone in the bidder's signal. As a result, the structural estimation method of the first-price auction model cannot

[1] In a FS auction, the successful bidder receives a payment equal to its price bid and provides the quality level specified in its quality bid. See Section 5.2 for more details.

[2] The Ministry of Land, Infrastructure and Transportation in Japan allocates most public construction project contracts through scoring auctions based on PQR awarding rules.

directly apply to scoring auction data with a nonquasilinear scoring rule.

In this article, we propose a structural estimation procedure of the FS auction model where the scoring rule accepts a nonquasilinear form. The model is established by Hanazono et al. (2013), which is an extension of Che (1993), allowing a broader class of scoring rules,[3] including nonquasilinear forms and binding reservation prices. In addition, imposing a condition on the bidder's cost function, the model guarantees the existence of the monotone pure equilibrium in scoring auctions in which bidders have multi-dimensional signals. Thus, the model fits the typical FS auction data in which price and quality are scattered in the price quality space. Based on the model, we establish a procedure for identifying the bidder's multi-dimensional signals from FS auction data. Our framework allows for a wide variety of scoring auction data to be used in empirical studies.

Several assumptions are made in the FS auction model. First, multi-dimensional signals are separable and monotone in conjunction with both the bidder's cost function and score function (Assumption 2). More specifically, each bidder with an $L+1$-dimensional type is asked to submit a price bid as well as an L-dimensional quality bid. For any quality, the bidder's marginal cost of providing an additional unit of l-th dimensional quality hinges solely on the bidder's dimension l signal for all $l = 1, \ldots, L$. The remaining dimension of the signal, *i.e.,* $l = 0$, affects the bidder's total costs; the total cost is strictly increasing in the dimension-zero signal given quality. This specification simplifies the mechanism design problem with multi-dimensional signals in a way that the bidder's information rent hinges solely on the distribution of dimension zero signal. The bidder's strategic interaction in the selection of the score is thus reduced to a single dimensional problem with single dimensional private information. In addition, Assumption 2 constitutes a sufficient condition for the identification of the FS auction model

[3] A scoring rule is *interdependent* if the bidder's score is determined not only by his/her p and q but also other bidders' p and q such that $S^i(p^1, \ldots, p^n, q^1, \ldots, q^n)$. In this article, we restrict attention to independent scoring rules. See Albano et al. (2009) for the classification of scoring rules.

with multi-dimensional signals. Note that Assumption 2 is satisfied, for instance, under a PQR scoring rule if a set of cost functions with an identical dimension-zero signal are homothetic with each other.

Second, the bidder's expected payoff function satisfies the log-supermodularity condition (Assumption 3). By Assumption 2, the bidder's choice in quality components (non-price attributes) is thoroughly endogenous in the score. Hence, the strictly positive cross partial derivative of the log of the bidder's expected payoff with respect to both score and dimension-zero signal implies the log-supermodularity condition, which guarantees the existence of the pure monotone strategy in the first-score auction.

To identify the bidder's multi-dimensional signals from observed multi-dimensional bid data, we choose a semi-parametric estimation methodology. The bidder's cost function is assumed to be known except for the $L + 1$-dimensional parameters (signals). In the FS auction, the bidder's signals are implicitly included in the bidder's first-order condition, in general. In other words, the first-order condition just constitutes an implicit function of the bidder's multi-dimensional signals. We, thus, exploit the monotonicity condition given in Assumption 1 to indicate that the implicit function is monotone in the dimension-zero signal. The identification of the remaining L dimensional signals is straightforward given the assumption of the bidder's cost function, *i.e.,* the marginal cost of l-th dimensional q is monotone in l-th dimensional signal. Finally, we identify the distribution of the bidder's multi-dimensional private signals. The structural estimation method of first-price auctions has been developed by Laffont et al. (1995), Guerre et al. (2000), and Li et al. (2002); and a growing number of empirical analyses of first-price auction data have been provided in the literature. Our methodology is an extension of Guerre et al. (2000) to the scoring auction model.

We conduct a Monte Carlo study to investigate the consistency and the finite sample property of our estimation method. Simulated bid data samples are created with the dimension zero signal following a uniform distribution, which indicates that the symmetric monotone equilibrium bidding strategy of the FS auction is explicitly obtained. Then, our structural estimation method is applied to the simulated scoring bid data to recover the distribution of the bidder's signal. The recovered cumulative distribution functions are presented in Section 5.3.

As an empirical application, we conduct a series of counterfactual analyses using the scoring auction data. The data is from public procurement auctions for construction projects in Japan, where the scoring rule is PQR. Throughout the article, we assume that the procurement buyer's true preference is represented by the observed PQR scoring rule. In addition, bidders' true cost functions are either quadratic, cubic, and quartic polynomials. All three functions satisfy the conditions that guarantees the existence of a unique monotone equilibrium under the PQR scoring rule. Then, the impact of the change in scoring rules or auction formats on both the procurement buyer's and suppliers' utilities is measured. Furthermore, the extent to which the utility of the buyer using scoring auctions would change by the use of price-only auctions is quantified.

The results of our empirical application are as follows. First, a change in the auction format has a very small impact on welfare; under the PQR scoring rule, the procurement buyer has an approximately .003 to .004 percent lower utility (higher exercised score) when using FS rather than SS auctions, whereas the winning bidder earns a payoff greater by approximately .15 to .26 percent in FS, as opposed to SS, auctions. Note that Hanazono et al. (2013) suggests that nonequivalence stems from the overproduction in quality in FS auctions. Accordingly, we observe that the expectation of the winner's quality provision is .001 to .002 percent larger in FS, as opposed to SS, auctions. Second, there is a QL FS auction that dominates the currently used PQR FS auction. With a well-designed QL scoring rule, the procurement buyer improves utility by approximately .29 percent while bidders earn lower payoffs by 3.4 to 4.2 percent. Finally, the outcome of a price-only auction is compared with that of the currently used PQR FS auctions. In simulated price-only auctions, bidders' payoffs vary, ranging from -41.2 to 1.34 percent, whereas the procurement buyer's utility is consistently 1 to 36 percent lower than with the PQR FS auction. These results suggest that a procurer can obtain an almost equivalent (slightly lower) gain with the use of a price-only auction with a well-designed fixed quality standard.

The remaining part of this article is organized as follows. Section 5.2 describes the theoretical consideration of scoring auctions with general scoring rules. Section 5.3 discusses the identification of the distribution of bidders' cost schedule parameters. Section 5.4 conducts empirical examinations using the structural estimation method. The final section is the conclusion.

5.2 A Theoretical Consideration

5.2.1 The Model

A procurement buyer auctions a project contract to n risk-neutral bidders.[4] The scoring function $S(p, \boldsymbol{q}) : \mathbb{R}_+^{L+1} \to \mathbb{R}$ is common knowledge, mapping the bidder's price-bid $p \in \mathbb{R}$ and a quality level $\boldsymbol{q} = (q^1, \ldots, q^L) \in [\underline{q}^1, \bar{q}^1] \times \cdots \times [\underline{q}^L, \bar{q}^L] \equiv \mathbf{Q}$ with $q^\ell > 0$ for all $\ell = 1, \ldots, L$ into a single dimensional value, the score, denoted by $s \in \mathbb{R}$. The scoring function is smooth and strictly monotone, *i.e.,* $S_p(p, \boldsymbol{q}) > 0$, $S_{q^\ell}(p, \boldsymbol{q}) < 0$, and $S_{q^\ell q^{\tilde{\ell}}}(p, \boldsymbol{q}) = 0$ for all $\ell = 1, \ldots, L$ and $\tilde{\ell} \neq \ell$. For instance, the PQR scoring rule with an unbinding reservation price is $S(p, \boldsymbol{q}) = p/V(\boldsymbol{q})$ where $V > 0$ for all $\boldsymbol{q}$ and $V_{q^\ell} > 0$ for all $\ell = 1, \ldots, L$. In addition, the QL scoring rule with an unbinding reserve is $S(p, \boldsymbol{q}) = p - V(\boldsymbol{q})$. The procurement buyer's utility function is represented by the scoring function, namely $U(p, \boldsymbol{q}) = -S(p, \boldsymbol{q})$.[5]

At the bid preparation stage, each bidder obtains an L-dimensional signal $\boldsymbol{\theta} \in [\underline{\theta}^0, \bar{\theta}^0] \times \cdots \times [\underline{\theta}^L, \bar{\theta}^L] \equiv \mathbf{\Theta}$ distributed following the publicly known cumulative joint distribution $F(\boldsymbol{\theta})$. We allow for θ^ℓ and $\theta^{\tilde{\ell}}$ with $\tilde{\ell} = 0, \ldots, L$ and $\tilde{\ell} \neq \ell$ to be correlated

[4] The argument in this section follows Hanazono et al. (2013).
[5] We relax this assumption in Section 5.4.

with each other; however, $\boldsymbol{\theta}$ is identically and independently distributed for every bidder. Finally, we denote by $F_\ell(\theta^\ell)$ the marginal distribution of θ^ℓ with $\ell = 0, \dots, L$.

The bidder's cost function $C(\boldsymbol{q}|\boldsymbol{\theta})$ is increasing and strictly convex in $\mathbf{Q}$ and is smooth in $\boldsymbol{Q}$ and $\boldsymbol{\Theta}$. Furthermore, we normalize the cost function such that i) $C(\boldsymbol{q}|\boldsymbol{\theta})$ is strictly increasing in θ^0, ii) C_{q^ℓ} is strictly decreasing in θ^ℓ for all $\ell = 1, \dots, L$, and iii)C_{q^ℓ} is constant in $\theta^{\tilde{\ell}}$ for any $\tilde{\ell} = 1, \dots, L$ and $\tilde{\ell} \neq \ell$. The interpretation of this specification is that the dimension-zero signal, θ^0, represents the bidder's overall productivity that affects total cost, whereas the rest of the signal dimensions, θ^ℓ with $\ell = 1, \dots, L$, are scale parameters in technology; the bidder with a larger θ^ℓ with $\ell = 1, \dots, L$ has a lower marginal cost to make an additional provision of ℓth-dimension quality.

Two auction formats are considered. In a FS auction, the successful bidder receives a payment, p. In a SS auction, the successful bidder can freely choose the contracted p and $\boldsymbol{q}$ as long as the score stemming from the contracted p and $\boldsymbol{q}$ equals the second-lowest score in the auction.

The scoring auction game can be equivalently considered as follows. Bidders are asked to submit a scoring bid, $s \in \mathbb{R}$. The lowest-score bidder wins the contract. Only the winner chooses a quality vector, $\boldsymbol{q}$, with which the winner performs the project work. The monotonicity of the scoring function implies the existence of the inverse function with respect to p. That is, for a score value s, the payment function, $P(s, \boldsymbol{q})$, is defined such that

$$S(P(s, \boldsymbol{q}), \boldsymbol{q}) \equiv s,$$

for any relevant score $s \in S(p, \mathbf{Q})$ with $p \in \mathbb{R}$.

