

## 論文の内容の要旨

論文題目                      Extended Multiroute Flow for Multilink Attack Networks

(拡張マルチルートフローとマルチリンク攻撃問題への応用)

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There are many polynomial-time algorithms that solve the maximum network flow problem (max-flow), including the recent results of Orlin (2013) where the max-flow of a network with  $n$  nodes and  $m$  links is solved in  $O(mn)$ . However, the amount of flow we get from a solution to those algorithms can drop significantly if some of the links in the network are attacked (or fail). Such a network flow that is robust against attacks is actively studied in various contexts.

This thesis covers a family of natural network flow problems against  $k$  attacks qualified as Multilink-Attack (MLA) Flow: MLA-Robust flow, MLA-Reliable flow, and MLA-Decomposition. Although the MLA-robust flow is proved to be intractable, a maximum multiroute flow of  $k+1$  routes is a  $(k+1)$ -approximation to the three MLA-network flow variants. An extended version of the multiroute flow is used to solve exactly in polynomial time MLA-robust flow and MLA-reliable flow. Furthermore, it also leads to tighter bounds for the approximation algorithm.

A  $h$ -route flow, a multiroute flow (MRF) of  $h$  routes, is a non-negative linear combination of  $h$  link-disjoint paths. It was introduced by Kishimoto and Takeuchi (1993), they extended the max-flow/min-cut duality property to the multiroute flow context and provided an algorithm to compute a max- $h$ -route flow in  $h$  iterations of a classical max-flow. Aggarwal and Orlin (2002) showed that a max- $h$ -route flow can be computed in less than  $h$  steps for some graphs by using a parametric max-flow where the parameter is a restriction over the links capacities. Aneja, Chandrasekaran, and Nair (2003) considers a first variant of MRF where the number of route is a parameter. The MRF is extended in another direction by consider the number of route as any non-negative real number by Aneja, Chandrasekaran, Kabadi, and Nair (2007).

The extended multiroute flow (EMRF) in this work combines into a modified Eisner-Severance (ES) algorithm the parametric scheme of Aggarwal and Orlin for restricted max-flow

and the two extensions concerning the route number by Aneja, Chandrasekaran, and Nair (2003) and Aneja, Chandrasekaran, Kabadai and Nair (2007). In  $O(\lambda \cdot mn)$  –where  $\lambda$  is the connectivity of the network–, the EMRF compute the whole max-h-route flow function for any non-negative real number of route  $h$ . This method outputs more general results than the MRF with a similar complexity when the  $h = \Omega(\lambda)$  and solve faster the problem of maximization of  $h$  for a given max-h-route flow value.

Robust network flows can be defined in many ways. MLA network flows were designed by considering the simplest theoretical approach. The general problem was divide into three necessary variants, depending on the information of the attacker. The MLA-robust flow problem is to find the minimum max-flow value among  $m$  choose  $k$  networks obtained by deleting each set of  $k$  links. In this setup, we suppose that the attacker first destroy a set of  $k$  links, and then we can reroute the flow through the network. The MLA-reliable flow problem is to find a max-flow of the network such that the flow value is maximum against any set of  $k$  link failures, when deleting the corresponding flow to those  $k$  links in the original flow. In this case, the attacker knows how much is routed on each link, and we cannot reroute the flow after the attack. Finally, the MLA-decomposition problem is quite similar to the reliable one, except that the attacker even knows the routing information at each node.

The multiroute flow is shown to be a  $(k+1)$ -approximation algorithm to MLA-robust flow, MLA-reliable flow, and MLA-decomposition. The relation between EMRF and MLA-robust flow can be pushed further. The restricted max-flow described before is a piecewise linear function with at most  $\lambda+1$  parts. If a linear part with slope  $k$  exists, then there exists a well chosen  $h$  such that the max-h-route flow value is equal to the MLA-robust flow and the MLA-reliable flow.

The networks can be classified into several categories if they are, for instance, EMRF-solvable or NP-hard. The NP-hard instances are inside a class of networks that possess a flying cut, that is a MLA-robust cut that is not catch by the parametric scheme of Aggarwal and Orlin. The approximation ratio of MRF algorithm can be improved heuristically by the iterative multiroute flow method, but also directly by a the pseudo-tangent method or by application of MLA-robust flow and MLA-robust cut duality.

The success rate of EMRF, and the gap between the upper and lower bounds are evaluated on a set of instances: (almost-)random shape networks, 2D and 3D grids, complete graphs, and complex networks generated by the R-MAT method. Although the distribution of the capacities of the links alone seems to have almost no impact on the success rate, the structure of the network has a huge impact, as on R-MAT generated instances. Finally, real numbers capacities mapped into integral ones can be as efficient as the originals when the interval sized is big enough – $m$  or  $m_2$  in the experiments.