論 文 の 内 容 の 要 旨

論文題目　The Linear Complementarity Problem: Complexity and Integrality
（線形相補性問題： 計算複雑度と整数性）

氏　名　澄田　範奈

Since the 1950’s, the theory of mathematical programming and game theory have been developed rapidly and extensively. The literature shows the rich theory of linear programming, convex quadratic programming, and bimatrix game, which are fundamental subjects in these areas. As a unifying framework of such problems, the linear complementarity problem was introduced in mathematical programming in the mid 1960’s.

The linear complementarity problem is to find, for a square matrix and a vector, a vector satisfying linear constraints and complementarity conditions. The linear complementarity problem has been studied from both theoretical and practical points of view, as it has applications in many areas such as computer science, economics and physics. While the linear complementarity problem is hard to solve in general, each of the above applications reduces to the linear complementarity problem in which input matrices have a certain property. There- fore, there exist many results on the computational complexity of the linear complementarity problem in terms of matrix classes. Furthermore, motivated by applications, finding a solution with a certain property such as an integral solution or a minimum norm solution has been attracting attention.

This thesis aims to study the theory of linear complementarity problems. In particular, we focus on two points: the computational complexity and the integrality of the problem.

To investigate the computational complexity, we focus on sparsity of a given matrix. It is known that we can efficiently find a vector satisfying a system of linear inequalities. Furthermore, if the inequalities have the highest sparsity, that is,

if each inequality involves at most two variables, then the problem can be solved in a combinatorial way. In this thesis, we classify the computational complexity of the linear complementarity problem in terms of sparsity of a given matrix. We also give an efficient algorithm based on a procedure for sparse linear inequalities, to solve the sparse linear complementarity problem.

As a further study, we investigate the parameterized complexity of the linear complementarity problem. While the classical complexity theory analyzes the time necessary to solve a problem exclusively in terms of the input data size, the parameterized complexity theory takes a specified parameter into account in addition to the input data. Intuitively, the parameterized complexity theory aims to find out an efficient algorithm when a parameter takes a small value. There exist results on parameterized complexity of the bimatrix game with some parameters. We analyze the parameterized complexity of the linear complementarity problem by using the idea for the bimatrix game. We note that some existing results cannot be extended to the linear complementarity problem.

We also study the complexity of finding a solution with a certain property to the linear complementarity problem. We introduce the problem of finding a solution to an instance of the linear complementarity problem whose basis is identical to the paired instance. This problem is called the linear complementarity problem with orientation. We present two applications of the problem, and show the computational complexity in terms of matrix classes.

The last part of this thesis is concerned with the existence of integral solutions to the linear complementarity problem. There exist two sufficient conditions for the existence of an integral optimal solution to the linear programming problem. One is total unimodularity of a matrix, and the other is total dual integrality of linear constraints, which means that existence of an integral solution to a dual problem implies that of the primal problem. In the study of the linear complementarity problem, it was shown that a basic solution (a solution of a special form) to the problem with a generalization of totally unimodular matrices is integral. In this thesis, we introduce the notion of total dual integrality to the linear complementarity problem. Then we show that the total dual integrality gives a sufficient condition for the existence of an integral solution. To define total dual integrality, we use the linear complementarity problem with orientation.