Let s^e be the exercised score. In a FS auction, the exercised score is the winning bidder's score, *i.e.,* $s^e = s$. In a SS auction, it is equal to the second-lowest score. Then, the bidder's problem in a scoring auction is given by

$$\max_{s,\boldsymbol{q}} \left[P(s^e, \boldsymbol{q}) - C(\boldsymbol{q}|\boldsymbol{\theta})\right] \Pr\{\text{win}|s\}.$$

We assume that, for any s^e, there exists a unique internal solution of $\boldsymbol{q}$ that maximizes the bidder's payoff upon winning, *i.e.,* $P(s^e, \boldsymbol{q}) - C(\boldsymbol{q}|\boldsymbol{\theta})$. Let $q^\ell(s^e, \boldsymbol{\theta})$ denote the maximizer of the bidder's payoff upon winning for each $\ell = 1, \ldots, L$ dimension such that

$$q^\ell(s^e, \boldsymbol{\theta}) = \arg\max_{q^\ell} P(s^e, \boldsymbol{q}) - C(\boldsymbol{q}|\boldsymbol{\theta}). \tag{5.1}$$

A sufficient condition for the uniqueness of the optimal quality choice is that, for all $\ell = 1, \ldots, L$, $P_{q^\ell}(s^e, \boldsymbol{q}(s^e, \boldsymbol{\theta})) - C_{q^\ell}(\boldsymbol{q}(s^e, \boldsymbol{\theta})|\boldsymbol{\theta}) = 0$ with $P_{q^\ell q^\ell}(s, \boldsymbol{q}(s, \boldsymbol{\theta})) - C_{q^\ell q^\ell}(\boldsymbol{q}(s, \boldsymbol{\theta}), \boldsymbol{\theta}) < 0$. For notational convenience, we define $u(s^e, \boldsymbol{\theta}) = P(s^e, \boldsymbol{q}(s^e, \boldsymbol{\theta})) - C(\boldsymbol{q}(s^e, \boldsymbol{\theta})|\boldsymbol{\theta})$. Then, the bidder's maximization problem is reduced into the following one-dimensional optimization problem:

$$\max_{s} u(s^e, \boldsymbol{\theta}) \Pr\{\text{win}|s\}. \tag{5.2}$$

Recall that we have normalized that $P_s(\cdot) = 1/S_p(\cdot) > 0$ and $C_{\theta^0} > 0$. Therefore, the derivatives of $u(\cdot)$, with respect to s^e and θ^0, are given by

$$\begin{aligned} u_s(s^e, \boldsymbol{\theta}) &= P_s(s^e, q(s^e, \boldsymbol{\theta})) > 0, \\ u_{\theta^0}(s^e, \boldsymbol{\theta}) &= -C_{\theta^0}(q(s^e, \boldsymbol{\theta})|\boldsymbol{\theta}) < 0. \end{aligned}$$

It suggests that the scoring auction game is a single-dimensional auction game in which bidders with nonlinear utility functions submit scores.

5.2.2 Equilibrium in a FS Auction

A symmetric monotone equilibrium in a FS auction with the multi-dimensional type space is analyzed by adding the following two technical assumptions to the bidder's utility function. The first assumption (Assumption 2) simplifies the analysis of the scoring auction with the multi-dimensional type space, whereas the second assumption (Assumption 3) is required for the existence of an equilibrium in a FS auction.

Let $\underline{u}(s, \theta^0)$ be the payoff of the smallest-scale bidder whose efficiency level is θ^0, so that $\underline{u}(s, \theta^0) = u(s, \theta^0, \underline{\theta}^1, \ldots, \underline{\theta}^L)$. Then, Assumption 1 is summarized as follows.

Assumption 2 (Separability)**.** *There exists a monotonic function $h(\boldsymbol{\theta}) = h^1(\theta^0, \theta^1)h^2(\theta^0, \theta^2) \cdots h^L(\theta^0, \theta^L)$ with $h^\ell(\theta^0, \theta^\ell) \geq 1$ and $h(\theta^0, \underline{\theta}^1, \ldots, \underline{\theta}^L) = 1$ such that, for any θ^0, $dh^\ell(\theta^0, \theta^\ell)/d\theta^\ell > 0$ for all $\ell = 1, \ldots, L$ and for all s and θ^0,*

$$u(s, \boldsymbol{\theta}) = h(\boldsymbol{\theta})\underline{u}(s, \theta^0). \tag{5.3}$$

Assumption 2 ensures that the equilibrium bidding strategy is a sole function of θ^0, *i.e.,* $s_{\mathrm{I}}(\theta^0)$. Together with the specification of the cost function such that C_{q^ℓ} is decreasing in θ^ℓ for all $\ell = 1, \ldots, L$, Assumption 2 implies that bidders with an identical θ^0 but different θ^ℓ in any ℓ never choose the same quality set in equilibrium. The monotonicity of the marginal cost is needed for the identification of the bidder's type from observables s and $\boldsymbol{q}$. A detailed discussion is delivered in Section 5.3.2.

To see that Assumption 2 is sufficient for the bidding strategy $s_{\mathrm{I}}(\cdot)$ to be independent of θ^ℓ with $\ell = 1, \ldots, L$, suppose that two bidders have an identical θ^0 but different θ^ℓ for some or all $\ell = 1, \ldots, L$. Let $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{\theta}}$ be their $L + 1$ dimensional signals. The equilibrium bid strategy $s_{\mathrm{I}}(\cdot)$ maximizes the bidder's expected payoff. The bidders' objective functions are given by

$$\max_s h(\boldsymbol{\theta})\underline{u}(s, \theta^0) \Pr\{win|s\}, \; \max_s h(\tilde{\boldsymbol{\theta}})\underline{u}(s, \theta^0) \Pr\{win|s\}.$$

Since the two maximization problems are monotonic transforms of each other, the two objective functions are maximized at the same s. This implies that the equilibrium bid strategy $s_{\text{I}}(\cdot)$ depends solely on θ^0.

Assumption 2 is interpreted as a generalization of the homothetic cost function. If the scoring function is PQR such that $S(p, \boldsymbol{q})$, the cost function $C(\boldsymbol{q}|\boldsymbol{\theta})$ is a homothetic function of $C(\boldsymbol{q}|\theta^0, \underline{\theta}^1, \ldots, \underline{\theta}^L)$, where $h(\boldsymbol{\theta})$ is a multiplier. In other words, $C(\boldsymbol{q}|\boldsymbol{\theta})$ is homogeneous of degree zero such that

$$C(q^1(s,\boldsymbol{\theta})h^1(\theta^0,\theta^1), \ldots, q^L(s,\boldsymbol{\theta})h^L(\theta^0,\theta^L)|\boldsymbol{\theta}) = h(\boldsymbol{\theta})C(\boldsymbol{q}(s,\boldsymbol{\theta})|\theta^0, \underline{\theta}^1, \ldots, \underline{\theta}^L),$$

if the scoring rule is PQR.

Given Assumption 2, only one dimension of the bidder's multi-dimensional signal, θ^0, associates the strategic interaction in the score choice game. Therefore, the existence of a Bayesian Nash equilibrium in a FS auction only requires that the cross-partial derivative of the log of $u(s^e, \boldsymbol{\theta})$ with respect to s^e and θ^0 is strictly positive.

Assumption 3 (Log-Supermodularity)**.** *The smallest-scale bidder's utility, $\underline{u}(s, \theta^0)$, is log-supermodular, namely*

$$\frac{\partial^2}{\partial s \partial \theta^0} \log \underline{u}(s, \theta^0) > 0.$$

Note that Assumption 3 is required only in the analysis of a FS auction, since, as will be seen in the next subsection, a dominant strategy equilibrium exists in a SS auction. Also note that, given Assumption 2, the expected payoff of any bidder is log-supermodular, because Assumption 2 ensures that the cross partial derivative of the log of $u(s^e, \boldsymbol{\theta})$ is independent of θ^ℓ with $\ell = 1, \ldots, L$:

$$\frac{\partial^2 \log u(s^e, \boldsymbol{\theta})}{\partial s \partial \theta^0} = \frac{\partial}{\partial \theta^0}\left(\frac{u(s^e, \boldsymbol{\theta})}{u_s(s^e, \boldsymbol{\theta})}\right) = \frac{\partial}{\partial \theta^0}\left(\frac{h(\boldsymbol{\theta})\underline{u}(s^e, \theta^0)}{h(\boldsymbol{\theta})\underline{u}_s(s^e, \theta^0)}\right) = \frac{\partial^2 \log \underline{u}(s^e, \theta^0)}{\partial s \partial \theta^0}. \quad (5.4)$$

Given these assumptions, a symmetric, increasing equilibrium strategy in a FS auction is characterized as follows. Let $s_{\rm I}(\theta^0)$ be a symmetric, increasing equilibrium in a FS auction. The log-supermodularity of the bidder's utility function is sufficient to guarantee the existence of a strictly increasing Bayesian Nash equilibrium as shown by Athey (2001). Then, the bidder's problem (5.2) is given by

$$\max_s u(s,\boldsymbol{\theta})\left[1-F_0(s^{-1}(s))\right]^{n-1}$$

in equilibrium. By imposing the symmetric condition, the first-order condition is given by

$$u_s(s_{\rm I}(\theta^0),\boldsymbol{\theta})s'(\theta^0)\left[1-F_0(\theta^0)\right]^{n-1}=u(s_{\rm I}(\theta^0),\boldsymbol{\theta})(n-1)f_0(\theta^0)\left[1-F_0(\theta^0)\right]^{n-2}. \quad (5.5)$$

Let $\boldsymbol{\theta}^{-0}=(\theta^1,\ldots,\theta^L)$. Solving the differential equation for $u(s_{\rm I}(\theta^0),\boldsymbol{\theta})$ yields

$$\begin{aligned} P(s_{\rm I}(\theta^0),\boldsymbol{q}(s_{\rm I}(\theta^0),\boldsymbol{\theta})) &= C(\boldsymbol{q}(s_{\rm I}(\theta^0),\boldsymbol{\theta})|\boldsymbol{\theta}) \\ &+\int_{\theta^0}^{\bar{\theta}^0} C_{\theta^0}(\boldsymbol{q}(s_{\rm I}(\tau),\tau,\boldsymbol{\theta}^{-0})|\tau,\boldsymbol{\theta}^{-0})\left[\frac{1-F_0(\tau)}{1-F_0(\theta^0)}\right]^{n-1}d\tau, \end{aligned} \quad (5.6)$$

which characterizes the equilibrium strategy $s_{\rm I}(\theta^0)$ in a FS auction.

5.2.3 Equilibrium in a SS Auction

Let $s_{(2)}$ be the second-lowest score in a SS auction. Then, the bidder's payoff upon winning in a SS auction:

$$u(s_{(2)},\boldsymbol{\theta}),$$

is independent of his own scoring bid. Because the winning bidder has a non-negative payoff, bidding the break-even score (the minimum score the bidder with type $\boldsymbol{\theta}$ makes with a non-negative utility) is a dominant strategy in a SS auction. Therefore, a dominant strategy equilibrium $s_{\mathrm{II}}(\cdot)$ in a SS auction satisfies

$$u(s_{\mathrm{II}}(\theta^0), \boldsymbol{\theta}) = 0. \tag{5.7}$$

As in the case of a FS auction, the equilibrium strategy in a SS auction is independent of θ^ℓ with $\ell = 1, \dots, L$ since $u(s_{\mathrm{II}}(\theta^0), \boldsymbol{\theta}) = \underline{u}(s_{\mathrm{II}}(\theta^0), \theta^0) = 0$ for any $\boldsymbol{\theta}$.

In the scoring auction, the profit-maximizing quality is first-best if the exercised score is equal to the bidder's break-even score.[6] Therefore, the bidder's quality choice at bidding is always equal to first-best in a SS auction. Let $\boldsymbol{q}^{FB}(\boldsymbol{\theta})$ be the first-best quality. Under the PQR scoring rule, for instance, $\boldsymbol{q}^{FB}(\boldsymbol{\theta})$ satisfies

$$C_{q^\ell}(\boldsymbol{q}^{FB}(\boldsymbol{\theta})|\boldsymbol{\theta})q^{FB,\ell}(\theta) = C(\boldsymbol{q}^{FB}(\boldsymbol{\theta})|\boldsymbol{\theta}). \tag{5.8}$$

5.2.4 Revenue Ranking

Revenue ranking is possible in scoring auctions. The exercised score s^e represents the auctioneer's utility from the scoring auction. Hanazono et al. (2013) showed that the equivalence regarding expected exercised scores (revenue) does not generally hold in the scoring auction. If we restrict attention to a class of scoring rules that are linear in price *e.g.,* PQR and QL, then the expected exercised score is weakly greater in FS than in SS auctions. In particular, if the scoring rule is PQR, a FS procurement auction creates a higher expected score than does a SS counterpart. Thus, the auctioneer prefers a SS auction if his true preference is PQR.

Using the characterization of the equilibrium strategies as well as the equilibrium properties in FS and SS auctions, a series of empirical examinations are highlighted in Section 5.4. As in the theoretical model, risk-neutral bidders and independently and identically distributed signals are assumed.

