

# 論文の内容の要旨

## Complex Langevin simulation of Bose-Einstein condensate under extreme conditions

(極限状態に置かれたボースアインシュタイン凝縮の  
複素ランジュバン法を用いた数値解析)

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In this thesis, we perform ab-initio lattice simulations of the nonrelativistic and relativistic Bose-Einstein condensates under strong external fields such as the electric field, the magnetic field, and the rotation. All the lattice actions discussed in this thesis are complex, so that the conventional Monte Carlo method suffers from the notorious sign problem. The path integral measure is complex due to the complex action, so that the important sampling algorithm cannot be applied. To overcome this difficulty, we adopt the complex Langevin method, which has been developed in the context of finite density quantum chromodynamics to attack the fermionic sign problem.

In particular, we perform an ab-initio simulation of the quantum vortex nucleation by the magnetic field or the rotation in nonrelativistic and relativistic systems. In dilute and low temperature systems, quantum and thermal fluctuations are negligible. The system can be remarkably well described by the mean field approximation. However, when quantum or thermal fluctuation becomes large, it is highly nontrivial how the quantized vortices behave. In particular, around the critical values of temperature, chemical potential, magnetic field, or angular velocity, the fluctuation grows and the mean-field description inevitably breaks down. To understand the quantum vortex nucleation in such situations, the ab-initio simulation is necessary as discussed in this thesis.

The direct evidence of the quantum vortex nucleation is the quantization of circulation. We show that the circulation is quantized in the superconducting (superfluid) phase far from the critical chemical potential, but it is not just above the critical chemical potential. We observe that the quantized circulation is blurred by the quantum fluctuation as the chemical potential is getting close to its critical value. In our simulation, the quantum fluctuation of vortex number is observed, while the averaged circulation is clearly quantized. At first glance, these two facts seem to be incompatible. However, we show that the fluctuation of vortex number behaves as Gaussian and, as a result of cancellation, the average circulation becomes exact integer.