論文の内容の要旨

論文題目：
Symmetries and quantum phases in one-dimensional spin systems
（１次元スピン系における対称性と量子相）

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要旨：

The Landau theory exhaustively classifies all possible symmetry-breaking phases in classical and quantum systems. In this framework, distinct phases are fully characterized by local order parameters. It also tells us that all disordered states, which do not break any symmetry of the corresponding Hamiltonian, are classified into a single phase. However, recent understanding on topological phases, which cannot be characterized by any local quantity, signals that the Landau theory fails to completely classify quantum phases. Even among disordered phases, phase transitions are known to occur in quantum systems.

A notable example of such disordered phases is the Haldane phase in the spin-1 antiferromagnetic Heisenberg chain. Its properties are qualitatively understood in terms of the Affleck-Kennedy-Lieb-Tasaki (AKLT) state. Although there is no local order parameter due to the absence of symmetry breaking, the spin-1 Haldane phase has a nonlocal (string) order parameter associated with the hidden $Z_2 \times Z_2$ symmetry breaking. Furthermore, spin-1/2 gapless excitations appear at the ends of an open chain. These features are actually traced back to an entangled nature of the state. Indeed, the spin-1 Haldane phase is composed of entangled pairs (spin singlets), which cannot be resolved as long as we keep one of three symmetries: time reversal, bond-centered inversion, and dihedral group of π rotations around two orthogonal spin axes. As a consequence, there must be a phase transition between the Haldane phase and another disordered phase which can be adiabatically connected to a direct-product state. The spin-1 Haldane phase is now recognized as one of the symmetry-protected topological (SPT) phases, which are disordered phases distinguished only under a certain set of symmetries. The symmetry-protected nature of the Haldane phase has been discussed in terms of the matrix-product state (MPS) representation of the wave function.

In this thesis, we propose an alternative approach to show the symmetry-protected nature of the Haldane phase in terms of field theory. This approach is based on the effective field-theoretical description of spin-S antiferromagnetic chains, which was originally derived by Schulz by using Abelian bosonization [Phys. Rev. B **34**, 6372 (1986)]. This approach enables us to describe various phase transitions among disordered phases, such as the valence-bond-solid (VBS) phases, realized in one-dimensional (1D) spin systems. We found that, in addition to the above three symmetries known to protect the Haldane phase, there is another symmetry protecting the phase transition between VBS phases: the site-centered inversion symmetry combined with a π

rotation around a spin axis. This is also confirmed by numerical simulations for a spin-1 model and proven by the MPS formalism. In contrast to the known three symmetries, this additional symmetry is not associated with any entangled nature of the state. This result tells us that there exist distinct SPT phases which cannot be characterized by their nontrivial entanglement structure, in the presence of point-group symmetries.

The organization of this thesis is as follows. After a brief introduction of this thesis, we review three basic facts underlying the thesis in Chapter 2. We first review the failure of the Landau symmetry-breaking theory on quantum systems and introduce a concept of the local unitary transformation for the classification of gapped quantum phases. After an intuitive argument based on many-body entanglement, we reach a definition of the SPT phase. Second, we give a short historical review on 1D quantum spin systems. In particular, we explain their field-theoretical descriptions using the non-linear sigma model and the physical properties of the Haldane phase from the VBS picture. Third, we give a short course on the MPS formalism. Starting from the definition of the MPS, we explain its pureness as a physical requirement and how to represent the symmetries on translation-invariant MPSs. Then we review the classification of one-dimensional SPT phases and its application to the AKLT state.

In Chapter 3, we revisit the field-theoretical description of spin ladder systems by Abelian bosonization, following Schulz. We derive an effective Hamiltonian only with a single bosonic field, which captures essential low-energy properties of 1D spin systems. However, we not only follow his derivation based on perturbation theory but also provide a non-perturbative argument focusing on the compactification of the bosonic field and symmetries. We also give an extension of the Lieb-Schultz-Mattis theorem for 1D spin systems, which states that half-odd-integer spin systems with site-centered inversion symmetry and either dihedral group or time reversal symmetries cannot have a unique gapped ground state.

In Chapter 4, we clarify a physical meaning of the effective field theory on various 1D spin systems. Starting from a review on the two-leg ladder which has been extensively studied, we further consider spin-S XXZ chains, N-leg spin-1/2 ladders, and dimerized spin ladders. All these examples point out that the effective field theory faithfully describes phase transitions between VBS phases, which have already been known by several numerical studies.

Chapter 5 is the main part of this thesis. Based on the effective field theory, we show that the phase transition between different VBS phases is protected by one of four symmetries: time reversal, bond-centered inversion, dihedral group of spin rotations, and site-centered inversion combined with a spin rotation. As mentioned above, while the first three are already known, we do not know what the last one indicates. To investigate its consequence, we propose a microscopic model which breaks the first three symmetries but preserves the last one, that is, a spin-1 chain with a staggered magnetic field. By a simple perturbative argument and numerical simulations, we find that the site-centered inversion symmetry combined with a spin rotation preserves the distinction between two trivial phases, both of which can be connected to direct-product states. We further confirm this fact by the MPS formalism without assuming translational invariance. We also give a proof of the Lieb-Schultz-Mattis theorem for 1D spin systems with the site-centered inversion symmetry in terms of the MPS.

We finish this thesis by summarizing our results in Chapter 6. Some future prospects on point-group symmetries in many-body quantum systems are also mentioned.