[6]The profit-maximizing quality is always first-best under the QL scoring rule even if the bidder's score is strictly greater than the break-even score (Che (1993)).

5.3 Structural Estimation of the Scoring Auction Model

5.3.1 Outline

In the scoring auction model, the bidder's induced utility $u(s, \boldsymbol{\theta})$, is generally an unknown nonlinear function, although the bidder is risk neutral. This situation is somewhat similar to auctions with risk-averse bidders. Guerre et al. (2009) showed that the model of auctions with risk-averse bidders is generally unidentified from bid data. A question might be whether the scoring auction model is identified only from bid data.

Because observed bids are $L+1$ dimensions in the scoring auction model, up to $L+1$ dimensions of parameters can be identified if bidders are symmetric and homogeneous goods or services are auctioned. Therefore, a possible way to identify the scoring auction data should be to assume that the bidder's cost function is parametric with $L+1$ dimensional latent parameters. Given the specification, the bidder's nonlinear utility function becomes parametric.

In the next subsection, we show that, if Assumption 2 is satisfied, then the derivative of the bidder's objective function with respect to each dimension of the bid is obtained parametrically and that it is strictly monotone in each dimension of the latent parameters. From the next subsection, we deliver a more detailed argument on the identification and the estimation procedure of the $L+1$ dimensional latent parameters from the $L+1$ dimensional scoring auction data.

5.3.2 Identification of the Bidder's Cost Function in a FS Auction

First, we show that, given the assumptions discussed in the previous section, an identification of the cost function parameters, $\boldsymbol{\theta} = (\theta^0, \ldots, \theta^L)$, is possible as follows. Since the equilibrium strategy depends only on θ^0 in a FS auction, and since $s_{\mathrm{I}}(\theta^0)$ is a strictly increasing function of θ^0, the inverse function of the equilibrium strategy, $s^{-1}(\cdot)$, exists. Therefore, the distribution of parameter θ^0 is identified from observed score s. Next, from (5.1), ignoring the boundary solution (*i.e.*, $q^\ell = \underline{q}^\ell$ or $\bar{q}^\ell$ for some $\ell = 1, \ldots, L$), the optimal quality $\boldsymbol{q}$ satisfies $P_{q^\ell}(s, \boldsymbol{q}) = C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$ for all $\ell = 1, \ldots, L$.

For notational convenience, define

$$\boldsymbol{\theta}^{-\mathbf{0}} = (\theta^1, \ldots, \theta^L).$$

Note that the value of $P_{q^\ell}(s, \boldsymbol{q})$ is observable from observed score s and observed quality $\boldsymbol{q}$. Furthermore, as θ^0 can be recovered from observed score s, $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$ is known up to $\boldsymbol{\theta}^{-\mathbf{0}}$. If $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$ is a strictly decreasing function of θ^ℓ, the function $y(\theta^\ell) = C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$ has its inverse. Therefore, if $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$ is a strictly decreasing function of θ^ℓ for all $\ell = 1, \ldots, L$, parameter θ^ℓ is also identified from observed score s and quality $\boldsymbol{q}$. The following proposition summarizes this point.

Proposition 4. *We define that a distribution $G(\cdot)$ of observed scores $(s_1, \ldots, s_n)$ is rationalized by the distribution of the bidder's multi-dimensional private signal $F(\cdot)$ in the scoring auction if $G(\cdot)$ is the distribution of the equilibrium score bid. Then, the model of scoring auctions with symmetric risk-neutral bidders is identified if the bidder's utility function is i) separable (Assumption 2) and ii) log-supermodular (Assumption 3) and iii) if the bidder's marginal cost for q^ℓ is monotone in θ^ℓ for all $\ell = 1, \ldots, L$.*

We have two remarks on the monotonicity condition of $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$. First, in the PQR scoring rule, Assumption 2 is sufficient for implying the monotonicity condition. To see this, we have

$$mC(\boldsymbol{q}|\boldsymbol{\theta}) = C(m\boldsymbol{q}|\theta^0, m\boldsymbol{\theta}^{-\mathbf{0}})$$

for all $m > 0$ in the PQR scoring rule. Therefore, we gain

$$C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta}) = C_{q^\ell}(m\boldsymbol{q}|\theta^0, m\boldsymbol{\theta}^{-\mathbf{0}}), \tag{5.9}$$

for all $\ell = 1, \ldots, L$. If $m > 1$, because $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$ is a strictly convex function of $\boldsymbol{q}$, $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$

is a strictly increasing function of $\boldsymbol{q}$. That is,

$$C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta}) < C_{q^\ell}(m\boldsymbol{q}|\boldsymbol{\theta}), \tag{5.10}$$

for all $\ell = 1, \ldots, L$. From (5.9) and (5.10),

$$C_{q^\ell}(m\boldsymbol{q}|\boldsymbol{\theta}) > C_{q^\ell}(m\boldsymbol{q}|\theta^0, m\boldsymbol{\theta^{-0}}) = C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta}),$$

for all $\ell = 1, \ldots, L$. Therefore, $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$ is strictly decreasing in θ^ℓ. Similarly, if $m \in (0, 1]$, $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$ is strictly decreasing in $\boldsymbol{q}$. Thus, Assumption 2 is sufficient for identifying $\boldsymbol{\theta}$.

Second, Assumption 2, in general, does not imply the monotonicity of $C_{q^\ell}(\boldsymbol{q}|\boldsymbol{\theta})$. In other words, the monotonicity condition, in general, is required. A typical example is seen under a QL awarding rule.

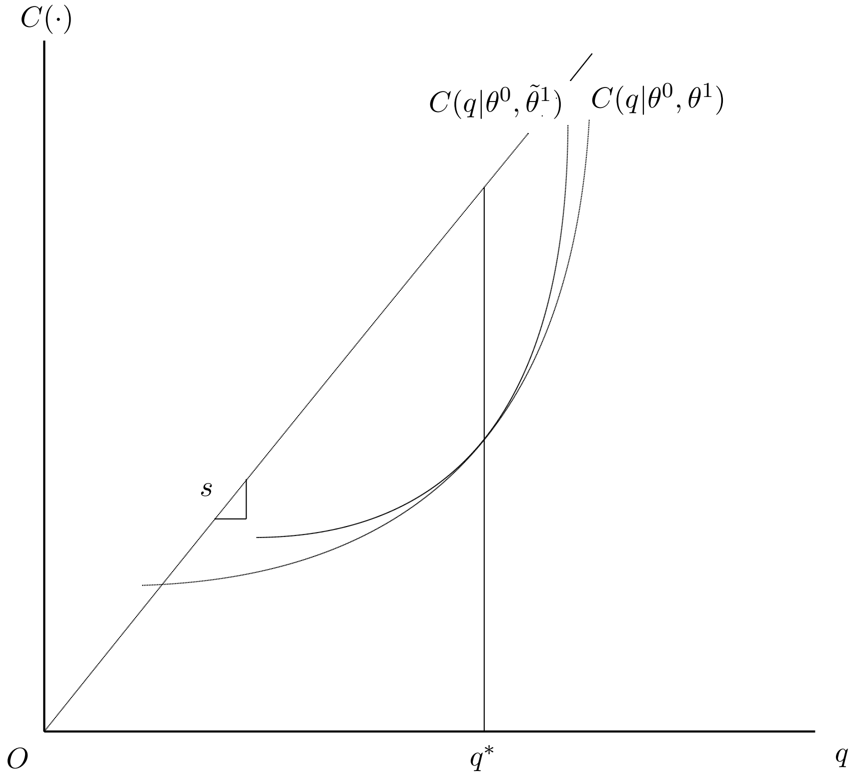

Figure 5.1: Example of nonidentifiable parameter (PQR scoring rule)

Figure 5.1 shows an example in which parameter $\boldsymbol{\theta} = (\theta^0, \theta^1)$ is not identifiable from the observed score s and quality $q \in \mathbb{R}_+$. In this example, a bidder with cost function $C_q(q|\theta^0, \theta^1)$ submits score s and quality q^*, whereas another bidder with cost function $C_q(q|\theta^0, \tilde{\theta})$ with $\theta^1 \neq \tilde{\theta}^1$ also submits s and q^*. Therefore, from the observed score s and quality q^*, parameter θ^1 is not identified.

5.3.3 Estimation for the Distribution of Cost Function Parameter Vector θ

The estimation of $\boldsymbol{\theta}$ from the observed data $(s_{i,t},\ \boldsymbol{q}_{i,t})$ proceeds as follows. By the equilibrium bidding function, we have

$$s_{\mathrm{I}}(\theta^0) = s_{i,t}. \tag{5.11}$$

In addition, from (5.1), ignoring boundary solution (*i.e.,* $\boldsymbol{q} = \underline{\boldsymbol{q}}$ or $\bar{\boldsymbol{q}}$), observed quality $\boldsymbol{q}_{i,t}$ satisfies Equation (5.1). Therefore, we obtain

$$P_{q^\ell}(s_{i,t}, \boldsymbol{q}_{i,t}) = C_{q^\ell}(\boldsymbol{q}_{i,t}|\boldsymbol{\theta}_{i,t}) \text{ with } \ell = 1, \ldots, L \tag{5.12}$$

as an empirical counterpart of (5.1).

Let $\hat{\boldsymbol{\theta}}_{i,t} = (\hat{\theta}^0_{i,t}, \ldots, \hat{\theta}^L_{i,t})$ be the solution of the simultaneous equations (5.11) and (5.12). Because $s_{\mathrm{I}}(\theta^0)$ is strictly increasing, parameter θ^0 is possibly obtained, using the inverse function, $s^{-1}(\cdot)$. Furthermore, the assumption that $C_q(\boldsymbol{q}|\boldsymbol{\theta})$ is monotone in θ^ℓ for all $\ell = 1, \ldots, L$ implies that, for given $s_{i,t}$, $q_{i,t}$, and $\theta^0_{i,t}$, parameter $\theta^\ell_{i,t}$ is obtained for all $\ell = 1, \ldots, L$. Therefore, $\hat{\boldsymbol{\theta}}$ would be estimated.

Unfortunately, the inverse function $s_{\mathrm{I}}^{-1}(\cdot)$ cannot be obtained analytically in general. It could be possible to obtain the inverse function $s^{-1}(\cdot)$ directly from (5.6) with a numerical computation; however, given the fact that the distribution of $\boldsymbol{\theta}$ is unknown, it is a computational burden. Therefore, we estimate θ from the first-order condition instead of solving the equilibrium strategy explicitly.

Let $G(s)$ be the cumulative distribution function of $s_{\mathrm{I}}(\theta^0)$ and $g(s)$ be its density. Then, letting $s_{\mathrm{I}}^{-1}(\cdot)$ be the inverse function of $s_{\mathrm{I}}(\cdot)$ such that $s_{\mathrm{I}}^{-1}(s_{\mathrm{I}}(\theta^0)) = \theta^0$, we have $G(s) = 1 - F_0(s_{\mathrm{I}}^{-1}(s))$. By the inverse function theorem, $g(s) = f_0(s_{\mathrm{I}}^{-1}(s))/s'_{\mathrm{I}}(\theta^0)$ holds. From (5.5), *i.e.,* the bidder's first-order condition in a FS auction, and given an optimal quality $\boldsymbol{q}(s, \boldsymbol{\theta})$ and parameter vector $\boldsymbol{\theta}$, we obtain

$$J(s, \theta^0) \equiv \frac{u(s, \boldsymbol{\theta})}{u_s(s, \boldsymbol{\theta})} - \frac{1}{n-1} \frac{1 - G(s)}{g(s)} = 0. \tag{5.13}$$

Note that Assumption 2 (separability) ensures that $J(\cdot)$ is independent of $\boldsymbol{\theta}^{-\mathbf{0}}$. It follows that $u(s, \boldsymbol{\theta}))/u_s(s, \boldsymbol{\theta})) = u(s, \theta^0, \tilde{\boldsymbol{\theta}}^{-0})/u_s(s, \theta^0, \tilde{\boldsymbol{\theta}}^{-0})$ for all $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{\theta}}^{-0}$. Therefore, J is expressed as a function of s and θ^0. Moreover, (5.13) satisfies $J(s_{\mathrm{I}}(\theta^0), \theta^0) = 0$ for all θ^0 in equilibrium. These suggest that the first-order condition, i.e, $J(s(\theta^0), \theta^0) = 0$, constitutes an implicit function that uniquely defines the inverse of a strictly increasing function $s_{\mathrm{I}}(\theta^0)$. Therefore, using (5.13), we can estimate θ^0 from the observed score s. The following proposition summarizes this result.

