博士論文（要約）

Theoretical Study on Topological Aspects in Superconducting Systems

(超伝導におけるトポロジーの理論的研究)

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ABSTRACT

THEORETICAL STUDY ON TOPOLOGICAL ASPECTS
IN SUPERCONDUCTING SYSTEMS

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This dissertation is mainly based on the following three projects which I worked on during my Ph. D program.

1. Sho Nakosai, Yukio Tanaka, and Naoto Nagaosa
   "Topological Superconductivity in Bilayer Rashba System"
   Physical Review Letters **108,** 147003 (2012)

2. Sho Nakosai, Jan Carl Budich, Yukio Tanaka, Björn Trauzettel, and Naoto Nagaosa
   "Majorana Bound States and Nonlocal Spin Correlations in a Quantum Wire on an Unconventional Superconductor"
   Physical Review Letters **110,** 117002 (2013)

3. Sho Nakosai, Yukio Tanaka, and Naoto Nagaosa
   "Tow-dimensional *p*-wave Superconducting states with Magnetic Moments on a conventional *s*-wave Superconductor"
   Physical Review B **88,** 180503 (2013)

4. Sho Nakosai, Yukio Tanaka, and Naoto Nagaosa
   "Theoretical Modeling and Properties of Class DIII Topological Superconductors"
   Physica E: Low-dimensional Systems and Nanostructures **55,** 37 (2014)

Topological aspects in condensed matter physics have been highlighted in these decades since the hallmark discovery of the "topological description" of quantized Hall conductivity in two-dimensional systems under magnetic field. They provide us a new scheme to classify systems into different phases. For a long time the phases of matters were always understood in the context of symmetries. However, one cannot apply this point of view to quantum Hall states at different filling factors with different conductivities even though it is apparent that phase transitions occur among them, because they are equivalent with respect to the symmetries. Based on the concept of topological nature, on the other hand, one can conclude that two states belong to distinct phases if they cannot be connected adiabatically. Following this new general scheme, new phases of matters including topological insulators and superconductors have been theoretically predicted and also experimentally observed. Topological nature especially manifests at the boundaries of the systems as gapless zero energy states. Topologically nontrivial superconducting states possess so-called Majorana bound states at the boundaries, which are robust against local perturbations due to topological protection. Majorana fermions are originally investigated in high-energy physics for the description of neutrinos, but interestingly they also emerge in condensed matter physics. Their singular feature — they are their own antiparticles — leads to the noncommutative statistics. When one interchanges some Majorana modes, the state of the system changes depending on the order of the interchange. Two Majorana modes build a single ordinary fermion, which forms a qubit. Thus they provide resources for storing the information, and operations can be done by interchanging them. They are considered to be a potential candidate for a platform of quantum computation. Motivated by these fascinating properties of Majorana modes, topological superconductors have been intensively studied, but many issues still remain to be solved. One of the obstacles is the fact that no materials, with very few exceptions, exhibit topologically nontrivial superconducting states.

This leads us to consider how can we design nontrivial states by combining ordinary materials? The first part of this Dissertation is the new proposals to realize topological superconducting states in (1) an interface system of transition metal oxides, and (2) a system where superconductivity and magnetic order coexist. We theoretically construct models based on experimental results, and analyze their topological properties. Another issue is how to detect Majorana bound states when they are successfully created. The most important feature of Majorana modes, noncommutative statistics, has not been demonstrated yet. In the second part, we show a novel property of Majorana bound states in systems with the time reversal symmetry, which will offer a good clue for the observation of them.

The first model we show is a bilayer Rashba system. It consists of two contiguous interfaces of transition metal oxides. This model is constructed based on the following experimental observations; electrons in bulk insulating materials, $SrTiO_3$ and $LaAlO_3$, form two dimensional electron gas system at the interface of them; Rashba spin-orbit coupling is active at the interface since the inversion symmetry is naturally broken there, and also controllable by gating; superconductivity occurs at low temperature $T < 200$mK, and its nature is suggested to be two-dimensional and s-wave. The important point is that the time reversal symmetry is preserved in this case. From a topological viewpoint, our model is characterized by $\mathbb{Z}_2$ index according to the topological classification table. The index can be easily calculated by virtue of the crystal symmetry of the system, and it has a nontrivial value when (1) the system exhibits odd-parity superconductivity with full gap structure, and (2) Fermi energy is tuned to an appropriate range. We obtain rich phase diagram of superconductivity coming from spin-orbit interaction and layer degrees of freedom when two interfaces are close enough to hybridize and interact with each other. It is notable that Cooper pair with odd parity appear as the leading instability in some regions in the phase diagram. The sign of parity is determined by the strength of the interaction, which can be modified, for example, by changing the thickness of the intermediate oxide. This proposal points out that we can manipulate the topological nature of the system by artificially designing structures of materials.

The second model we treat is coupled system of magnetic materials and conventional superconductors. This study gives a promising proposal for the realization of topological superconductivity with broken time reversal symmetry. In our model, such a state is effectively generated in the gap of conventional superconductors. We provide an intuitive understanding, which enable us to directly connect the magnetic order and the topological nature of the generated superconductivity. It is known that a classical magnetic moment in a conventional superconductor produces an in-gap state localized around the moment, which is called a Shiba state. We use this Shiba state as a building block, and construct an effective lattice model made from aligned Shiba states. The finite overlap of the wave functions causes transfer integrals and pair hopping among neighboring Shiba states, resulting in a band structure with superconducting gap. Since the magnitude of the overlap, and hence the parameters in the effective lattice model, depend on relative direction of moments, the properties of the system are determined by the magnetic structure. It is found that non-coplanar magnetic ordering is needed for two-dimensional systems with full gap superconductivity. As a prominent example, skyrmion structures produce chiral p-wave superconductivity.

When a system preserves the time reversal symmetry, each state involves its time reversal partner. This is also correct for Majorana bound states. Therefore topological superconductors with the time reversal symmetry possess paired Majorana states at the boundaries. We focus on the one-dimensional case, and show an unusual correlation between two spins defined from Majorana states at the two ends. A quantum nanowire with one single channel deposited on a substrate with helical p-wave superconductivity extracts the topological nature in the substrate. The effective theory obtained by integrating out the degrees of freedom in the substrate becomes equal to two parallel copies of Kitaev chain model. We have two Majorana bound states at each end, and can define spin operator using the Pauli matrices. When the capacitance of an island of a superconducting substrate with a wire on it is finite and the system is gated, the ground state degeneracy is lifted into two twofold degenerate states from fourfold degenerate states depending on the Fermion parity of the system. We can choose the Fermion parity of the newly generated ground state by manipulating the gate voltage. The calculation of the spin-spin correlations in equilibrium of two ground states with different parities shows that they have opposite signs. This means that one can control the magnetic correlation using purely electric means. The result is a manifestation of the ground state degeneracy and nonlocality due to paired Majorana bound states.