Proposition 5. *Let $G(s_1, \ldots, s_n)$ be the joint distribution of $(s_1, \ldots, s_n)$ with support $[\underline{s}, \bar{s}]$. Then, there exists a distribution of bidders' private signal $F(\cdot)$ such that $G(s_1, \ldots, s_n)$ is the distribution of the equilibrium scores in a FS auction with symmetric, risk-neutral bidders if*

1. *$G(s_1, \ldots, s_n) = \Pi_{i=1}^n G(s_i)$.*

2. *The scoring rule and the true cost function satisfy Assumption 2 and 3.*

Moreover, the following implicit function,

$$J(s_{i,t}, \theta^0_{i,t}) \equiv \frac{\underline{u}(s_{i,t}, \theta^0_{i,t})}{\underline{u}_s(s_{i,t}, \theta^0_{i,t})} - \frac{1}{n-1} \frac{1 - G(s_{i,t})}{g(s_{i,t})} = 0,$$

uniquely defines a strictly increasing and differentiable function that coincides with the inverse bidding strategy $s^{-1}(s_{i,t}) = \theta^0_{i,t}$

Proof. See 5.A. $\square$

Two observations are made here. First, if we define

$$k(s_{\mathrm{I}}(\theta^0),\theta^0) \equiv s_{\mathrm{I}}(\theta^0) - \underline{u}(s,\theta^0)/\underline{u}_s(s,\theta^0),$$

the first-order condition (5.13) explicitly gives k:

$$k(s_{\mathrm{I}}(\theta^0),\theta^0) = s_{\mathrm{I}}(\theta^0) - \frac{1-G(s_{\mathrm{I}}(\theta^0))}{(n-1)g(s_{\mathrm{I}}(\theta^0))}.$$

As discussed in Hanazono et al. (2013), k is known as the bidder's *pseudotype* (Asker and Cantillon (2008)) if the scoring rule is QL and the reservation price is nonbinding.[7] In a nonquasilinear scoring rule, however, estimating k may not be sufficient for obtaining the parameter θ^0; if the second partial derivative of $u(\cdot)$ with respect to s is strictly negative, *i.e.*, $u_{ss} < 0$, then $k(s_{\mathrm{I}}(\theta^0),\theta^0)$ may not be strictly increasing in θ^0 in equilibrium.[8] In other words, a Bayesian Nash equilibrium is characterized in a FS auction with an independent scoring rule regardless of whether the equilibrium s is strictly increasing in the equilibrium $k(\cdot)$. Therefore, no one-to-one mapping is guaranteed from the estimated k to the private signal θ^0 in the scoring auction model.

Second, although function $J(s,\theta)$ includes an unknown parameter θ^{-0}, obtaining the functional form of $J(\cdot)$ only requires the values of $G(s)$ and $g(s)$, Assumption 2 allows us to obtain $J(\cdot)$ explicitly. In practice, we can set Θ^{-0} to be any arbitrary vector in Θ^{-0}, *e.g.*, $\boldsymbol{\theta}^{-0} = \underline{\boldsymbol{\theta}}^{-0}$ to obtain the functional form of J. A unique inverse function $s^{-1}(\cdot)$ is implied by the implicit function $J(s,\theta^0) = 0$, regardless of the value of $\boldsymbol{\theta}^{-0}$. Thus, given Assumption 2 and Assumption 3, using $J(\cdot) = 0$ is a general procedure for estimating θ^0 from scoring auction data.

The nonparametric estimation of the distribution of $\boldsymbol{\theta}$ is given as follows. Because s is observable, the cumulative distribution function, $G(s)$, and its density, $g(s)$, can be

[7]The *pseudotype* is also discussed in Che (1993) as *productive potential* as *generalized cost.*
[8]See Hanazono et al. (2013) for an example of the nonmonotonic $k(\cdot)$.

estimated by the standard kernel estimator. Let T be the number of scoring auction samples, each indexed by $t = 1, \dots, T$. Auction-specific heterogeneities, such as the number of bidders, project location, time, and the maximum quality level, are controlled; let n_t and x_t denote the number of bidders and the covariates of auction t, respectively. Let $g(s, n, \boldsymbol{x})$ denote the joint density function of s, n, and $\boldsymbol{x}$. Then, the kernel estimator for $G(s, n, \boldsymbol{x}) := \int_{-\infty}^{s} g(v, n, \boldsymbol{x}) dv$ is provided by

$$\hat{G}(s, n, \boldsymbol{x}) = \frac{1}{T h_{G_n} h_{G_x}^d} \sum_{t=1}^{T} \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(s_{i,t} \leq s) K_G\left(\frac{n - n_t}{h_{G_n}}, \frac{x_1 - x_{1,t}}{h_{G_x}}, \cdots, \frac{x_d - x_{d,t}}{h_{G_x}}\right), \tag{5.14}$$

where $\mathbf{1}(\cdot)$ is an indicator function, K_G is a kernel with a bounded support, and h_{G_n} and h_{G_x} are bandwidths. Similarly, the kernel density estimator for $g(s, n, \boldsymbol{x})$ is given by

$$\hat{g}(s, n, \boldsymbol{x}) = \frac{1}{T h_s h_{g_n} h_{g_x}^d} \sum_{t=1}^{T} \frac{1}{n} \sum_{i=1}^{n} K_g\left(\frac{s - s_t}{h_s}, \frac{n - n_t}{h_{g_n}}, \frac{x_1 - x_{1,t}}{h_{g_x}}, \cdots, \frac{x_d - x_{d,t}}{h_{g_x}}\right), \tag{5.15}$$

where K_g is a kernel with a bounded support and h_s, h_{g_n}, and h_{g_x} are bandwidths. In practice, the discrete variables, such as the number of bidders and the maximum quality level, are smoothed out in the way discussed in Li and Racine (2006).

The estimation for $F(\boldsymbol{\theta}, \boldsymbol{x}) := \int_{-\infty}^{\theta^0} \cdots \int_{-\infty}^{\theta^{L-1}} f(\boldsymbol{\tau}, \boldsymbol{x}) d\tau^0 \cdots d\tau^{L-1}$ is given by the standard kernel method:

$$\hat{F}(\boldsymbol{\theta}, \boldsymbol{x}) = \frac{1}{T h_{F_x}^d} \sum_{t=1}^{T} \mathbf{1}(\theta^0 \leq \theta_{i,t}^0, \dots, \theta^{L-1} \leq \theta_{i,t}^{L-1}) K_F\left(\frac{x_1 - x_{1,t}}{h_{F_x}}, \cdots, \frac{x_d - x_{d,t}}{h_{F_x}}\right),$$

where K_F is a kernel with bounded support and h_{F_x} is a bandwidth. Similarly, the kernel density estimator for the joint density function of $\boldsymbol{\theta}$ and the covariate vector $\boldsymbol{x}$ is given by

$$\hat{f}(\boldsymbol{\theta}, \boldsymbol{x}) = \frac{1}{Th_{f_{\boldsymbol{\theta}}} \cdots h_{f_L} h_{f_x}^d} \sum_{t=1}^{T} K_f \big(\frac{\theta^0 - \theta^0_{i,t}}{h_{f_0}}, \ldots, \frac{\theta^{L-1} - \theta^{L-1}_{i,t}}{h_{f_L}}, \frac{x_1 - x_{1,t}}{h_{f_x}}, \cdots, \frac{x_d - x_{d,t}}{h_{f_x}} \big)$$

where K_f is a kernel with bounded support, and h_{f_0}, h_{f_1}, and h_{f_x} are bandwidths. The property of the estimator $\hat{f}(\boldsymbol{\theta}, \boldsymbol{x})$ is examined in Guerre et al. (2000).

5.3.4 Simulation Experiments

To illustrate our identification procedure, we conduct a numerical simulation. Our Monte Carlo study consists of $R = 500$ replications with $T = 500$ auctions in each replication and two bidders in each auction. The cost function is specified as

$$C(q|\boldsymbol{\theta}) = (1 + \theta^1) \left[\left(\frac{q}{1 + \theta^1} - 1 \right)^2 + \theta^0 \right].$$

Signal $\boldsymbol{\theta}$ is independently and identically distributed with the marginal distributions of θ^0 and θ^1) being Uniform ($U(0,1)$) and Beta ($B(3,2)$), respectively, for each replication r = 1, 2, ..., 500. Given the specification, the equilibrium bidding function is explicitly obtained as [9]

$$s_{\mathrm{I}}(\theta^0) = -2 + \sqrt{2\theta^0 + 6},$$
$$q(s_{\mathrm{I}}(\theta^0), \boldsymbol{\theta}) = (1 + \theta^1) \left[\frac{s_{\mathrm{I}}(\theta^0)}{2} + 1 \right].$$

Substituting the random samples $\boldsymbol{\theta}$ into these equilibrium strategies, we generate a five hundred pairs of sample bids $s_{\mathrm{I}}(\theta^0)$. Then, using our estimation procedure, the private signal $\boldsymbol{\theta}$ is recovered. We follow Guerre et al. (2000) for the nonparametric estimation: the use of the triweight kernel and the selection of the bandwidth. The recovered distributions of θ^0 and θ^1 are given in Figures 5.2 and 5.3, respectively. The

[9] See 5.B for the derivation of the equilibrium strategy.

results imply that our nonparametric estimation method can identify the private signals from bid data.

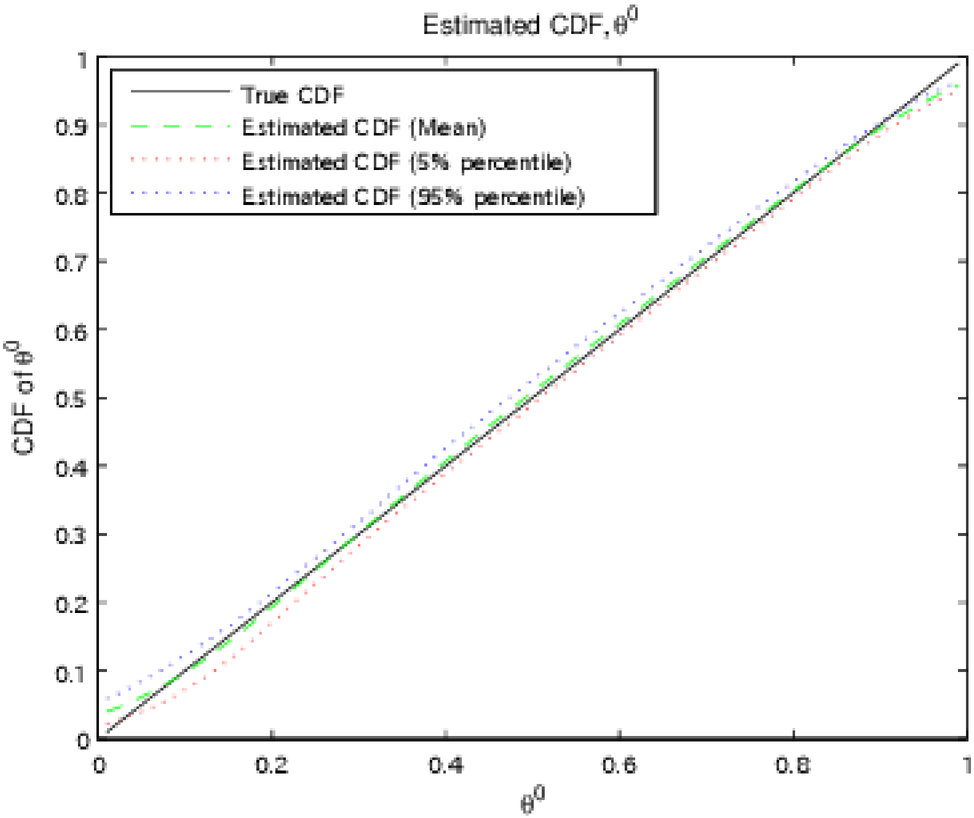

Figure 5.2: Estimated CDF of θ^0 [Uniform(0,1)]

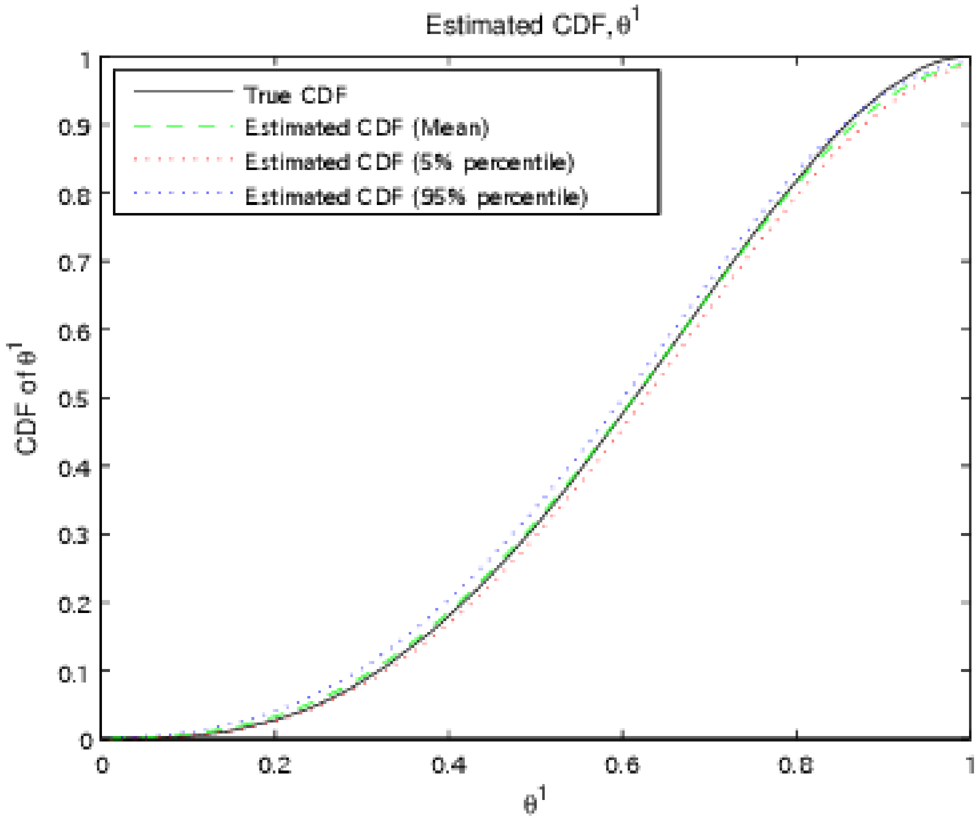

Figure 5.3: Estimated CDF of θ^1 [Beta(3, 2)]

5.4 Empirical Experiments

5.4.1 Data

The data used in our analysis contain the bid results of the procurement auctions for civil engineering projects from April 2010 through January 2013 by the Ministry of Land, Infrastructure, and Transportation (MLIT) in Japan. The number of contracts awarded during this period was 7,538. The bid results are posted on the Public Works Procurement Information Service (PPI) website.[10] The information available from PPI includes project names, project types, dates of auctions, engineers' estimates, scoring

[10]The address is `http//www.ppi.go.jp`.

auctions or not, and submitted bids with the bidder's identity. MLIT procures 21 types of construction work including civil engineering (or heavy and general construction work), buildings, bridges, paving, dredging, and painting. The civil engineering projects cost approximately 750 billion yen a year, which accounts for approximately 54 percent of the entire expenditure of the ministry, as well as for approximately 7 percent of the public construction investment in the country. Most of the procurement contracts for the civil engineering projects (7,489 out of 7,538) are allocated through scoring auctions. The data on price-only auctions have been removed from our samples.

Percentage Bids

In the scoring auctions held by the MLIT, the bidder with the highest-score wins the project. The scoring bid is calculated as the factor bid divided by the price bid. The factor bid consists of multiple components, such as noise level, completion time, and bidder experience.

The data set records each bidder's quality bid, Q, as a number. The lower bound of the factor bids is 100 for all auctions, and the upper bound is 110 to 200, depending on the auction. In practice, each bidder submits a technical proposal that is converted into the factor bid according to the publicly announced tender notice for the auction. The bidder proposing nothing has a factor bid equal to 100. The method of converting a technical proposal into a factor bid differs for each project. For instance, each one decibel reduction in noise accounts for five additional factor bid points.

We incorporate the scoring auction data into the model. Let B_i and Q_i be the values of the price and factor bids, respectively. Let S_i be bidder i's score. Under the price–factor (quality) ratio scoring rule, $S_i = B_i/Q_i$. To control for project size heterogeneity, we introduce the percentage score; let $\bar{B}$ and $\underline{Q}$ be the engineer's estimated cost and the factor bid evaluating nothing (the lowest possible factor bid), respectively. Then, a base score, $\bar{S}$, is defined such that $\bar{S} \equiv \bar{B}/\underline{Q}$. Then, bidder i's percentage score is defined as

$$s_i = \frac{S_i}{\bar{S}}, \tag{5.16}$$

where the bidder with the lowest percentage score wins.

Let T denote the number of procurement contracts to be auctioned off by the buyer. Furthermore, let $S_{(1),t}$, $\bar{S}_t$, and $\bar{B}_t$ be the winning bidder's score, the base score, and the engineer's estimated cost in auction $t = 1, \ldots, T$, respectively. Our model assumes that the scoring rule represents the procurement buyer's utility. Thus, a higher value-for-money contract (Q/B is higher) implies a contract with a lower *quality-adjusted* cost (B/Q is lower). The winning score is the quality-adjusted procurement cost. The effective procurement cost of purchasing T contracts is, thus, given by $\sum_{t=1}^{T} S_{(1),t}$. In our data, $\underline{Q}$ is normalized to be 100 for all T projects. Hence, the average percentage of the winning score is given by

$$\frac{1}{T}\sum_{t=1}^{T} s_{(1),t}, \tag{5.17}$$

where $s_{(1),t} = S_{(1),t}/\bar{S}_t$. In what follows, this value is considered to be the effective procurement cost.

Covariates

The sample auction data involve significant heterogeneity, such as in the number of bidders, the project size, and the maximum quality level. The percentage score somehow mitigates the project size heterogeneity but not perfectly. Therefore, we introduce a covariate vector x to control for the auction-specific effects. In our analysis, the covariates include the maximum quality level, the auction date, and the log of the engineer's estimated costs (as a proxy of project sizes).

5.4.2 Specifications under the PQR Scoring Rule with a Parametric Cost Function

Estimation of $\boldsymbol{\theta}$

Let us assume that the cost function we estimate is parameterized with the following two-dimensional signal $\boldsymbol{\theta} = (\theta^0, \theta^1)$ as

$$C(q|\boldsymbol{\theta}) = \theta^1 \left[\left(\frac{q}{\theta^1} - \alpha \right)^\beta + \theta^0 \right], \tag{5.18}$$

with $\underline{q} = \alpha\theta^1$. We fix $\alpha = 1$ but set β as being equal to either 2, 3, or 4, *i.e.,* the cost function being quadratic, cubic, or quartic polynomials, to see the robustness of our empirical examinations against the variations of the cost function specification.

Given these cost functions, θ^0 and θ^1 remain representing the efficiency and scale parameters, respectively; the lower θ^0 is, the lower the bidder's cost is given all other things are constant, whereas the higher θ^1 is, the greater the bidder's quality provision level is at the break-even (zero profit) score, even if the value of the break-even score is constant. Consequently, the marginal cost is monotonic in θ_1. With the PQR scoring rule, the separability of the θ dimension is achieved as long as cost functions with an identical θ^0 are homothetic with each other. All three parametric cost functions are homothetic.

The estimated implicit function that provides the inverse bidding function is given by

$$J(s, \theta^0) = s - \frac{\underline{\theta}^1}{q(s, \theta^0, \underline{\theta}^1)} \left[\left(\frac{q(s, \theta^0, \underline{\theta}^1)}{\underline{\theta}^1} - \alpha \right)^\beta + \theta^0 \right] - \frac{1 - G(s)}{(n-1)g(s).}$$

Here, $q(s, \theta^0, \underline{\theta}^1)$ is the solution of $\arg\max_q P(s_{i,t}, q) - C(q|\theta^0, \underline{\theta}^1)$, which is explicitly obtained by

$$q(s, \underline{\theta}^1) = \underline{\theta}^1 \left[\left(\frac{s}{\beta} \right)^{\frac{1}{\beta - 1}} + \alpha \right].$$

This equation is also used for estimating θ^1, with the fact that the observation, $q_{i,t}$, must satisfy $q(s_{i,t}, \theta_{i,t}) = q_{i,t}$. Therefore, with the observations $q_{i,t}$ and $s_{i,t}$ and the estimated distribution and density $\hat{G}$ and $\hat{g}$, we have

$$\hat{\theta}^0_{i,t} = \left[s_{i,t} - \frac{1 - \hat{G}(s_{i,t})}{(n-1)\hat{g}(s_{i,t})} \right] \cdot \left[\left(\frac{s_{i,t}}{\beta} \right)^{\frac{1}{\beta-1}} + \alpha \right] - \left(\frac{s_{i,t}}{\beta} \right)^{\frac{\beta}{\beta-1}},$$

$$\hat{\theta}^1_{i,t} = \frac{q_{i,t}}{\left(\frac{s_{i,t}}{\beta} \right)^{\frac{1}{\beta-1}} + \alpha}.$$

For estimating $\hat{G}$ and $\hat{g}$, the following triweight kernel is used:

$$K(u) = \frac{35}{32}(1-u^2)^3 \mathbf{1}(|u| < 1).$$

As usual, the bandwidths h_s and h_x are given by the so-called rule of thumb; $h_s = \eta_s(\sum_{k=1}^m n_k)^{-1/6}$ and $h_x = \eta_x(\sum_{k=1}^m n_k)^{-1/6}$, where $\eta_s = 2.978 \times 1.06\hat{\sigma}_s$ and $\eta_x = 2.978 \times 1.06\hat{\sigma}_x$, respectively. Both $\hat{\sigma}_s$ and $\hat{\sigma}_x$ are sample variances of the normalized scoring bids and the observed covariate, respectively.

From the pseudo-values of θ^0, we compute the quality-adjusted costs, $\hat{k}_{i,t} = s_{i,t} - u(s_{i,t}, \hat{\boldsymbol{\theta}})/u_s(s_{i,t}, \hat{\boldsymbol{\theta}})$. Corollary 1 in Hanazono et al. (2013) suggests that, under an IPV environment, the expectation of the lowest scoring bid will coincide with the expectation of the second-lowest bidder's k. The average of the obtained 6,088 pseudo-values of the second-lowest bidders' $\hat{k}_{i,t}$ is 0.583358. The average of the winning bidders' scores is 0.583358. Therefore, our estimation result is in line with the theoretical prediction.

The following figures are the estimated joint density functions assuming that the cost function is the quadratic polynomial ($\beta = 2$). Axes x (horizontal) and y (depth) represent θ^0 and θ^1, respectively. Recall that we parameterize the cost function so that the cost function shifts up vertically as the efficiency parameter θ^0 rises. Given this specification, a strong negative correlation is observed between θ^0 and θ^1 ($R^2 = -0.8657$), suggesting that more (less) efficient supplies tend to be larger (smaller).

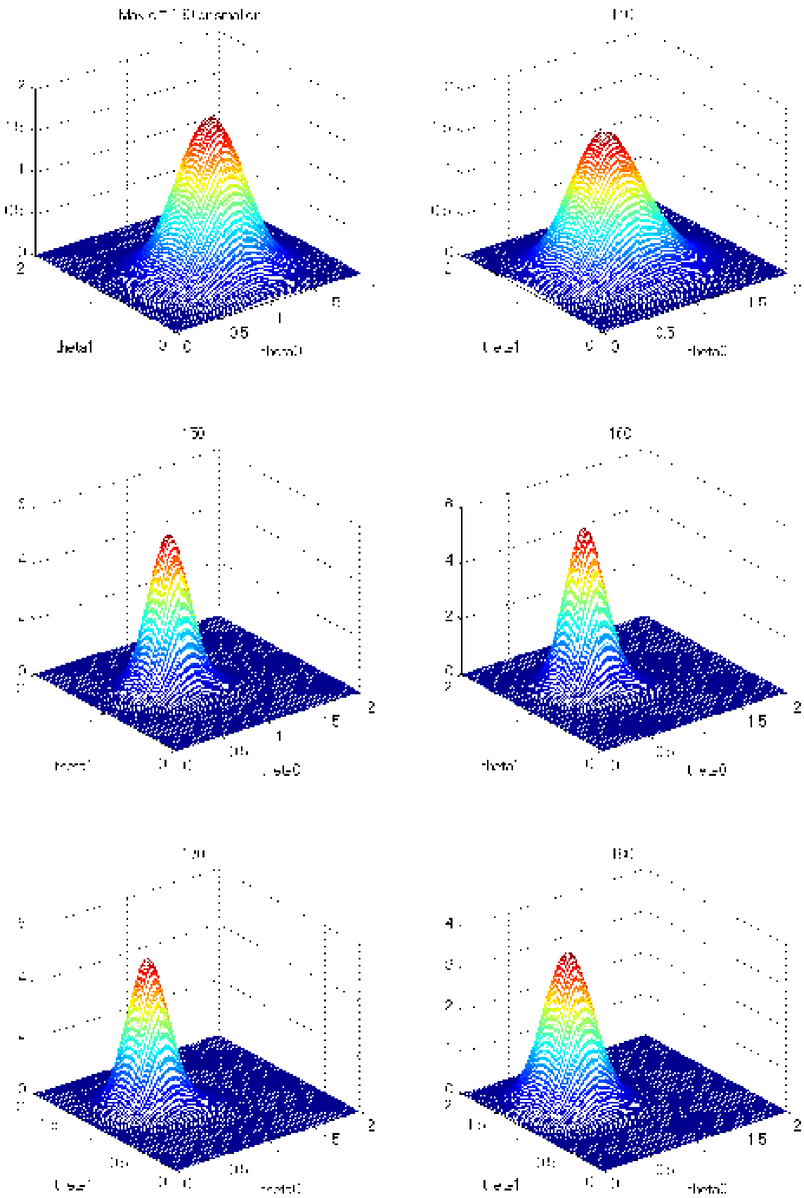

Figure 5.4: Estimated PDF (3D)

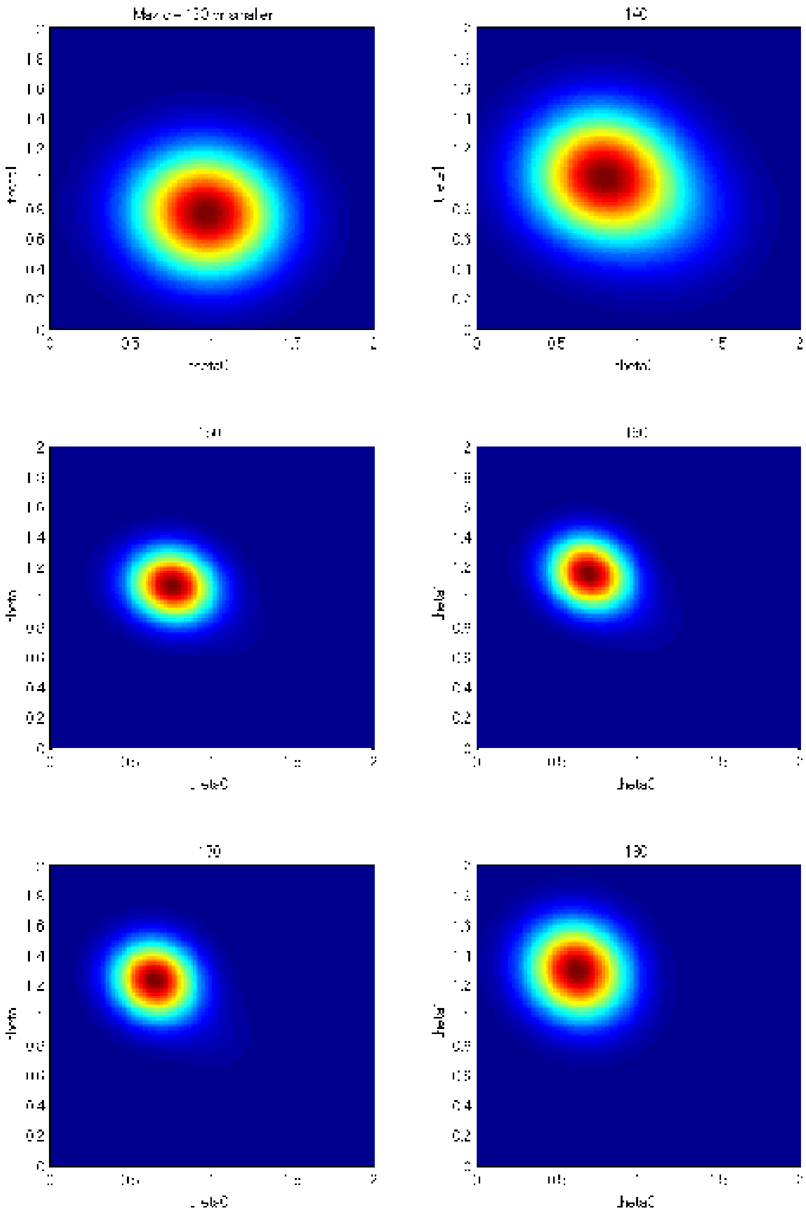

Figure 5.5: Estimated PDF (Pseudo Color)

Rationalizability

The scoring auction model imposes an additional restriction on the observations such that $J_s(s_{i,t}, \theta_{i,t}) > 0$. In this subsection, we show that the restriction is a necessary condition for the scoring auction data to be rationalizable. Because J_s contains the latent variable θ^0, the restriction is not directly obtained from the observations and their distributions. Furthermore, our observations include covariates, which also prevents us from obtaining the restriction explicitly from the data. Therefore, we choose to check whether the estimated $\theta_{i,t}$s are indeed strictly increasing in $s_{i,t}$ in each auction.

We have 6,115 auction samples from which θs have been effectively obtained. Of these, 22 auctions, accounting just for 0.36 % of all auction samples, exhibit nonmonotonic $\hat{\theta}^0$ with respect to s. Except for one auction, the nonmonotonicity is observed in a pair of bidders in which one bid a lower score but the bidder's θ^0 is estimated to be higher than that of the other. The observed scores that result in the nonmonotonic θ^0 are relatively close to the lower bound of the observed scores. Therefore, it is hard to conclude that the nonmonotonicity occurs simply because the auction samples are not rationalizable or because the non-parametric estimation suffers from biases close to the boundary.

On the other hand, the rest of the auction samples exhibit a strict monotonicity between the observed scores and the estimated θ^0. Hence, we conclude that our scoring auction data is rationalizable from the scoring auction model with symmetric risk-neutral bidders.

5.4.3 Counterfactual Analyses

Second-Price vs. FS Auctions

We first examine the welfare effect by using scoring auctions for government procurement. As Milgrom (2004) addressed, one of the appeals of multi-parameter auctions is that bidders increase profits without reducing the auctioneer's utility. Our first empirical examination thus focuses on measuring how much the use of scoring auctions raises the procurement buyer's utility $U(p, q)$, which is assumed to be represented by the observed PQR scoring rule, $S(p, q)$, namely $U(p, q) = p/q$.

We design a series of second-price auctions, in each of which the quality level is fixed at $q = 1$, 1.3, 1.4, 1.5, and 1.6, where $q = 1$ is the minimum quality level the bidder can propose in the observed scoring auctions, representing *no quality improvement*. Given the estimated bidder's private information, bidders' costs are computed for all $q = 1.0, \ldots, 1.6$, and the second-lowest costs are collected for all auction samples as the contract prices of the counterfactual second-price auctions. In a counterfactual second-price auction, the price quality ratio, p/q, no longer represent a score. Therefore, we denote by $-U(p, q) = p/q$ the procurement buyer's quality-adjusted procurement cost. The buyer's quality-adjusted procurement cost for each contract is measured by the second-lowest cost divided by q, where $q = 1.0, \ldots, 1.6$. Because the bidder's cost functions are differentiated by $\beta = 2$, 3, and 4, 15 types of counterfactual second-price auctions are created.

Table 5.1 compares the procurement buyer's quality-adjusted procurement costs in the observed FS auction versus those in cases where price-only auctions take place instead. The extent of the government's expected welfare gain from the scoring auction crucially depends on the fixed quality level of the counterpart second-price auction. The government utilities would drop quite trivially (approximately 1 to 2 percent) if a second-price auction with $q = 1.5$ were to be used while the drops would be nontrivial (greater than 30 percent) if a second-price auction with $q = 1.0$ were to be used. This suggests that a simple low-price auction works well if the design (a fixed quality standard) is appropriate.

The bidder's payoff also varies, depending on the quality standard in the price-only auction. Table 5.2 reports the winning bidders' payoffs. Bidders earn significantly lower payoffs upon winning in a first-price auction if the quality standard is less than 1.4. On the other hand, bidders earn larger payoffs in a price-only auction if the quality standard is greater than 1.5. The positive relationship between a larger payoff and a higher quality standard in a price-only auction stems from the fact that bidders with larger θ^1 are selected in price-only auctions with higher quality standards.

Although a price-only auction for a contract with an appropriate quality level still performs worse than does an observed PQR FS auction, the difference is not remarkably large. In addition, the bid preparation costs for a scoring auction may be greater than those for a simple price-only auction, which discourages potential bidders' entry into a scoring auction. Furthermore, the bid evaluation, with respect to quality proposals, is costly for a procurement buyer who is unfamiliar with the process. Taking into account these disadvantages in using a scoring auction, a price-only low-bid auction has still been a good mechanism to allocate the government contract if the quality standard of the contract is appropriate (in our case, $q = 1.5$).

Form	$C(q\|\cdot)$	q	Obs	Mean	Std. Dev.	Min	Max	Change[*3]
FS[*1]	-		6,063	0.5800	0.0680	0.2623	0.9945	-
		1.0	6,043	0.7638	0.0600	0.2907	1.1245	31.68%
		1.3	6,050	0.6097	0.0593	0.2401	1.0266	5.124%
	Quadratic	1.4	6,049	0.5909	0.0644	0.2488	1.0746	1.877%
		1.5	6,049	0.5857	0.0709	0.2680	1.1345	0.975%
		1.6	6,049	0.5917	0.0783	0.2825	1.2061	2.014%
		1.0	6,045	0.7758	0.0638	0.2998	1.1770	33.76%
		1.3	6,047	0.6160	0.0598	0.2460	1.0800	6.200%
SP[*2]	Cubic	1.4	6,049	0.5941	0.0659	0.2502	1.2068	2.435%
		1.5	6,049	0.5885	0.0767	0.2682	1.3779	1.466%
		1.6	6,048	0.5995	0.0923	0.2825	1.5932	3.354%
		1.0	6,045	0.7906	0.0655	0.3132	1.2092	36.31%
		1.3	6,047	0.6224	0.0600	0.2514	1.1592	7.313%
	Quartic	1.4	6,048	0.5975	0.0686	0.2515	1.4016	3.021%
		1.5	6,049	0.5918	0.0866	0.2684	1.7635	2.036%
		1.6	6,040	0.6077	0.1075	0.2825	1.8473	4.777%

[*1] Observed FS auctions. [*2] Counterfactual second-price auctions. *3 Change in mean from FS to SP auction. [*] Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 5.1: Quality-adjusted procurement costs

Form	$C(q\|\cdot)$	q	Obs	Mean	Std. Dev.	Min	Max	Change[*3]
FS[*1]	-		6,034	0.0640	0.0750	0.0018	0.7103	-
		1.0	6,027	0.0376	0.0601	0.0000	0.7684	-41.20%
		1.3	6,034	0.0506	0.0726	0.0000	0.8279	-20.92%
	Quadratic	1.4	6,034	0.0568	0.0787	0.0000	0.8512	-11.26%
		1.5	6,034	0.0635	0.0861	0.0000	0.8761	-0.80%
		1.6	6,034	0.0707	0.0949	0.0000	0.9029	10.40%
		1.0	6,032	0.0445	0.0657	0.0000	0.8108	-30.69%
		1.3	6,033	0.0511	0.0733	0.0000	0.8290	-20.22%
SP[*2]	Cubic	1.4	6,034	0.0570	0.0816	0.0000	0.8483	-10.99%
		1.5	6,032	0.0649	0.0933	0.0000	0.9875	1.34%
		1.6	6,020	0.0736	0.1017	0.0000	0.9954	14.96%
		1.0	6,032	0.0448	0.0661	0.0000	0.8155	-30.02%
		1.3	6,033	0.0507	0.0740	0.0000	0.8278	-20.78%
	Quartic	1.4	6,034	0.0571	0.0864	0.0000	0.9976	-10.83%
		1.5	6,018	0.0643	0.0924	0.0000	0.9772	0.54%
		1.6	5,996	0.0747	0.0976	0.0000	0.9996	16.75%

[*1] Observed FS auctions. [*2] Counterfactual second-price auctions. [*3] Change in mean from FS to SP auction. [*] Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 5.2: Bidders' payoffs

SS vs. FS Auctions

Next, the extent to which the expected scores would be changed by introducing SS auctions is estimated. Given the parametric cost function, the bidder's induced utility function $u(s, \theta)$ is convex in s for any $\beta > 1$ if the scoring rule is PQR, as the second derivative of u is given by

$$u_{ss}(s, \theta) = \theta^1 \frac{1}{\beta - 1} \left(\frac{s}{\beta} \right)^{-\frac{\beta-2}{\beta-1}} > 0 \text{ with } \beta \geq 2. \tag{5.19}$$

Therefore, the expected exercised score will be lower in SS than FS auctions as suggested by Theorem 3 in Hanazono et al. (2013). In this subsection, we conduct a counterfactual analysis to empirically measure the difference between FS and SS auctions regarding expected exercised scores (the buyer's welfare), bidders' payoffs, and quality levels.

The counterfactual samples related to the SS auction is created from the estimated parameters, $\hat{\boldsymbol{\theta}}_{i,t}$. First, the pseudo-samples of the first-best quality $q^{FB}(\theta)$ is created from (5.8), which is given by

$$\left(\frac{q^{FB}}{\theta^1_{i,t}} - \alpha\right)^{\beta} + \theta^0_{i,t} = \frac{q^{FB}}{\theta^1_{i,t}}\beta \cdot \left(\frac{q^{FB}}{\theta^1_{i,t}} - \alpha\right)^{\beta-1},$$

under the specific cost function. Thus, the first-best quality of bidder i in auction t is created as

$$\hat{q}^{FB}(\hat{\theta}_{i,t}) \equiv \left\{q : (1-\beta)r^{\beta}_{i,t}(q) - \alpha\beta r^{\beta-1}_{i,t}(q) + \hat{\theta}^0_{i,t} = 0\right\}, \tag{5.20}$$

where $r_{i,t}(q) = q/\hat{\theta}^1_{i,t} - \alpha$. Next, the counterfactual samples of the bidder's break-even score is created. From (5.7), the first-best quality, and the observed data, the break-even score of the bidder whose type is equal to $\theta_{i,t}$ is predicted as

$$k^{-}(\hat{\theta}^0_{i,t}) = \frac{\hat{\theta}^1_{i,t}}{\hat{q}^{FB}_{i,t}}\left[\left(\frac{\hat{q}^{FB}_{i,t}}{\hat{\theta}^1_{i,t}} - \alpha\right)^{\beta} + \hat{\theta}^0_{i,t}\right], \tag{5.21}$$

under the PQR scoring rule.

The awarded bidder's quality choice in the SS auction is also estimated. Let $\hat{\boldsymbol{\theta}}_{(i),t}$ be the signal of the bidder whose score is the ith lowest in auction t. In SS auctions, the exercised score is the second-lowest bidder's break-even score $k^{-}(\theta^0_{(2)})$. Thus, the winning bidder chooses the optimal quality level $q(k^{-}(\hat{\theta}^0_{(2),t}), \hat{\boldsymbol{\theta}}_{\mathbf{(1)},\boldsymbol{t}})$. Let $\hat{q}^{\mathrm{II}}_t$ denote the quality level. The first-order condition of the bidder's quality choice given s suggests

$C_q(\hat{q}_t^{\mathrm{II}}|\hat{\boldsymbol{\theta}}_{(1),t}) = \hat{k}^-(\theta^0_{(2),t})$, which is expressed as

$$q_t^{\mathrm{II}} = \hat{\theta}^1_{(1),t} \cdot \left[\left(\frac{k^-(\hat{\theta}^0_{(2),t})}{\beta} \right)^{\frac{1}{\beta-1}} + \alpha \right], \tag{5.22}$$

given our parametric cost functions. Thus, the awarded bidder's payoff, $u(\hat{k}^-(\theta^0_{(2),t}), \boldsymbol{\theta}_{(1),t})$, is given by

$$u(k^-(\hat{\theta}^0_{(2),t}), \hat{\boldsymbol{\theta}}_{(1),t}) = \hat{q}_t^{\mathrm{II}} \left[k^-(\hat{\theta}^0_{(2),t}) - k(\hat{q}_t^{\mathrm{II}}, \boldsymbol{\theta}_{(1),t}) \right]. \tag{5.23}$$

In a SS auction, the score in the final contract equals the break-even score of the lowest losing bidder, denoted by $k^-(\theta^0_{(2),t})$. The data on $k^-(\theta^0_{(2),t})$ is shown in Table 5.3. The expected score declines approximately by .04 percent (when $\beta = 2$) and .02 percents (when $\beta = 4$) if the auction format alters from FS to SS mechanisms. The variances are greater than that in the FS auction similar to the difference in the variance of first- and second-price auctions. Table 5.4 shows that the quality level finalized in the contract is, on average, declined approximately by 3 to 4 percents if SS auctions are used.

	$C(q\|\theta)$	Obs	Mean	Std. Dev.	Min	Max	Change
FS*1	-	6,004	0.5803	0.0495	0.4624	0.9142	-
	Quadratic	6,004	0.5801	0.0680	0.2399	0.9945	-0.0417%
SS*2	Cubic	6,004	0.5801	0.0679	0.2458	0.9945	-0.0288%
	Quartic	6,005	0.5802	0.0679	0.2497	0.9945	-0.0257%

*1 Observed FS auctions (PQR). *2 Hypothetical SS auctions with the PQR rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 5.3: Exercised scores (quality adjusted procurement cost) in FS and SS auctions

Form	$C(q\|\theta)$	Obs	Mean	Std. Dev.	Min	Max	Change
FS*1	-	6,004	1.5391	0.0968	1.3100	1.9000	-
	Quadratic	6,004	1.5389	0.1002	1.2968	1.9078	-0.012%
SS*2	Cubic	6,004	1.5387	0.0986	1.3170	1.8988	-0.028%
	Quartic	6,005	1.5388	0.0980	1.3177	1.8985	-0.024%

*1 Observed FS auctions (PQR). *2 Hypothetical SS auctions with the PQR rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 5.4: Contracted quality level in FS and SS auctions

Form	β	Obs	Mean	Std. Dev.	Min	Max	Change
FS*1	-	6,004	0.0637	0.0749	0.0018	0.7103	-
	Quadratic	6,004	0.0638	0.0820	0.0002	0.8863	0.255%
SS*2	Cubic	6,004	0.0638	0.0818	0.0002	0.8851	0.194%
	Quartic	6,005	0.0638	0.0817	0.0002	0.8845	0.151%

*1 Observed FS auctions (PQR). *2 Hypothetical SS auctions with the PQR rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 5.5: Bidder's payoffs in FS and SS auctions

QL vs. PQR Rules

Finally, we explore a QL scoring rule that dominates the current PQR scoring rule. Specifically, we suppose that the buyer uses a QL scoring rule that differs from the buyer's true preference $-U(p,q) = p/q$. To construct a well-performing QL rule, we relax the assumption that the quality price in the QL rule (the derivative of the score function with respect to q) is one such that, for some $\phi > 0$,

$$S(p,q) = p - \phi(\beta)q. \tag{5.24}$$

The lower utility caused by the use of a FS auction under PQR lies in over-provision in quality. In SS auctions, that upward distortion in quality provision is not observed. Therefore, a candidate of a QL rule that dominates the current PQR FS auction is such that the average of the winning bidders' first-best quality is equivalent to the average of the quality level to be chosen in a SS auction. We thus choose the following three values of the quality price: $\phi(2) = 0.6502278$, $\phi(3) = 0.6493106$, and $\phi(4) = 0.6477461$, each equal to the average of the exercised score in the counterfactual SS auction at $\beta = 2$, 3, and 4, respectively. Given $\phi(\beta)$, we predict the expected value of the wining score in SS auctions with the QL rule.

Under the QL rule, the bidder's quality-adjusted cost is given by $k(q, \boldsymbol{\theta}) = C(q|\boldsymbol{\theta}) - \phi(\beta)q$, and the first-best quality $q^{FB}(\boldsymbol{\theta})$ satisfies $C_q(q^{FB}(\boldsymbol{\theta})|\boldsymbol{\theta}) = \phi(\beta)$. Given the parametric cost function, the marginal cost is given by $C_q(q|\boldsymbol{\theta}) = \beta \cdot \big(q/\theta^1 - \alpha\big)^{\beta-1}$. Therefore, $q^{FB}(\boldsymbol{\theta})$ is given by

$$q^{FB}_{QL}(\hat{\boldsymbol{\theta}}) = \hat{\theta}^1 \cdot \left(\alpha + \left(\frac{\phi(\beta)}{\beta}\right)^{\frac{1}{\beta-1}}\right). \tag{5.25}$$

Using $q^{FB}_{QL}(\boldsymbol{\theta})$ and the estimated $\boldsymbol{\theta}$, we compute the bidder's break-even score, $k^{-}(\hat{\theta}^0)$, under QL rules. With our parameterized cost function, this is expressed as

$$k^{-}(\hat{\theta}^0) \equiv \hat{\theta}^1 \cdot \left[\left(\frac{q^{FB}_{QL}(\hat{\boldsymbol{\theta}})}{\hat{\theta}^1} - \alpha\right)^{\beta} + \hat{\theta}^0\right] - q^{FB}_{QL}(\hat{\boldsymbol{\theta}}). \tag{5.26}$$

Because bidders are symmetric, the bidder with the lowest $k(q^{FB}_{QL}(\hat{\boldsymbol{\theta}}), \hat{\boldsymbol{\theta}})$ is the awarder, receiving the payment $P_{QL} = C(q^{FB}_{QL}(\hat{\boldsymbol{\theta}}_{(1)})|\hat{\boldsymbol{\theta}}_{(2)})$ in the SS auction with the QL scoring rule. Thus, both the contract price and the quality level are given by P_{QL} and $q^{FB}_{QL}(\hat{\boldsymbol{\theta}}_{(1)})$. The buyer's utility is thus computed by

$$s_{QL} = \hat{p}_{QL}/q^{FB}_{QL}(\hat{\boldsymbol{\theta}}_{(1)}).$$

Table 5.6 reports the buyer's utility s_{QL} in counterfactual SS auctions with QL rules. In all cases, s_{QL}s drop on average approximately by 5 to 15 percent. The greater variances in SS auctions due to the non-negative variance of the conditional second-order statistic can be remedied by the use of FS auctions. Table 5.7 shows the bidder's profit. The bidder's profit drops by 1 to 12 percent. Hence, the use of an appropriate QL rule extracts more rents from bidders.

Form	β	Obs	Mean	Std. Dev.	Min	Max	Change
FS*1	-	6,004	0.5803	0.0495	0.4624	0.9142	-
	Quadratic	5,995	0.5786	0.0654	0.2650	0.9457	-0.295%
SS*2	Cubic	5,996	0.5786	0.0655	0.2653	0.9424	-0.286%
	Quartic	5,997	0.5786	0.0654	0.2655	0.9447	-0.288%

*1 Observed FS auctions (PQR). *2 Hypothetical SS auctions with the QL rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 5.6: Exercised scores (quality adjusted procurement cost) under simulated QL rules

Form	β	Obs	Mean	Std. Dev.	Min	Max	Change
FS*1	-	6,004	0.0637	0.0749	0.0018	0.7103	-
	Quadratic	5,971	0.0610	0.0763	0.0000	0.8693	-4.232%
SS*2	Cubic	5,972	0.0614	0.0768	0.0000	0.8707	-3.659%
	Quartic	5,972	0.0615	0.0770	0.0000	0.8714	-3.386%

*1 Observed FS auctions (PQR). *2 Hypothetical SS auctions with the QL rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 5.7: Payoffs under simulated QL rules

The additional rent extraction by the QL scoring rule stems from the downward distortion of the quality provision. Table 5.8 presents the contracted quality level in the observed FS auction and simulated QL scoring auctions. The quality levels would be sharply declined under the well-designed QL scoring rule. Although the well-designed QL scoring rule is not optimal, the lower contracted quality levels by the QL scoring rule limits the winner's informational rent, resulting in the greater welfare of the procurement buyer.

Form	β	Obs	Mean	Std. Dev.	Min	Max	Change
FS*[1]	-	6,034	1.5388	0.0968	1.3100	1.9000	-
	Quadratic	6,009	1.5135	0.1051	0.8414	1.8892	-1.648%
QL*[2]	Cubic	6,010	1.5209	0.1035	0.7695	1.8920	-1.163%
	Quartic	6,010	1.5249	0.1030	0.7334	1.8938	-0.907%

*[1] Observed FS auctions (PQR). *[2] Hypothetical SS auctions with the QL rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 5.8: Contracted quality levels in FS and QL scoring auctions

Summary of Empirical Experiments

Our counterfactual analyses suggest that FS auctions perform poor under PQR scoring rules. However, it does not mean that FS auctions never benefit procurement buyers whose preference is based on PQR. The performance of a price-only auction strongly depends on the choice of the fixed quality level. In many occasions, auctioneers have limited information regarding bidders' cost structures. Thus, only experienced buyers can choose the quality level that renders a higher expected utility to the buyer in a price-only auction than in a FS auction. For inexperienced buyers, the use of a FS auction is the best option even if their true preference is based on PQR. The same is true for QL scoring rules. We observed that, when a FS auction is used, a QL scoring rule may dominate the PQR rule in terms of the expected contracted score. However, for the procurement buyer with PQR preference, designing a well-performing QL scoring function, in particular, choosing the best quality price in a QL scoring function, requires accurate information on the bidders' cost structures. Less informed buyers with PQR preference will thus benefit from the use of his/her true preference as a scoring function since the quality price is determined in the market under a PQR scoring rule.

In scoring auctions, bidders' advantages in non-monetary attributes are evaluated. Therefore, the procurement buyer may obtain a better contract without reducing the bidder's profit. However, this is just an advantage of scoring auctions. Rather, the advantage of the use of scoring auction is in that even an inexperienced buyer can pursue the best value in procurement since he does not need to specify the quality level. If an inexperienced buyer is not familiar with bidders' advantages in non-monetary attributes rather than in costs, the scoring function selects the winner who provides the most value-for-money contract.

We found that bidders' earnings are greater in the FS auction than in the SS auction with the PQR scoring rule or than in the FS auction with the well-designed QL scoring rule. This result suggests that a major advantage of the currently adopted FS auction format lies in the promotion of bidder participation. The intensified competition by a FS auction will lower the quality-adjusted procurement cost even if the procurement buyer has limited information on bidders' cost structures. An interesting extension will be to take into account potential bidders' endogenous participation in the structural model.

5.5 Conclusion

In this research, we provided a structural estimation method for a scoring auction with generalized scoring rule. From the scoring auction data that typically include scores and quality bids, latent parameters in the bidder's cost function was estimated. From observed quality levels, the bidder's marginal costs are estimated through the bidder's profit maximization behavior such that the marginal cost equals the quality price. Bidders' costs were estimated through the first-order condition by the application of the non-parametric estimation methodology for the first-price auction model. It is obvious that the number of parameters capable to be identified is equal to or less than the number of dimensions of the observed data. Thus, for instance, the degree of concavity of the cost function is the one that is unable to be identified. We thus conducted a series of empirical experiments in which the parameters of the cost function vary to ensure the robustness of estimation results.

We also showed an simulation experiment to illustrate that our structural estimation method does identify the latent distribution of the bidder's signal. The recovered density and cumulative distribution functions were coincident with the true density and distribution functions except in the areas of boundaries. Therefore, our estimation method effectively identifies the bidder's multi-dimensional signal.

Furthermore, we applied our estimation technique to real world scoring auction data. Theory has suggested that the non-equivalence in the expected winning scores stems from the overproduction in quality in a FS auction with the PQR scoring rule. Accordingly, we observed that the expectation of the winner's quality provision is larger in FS than in SS auctions. Furthermore, with a well-designed QL scoring rule, we found that the procurement buyer improves utility while bidders earn lower payoffs. Generally, the optimal design problem is hard to be solved if the bid and signal are multi-dimensional. Therefore, our counterfactual analysis uses a standard FS or SS auction with a well-designed QL scoring rule as a suboptimal mechanism. Nevertheless, a flavor of the optimal design problem has been seen in our empirical result, the quality provision is distorted downward (allocative inefficiency) and the bidder's informational rents are limited.

In this article, we restrict attention to the independent scoring rule, in which the bidder's score depends only on his or her price and quality bids. In the real-world procurement auctions, however, a wider-variety of scoring rules are used including the one in which the bidder's score depends also on the other bidders' price and quality bids (an interdependent scoring rule). Literature suggests that an interdependent scoring rule involves some inefficiency when bidders choose optimal quality levels since the realized exercised score generally differs from the score predicted by the bidder when choosing the quality bid. As a result, the expected exercised score (the procurement buyer's utility) is greater (smaller) than the scoring auction with an independent scoring rule. Albano et al. (2009) suggests that the welfare loss of the procurement buyer is approximately 11 %. Theoretical literature, on the other hand, has so far been silent on the equilibrium in the scoring auction with such an interdependent scoring rule. An interesting future research may lie in the structural analysis of the scoring auction with an interdependent scoring rule. A counterfactual analysis would quantify the expected score difference between the FS and SS auctions with an interdependent scoring rule.

5.A Proof of Proposition 5

Proof. Assumption 3 (log-supermodularity) suggests that

$$\frac{\partial}{\partial \theta^0} \frac{u_s}{u} > 0,$$

for all θ^0. Therefore,

$$\frac{\partial}{\partial \theta^0} \frac{u}{u_s} = J_{\theta^0} < 0.$$

Assumption 3 also suggests the existence of a strictly increasing equilibrium strategy $s_{\mathrm{I}}(\theta)$ for all $\theta^0 \in [\underline{\theta}^0, \bar{\theta}^0]$. Therefore, for an arbitrary $\tilde{\theta}^0 \in [\underline{\theta}^0, \bar{\theta}^0]$ and a strictly increasing equilibrium strategy $s_{\mathrm{I}}(\theta)$, we have

$$J(s_{\mathrm{I}}(\tilde{\theta}^0), \theta^0) \equiv \frac{\underline{u}(s_{\mathrm{I}}(\tilde{\theta}^0), \theta^0)}{\underline{u}_s(s_{\mathrm{I}}(\tilde{\theta}^0), \theta^0)} - \frac{1}{n-1} \frac{1 - G(s_{\mathrm{I}}(\tilde{\theta}^0))}{g(s_{\mathrm{I}}(\tilde{\theta}^0))}.$$

Therefore, for all θ^0, we have

$$J(s_{\mathrm{I}}(\tilde{\theta}^0), \theta^0) \lesseqqgtr 0 \text{ if } \tilde{\theta}^0 \lesseqqgtr \theta^0.$$

Because $s_{\mathrm{I}}(\cdot)$ must be strictly increasing, we have

$$J(s, \theta^0) \lesseqgtr 0 \text{ if } s \lesseqgtr s_{\mathrm{I}}(\theta^0),$$

for all θ^0 in a neighborhood of $(\theta^0, s_{\mathrm{I}}(\theta^0))$. Hence,

$$J_s(s, \theta^0) > 0.$$

at $s = s_{\mathrm{I}}(\theta^0)$.

Applying the implicit function theorem indicates that there are a neighborhood U of θ^0 and a unique C^1 function φ such that $\theta^0 = \varphi(s)$ and $J(s, \varphi(s)) = 0$ for all $\theta^0 \in U$. The derivative of φ at θ^0 is

$$\varphi'(s) = -\frac{J_s(s, \theta^0)}{J_{\theta^0}(s, \theta^0)},$$

which is strictly positive, because $J_s > 0$ and $J_{\theta^0} < 0$ for all $\theta^0 \in [\underline{\theta}^0, \bar{\theta}^0]$. In addition, $\varphi(\bar{s}) = \bar{\theta}^0$. Thus, $\varphi(s)$ must be the inverse bidding strategy s^{-1} if the distribution of the observed score is rationalizable. □

5.B The Equilibrium Strategy in the Simulation Experiment

The optimal quality is given by $q(s, \theta) = (1 + \theta^1)(s/2 + 1)$. Therefore, we have

$$\frac{u(s, \boldsymbol{\theta})}{u_s(s, \boldsymbol{\theta})} = \frac{(s_{\mathrm{I}}(\theta^0))^2 + 4s_{\mathrm{I}}(\theta^0) - 4\theta^0}{2s_{\mathrm{I}}(\theta^0) + 4}.$$

Given the uniform distribution of θ^0, the first-order condition (5.5) is written as

$$s'(\theta^0) = \frac{1}{1 - \theta^0} \frac{(s_{\mathrm{I}}(\theta^0))^2 + 4s_{\mathrm{I}}(\theta^0) - 4\theta^0}{2s_{\mathrm{I}}(\theta^0) + 4}.$$

Because $s_{\mathrm{I}}(\theta^0)$ is strictly increasing in θ^0 and $u(s_{\mathrm{I}}(\theta^0), \boldsymbol{\theta}) = 0$ at $\theta^0 = 1$, we obtain $s_{\mathrm{I}}(1) = 2\sqrt{2} - 2$ as a boundary condition. Thus, the equilibrium bidding strategy is the solution of the differential equation

$$\begin{cases} s'(\theta^0) & = \frac{1}{1-\theta^0} \frac{(s_{\mathrm{I}}(\theta^0))^2 + 4s_{\mathrm{I}}(\theta^0) - 4\theta^0}{2s_{\mathrm{I}}(\theta^0)+4} \\ s_{\mathrm{I}}(1) & = 2\sqrt{2} - 2 \end{cases}$$

Solving the differential equation gives

$$(1-\theta^0)\left[(s_{\mathrm{I}}(\theta^0))^2 + 4s_{\mathrm{I}}(\theta^0) - 2(1+\theta^0)\right] = 0. \tag{5.27}$$

Applying the implicit function theorem ensures that (5.27) be the solution of the differential equation. Taking (5.27) as a quadratic equation, we obtain the equilibrium bidding function explicitly as

$$s_{\mathrm{I}}(\theta^0) = -2 + \sqrt{2\theta^0 + 6}.$$

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